Achieve ADP Algebra I End-of-Course Exam Content Standards with Comments & Examples

October 2008

O: Operations on Numbers and Expressions

Priority: 25%

Successful students will be able to perform operations with real numbers, including numerical expressions involving exponents, scientific notation and square roots, using estimation and an appropriate level of precision. Reasoning skills will be emphasized, including justification of results. There is a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems.

Content Benchmarks	Explanatory Comments and Examples
O1. Number Sense and Operations	
a. Use properties of number systems within the set of real numbers to verify or refute conjectures or justify reasoning and to classify, order, and compare real numbers.	• Define, give examples of, distinguish between and use numbers and their properties, from each of the following number sets: whole numbers, integers, rationals, irrationals, and reals. • Determine whether the square roots of whole numbers are rational or irrational. *Example*: Which of the following numbers are rational and which are irrational? Explain. $\sqrt{(10)(40)}, \quad \sqrt{4^3}, \quad 2\sqrt{2}, \quad \sqrt{49}$ • Compare and order real numbers, including determining between which two consecutive whole numbers the value of a square root lies. *Example*: Which of the following numbers comes closest to the value of π without exceeding it: $\sqrt{10}, \quad 3.14, \quad \text{and} \quad \frac{22}{7}$ • Provide counterexamples to refute a false conjecture. • Establish simple facts about rational and irrational numbers using logical arguments and examples. *Example*: Give an example to illustrate that if *r* and *s* are rational, then both *r* + *s* and *(r)(s)* are rational. *Sample Solution:* Both $\frac{3}{4}$ and 2.3 are rational; $\frac{3}{4}+2.3=\frac{3}{4}+\frac{23}{10}=\frac{15}{20}+\frac{46}{20}=\frac{61}{20}$ which

	is the ratio of two integers, hence rational. *Assessment Limitation*: Items involving radicals will be limited to square roots. Students will not be expected to produce formal proofs.
b. Use rates, ratios and proportions to solve problems, including measurement problems.	• Use dimensional analysis for unit conversion. • Solve problems using derived measures (Derived measures are those achieved through calculations with measurement that can be taken directly, e.g. percent change and density). • Solve problems involving scale factor (e.g., similar figures, scale drawings, map scales, dilations). • Solve applications related to proportional representation. *Example*: There are 223 students in the freshman class, 168 in the sophomore class, 173 in the junior class and 138 in the senior class. The student council has 30 members, with these seats allocated based on the number of students in each class. How many student council members should each class have? Explain your answer. • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision.
c. Apply the laws of exponents to numerical expressions with integral exponents to rewrite them in different but equivalent forms or to solve problems.	*Example:* Rewrite $\frac{7\left(3^{-3}\right)}{\left(2^{-4}\right)\left(3^{5}\right)}$ as a fraction having only positive exponents. *Sample Solution:* $\frac{7\cdot 3^{-3}}{2^{-4}\cdot 3^{5}}=\frac{7\cdot 2^{4}}{3^{3}\cdot 3^{5}}=\frac{7\cdot 2^{4}}{3^{8}}$ *Example:* Multiply, giving the answer without exponents: $\frac{2}{5^{-4}}\cdot\frac{3^{-3}}{5^{2}}\cdot\frac{3^{5}}{2^{5}}$ *Sample Solution:* $\frac{2}{5^{-4}}\cdot\frac{3^{-3}}{5^{2}}\cdot\frac{3^{5}}{2^{5}}=\frac{2\cdot 3^{5}\cdot 5^{4}}{3^{3}\cdot 5^{2}\cdot 2^{5}}=\frac{3^{2}\cdot 5^{2}}{2^{4}}=\frac{225}{16}$

	• Represent, compute and solve problems using numbers in scientific notation. Examples of applications may include determining national debt, astronomical distances or the distance between electrons and protons in an atom. • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision.
d. Use the properties of radicals to rewrite numerical expressions containing square roots in different but equivalent forms or to solve problems.	• Add, subtract, multiply, divide and manipulate numerical expressions with square roots. Results may be required to be given in exact form. *Example*: Show or explain how $\left(2\sqrt{2}\right)^2$ is equal to 8. *Sample Solution*: $\left(2\sqrt{2}\right)^2 = 4(2) = 8$ *Example:* Show or explain how $5\sqrt{2}$ is equal to $\sqrt{50}$. *Sample Solution:* Showing that the number on the left equals that on the right: $5\sqrt{2} = \sqrt{25}\cdot\sqrt{2} = \sqrt{50}$ *Example*: Rewrite the radicals to determine the sum of $\sqrt{8}+\sqrt{18}.$ *Sample solution*: $\sqrt{8}+\sqrt{18} = \sqrt{4\cdot 2}+\sqrt{9\cdot 2} = 2\sqrt{2}+3\sqrt{2} = 5\sqrt{2}$ *Example*: $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ *Example*: $\frac{\sqrt{6}+\sqrt{9}}{\sqrt{3}} = \sqrt{2}+\sqrt{3}$ • Use the distance formula, based on the Pythagorean Theorem, to solve problems. *Example*: Determine the perimeter of a quadrilateral with vertices (1, 1), (-1, 2), (2, 4) and (4, 3). *Example*: If the legs of a right triangle measure $\sqrt{5}$ units and $\sqrt{7}$ units, determine the exact measure of the hypotenuse in simplest form.

	Sample Solution: $\left(\sqrt{5}\right)^2+\left(\sqrt{7}\right)^2=5+7=12$ The length of the hypotenuse is $\sqrt{12}=2\sqrt{3}$ units. • For applications, this includes using and interpreting appropriate units of measurement for solutions. *Assessment Limitation*: When division by a radical or rationalization of a denominator is required, the denominator will be a monomial.
O2. Algebraic Expressions	
a. Apply the laws of exponents to algebraic expressions with integral exponents to rewrite them in different but equivalent forms or to solve problems.	*Example*: Write the expression in simplest form: $\left(2a^2b^3\right)^5=32a^{10}b^{15}$ *Example:* Write the expression in simplest form: $\frac{3a^2+6ab}{3a}=a+2b$ • Translate to expressions with only positive exponents. *Example*: Rewrite the expression with each variable appearing only once and with only positive exponents: $\frac{3x^{-2}y^3}{2x^{-5}y^{-3}}=\frac{3}{2}x^3y^6$ • Translate to expressions with variables appearing only in the numerator. *Example*: Rewrite the expression with variables only in the numerator: $\frac{3s^3}{2r^5}=\frac{3}{2}s^3r^{-5}$ • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision. *Assumption*: All algebraic expressions are defined.
b. Add, subtract and multiply polynomial expressions with or without a context.	*Example*: $3x^5(x-2)-2x^4(x^2+2)$ *Sample Solution*: $3x^5(x-2)-2x^4(x^2+2)=3x^6-6x^5-2x^6-4x^4=x^6-6x^5-4x^4$

	Example: Multiply: $(x+a)(x+b)$ *Sample Solution:* $(x+a)(x+b) = x^2 + ax + bx + ab = x^2 + (a+b)x + ab$ *Assessment Limitation:* Multiplication is limited to a monomial multiplied by a polynomial or a binomial multiplied by a binomial.
c. Factor simple polynomial expressions with or without a context.	*Example:* Factor completely $6u^5 - 15u^3$ *Sample Solution:* $6u^5 - 15u^3 = 3u^3(2u^2 - 5)$ *Example*: Factor completely $3x^3y + 21x^2y + 30xy$ *Sample Solution:* $3xy(x^2 + 7x + 10) = 3xy(x+2)(x+5)$ *Example*: Factor completely $6x^2 - x - 12$ *Sample Solution:* $(3x+4)(2x-3)$ *Assessment Limitation:* Factoring will be limited to factoring out common monomial factors, perfect-square trinomials, differences of squares and quadratics of the form $ax^2 + bx + c$ that factor over the set of integers. The factoring process may require more than one step.
d. Use the properties of radicals to convert algebraic expressions containing square roots into different but equivalent forms or to solve problems.	• Add, subtract, multiply, divide and manipulate algebraic expressions with square roots. Results may be required to be given in exact form. • When taking square roots of variable expressions, absolute values must be included when appropriate. *Example:* $\sqrt{x^3} = x\sqrt{x}$ because $\sqrt{x^3}$ was assumed real $\sqrt{x^2} = \lvert x \rvert$, $\sqrt{x^4} = x^2$, $\sqrt{x^6} = \lvert x^3 \rvert$ or $\lvert x \rvert^3$, $\sqrt{x^8} = x^4$, $\sqrt{x^{10}} = \lvert x^5 \rvert$ or $\lvert x \rvert^5$ *Example*: Explain how $\sqrt{25x^6}$ is equal to $5\lvert x^3 \rvert$ and to $5\lvert x \rvert^3$

	Sample Solution: $\sqrt{25x^6} = \sqrt{5^2 \cdot x^2 \cdot x^2 \cdot x^2}$ $= 5\lvert x \cdot x \cdot x \rvert = 5\lvert x^3 \rvert = 5\lvert x \rvert \lvert x \rvert \lvert x \rvert = 5\lvert x \rvert^3$ • For applications, this includes using and interpreting appropriate units of measurement and the appropriate level of precision. *Assumption*: All radical expressions represent real numbers. *Assessment Limitation:* Expressions under radicals will be limited to monomials. When rationalization of a denominator is required, the radical in the denominator will contain no variables.

Achieve ADP Algebra I End-of-Course Exam Content Standards with Comments & Examples

October 2008

L: Linear Relationships

Priority: 35%

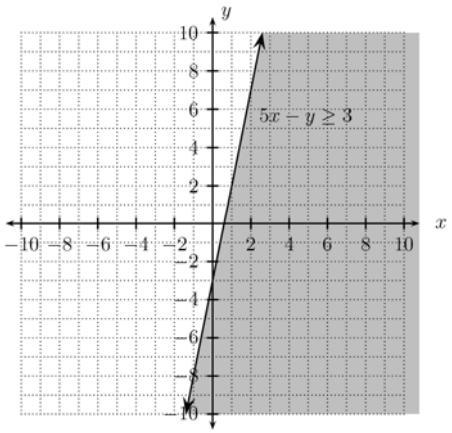
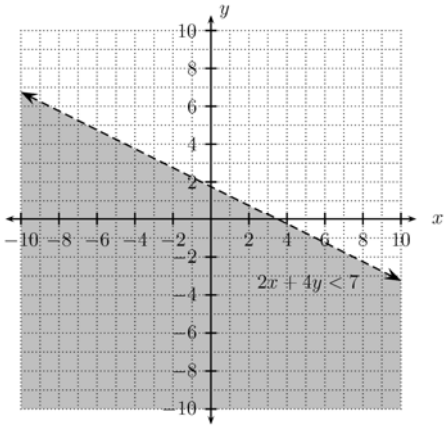
Successful students will be able to solve and graph the solution sets of linear equations, inequalities and systems of linear equations and to use words, tables, graphs and symbols to represent, analyze and model with linear functions. There is a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems, students may be required to graph and interpret their solutions in terms of the context. They should be able to apply such problem solving heuristics as: identifying missing or irrelevant information; testing ideas; considering analogous or special cases; making appropriate estimates; using inductive or deductive reasoning; analyzing situations using symbols, tables, graphs or diagrams; evaluating progress regularly; checking for reasonableness of results; using technology appropriately; deriving independent methods to verify results; and using the symbols and terms of mathematics correctly and precisely. On the Algebra I End-of-Course test, function notation may be used.

Content Benchmarks	Explanatory Comments and Examples
L1. Linear Functions	
a. Recognize, describe and represent linear relationships using words, tables, numerical patterns, graphs and equations. Translate among these representations.	• Use correct terminology and notation for functions (e.g., $f(x)$, independent and dependent variables, etc.). When equations are presented, any form of a linear equation can be used. *Example:* Explain how the relationship between length of the side of a square and its perimeter can be represented by a direct proportion. • Use models and algebraic formulas to represent and analyze linear patterns, including determining a formula for the general term of an arithmetic sequence and interpreting the constant difference as the slope of the line that represents the pattern. *Example:* Given the sequence: 5, 7, 9, 11, … If 5 is considered the first term when x=1, what linear equation could generate this pattern? *Example:* Express the following sentence in equation form: two times the quantity of a number increased by eight is equivalent to five less than the same number. *Sample Solution:* $2(x+8)=x-5$ • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). *Assessment Limitation*: Subscript notation will not be used or required for items involving sequences.

b. Describe, analyze and use key characteristics of linear functions and their graphs.	• Key characteristics include constant slope and x- and y-intercepts. • When equations are presented, any form of a linear equation can be used. • Interpret slopes of given lines to determine whether lines are parallel, perpendicular, intersecting or coincident. *Example*: Write an equation for a line parallel to the line through (1, -2) and (-3, 5). • Identify and distinguish among parameters and the independent and dependent variables in a linear relationship. • Describe the effects of varying the parameters m and b in linear functions of the form $f(x) = mx + b$ or $y = mx + b$. *Example*: Compare and contrast the positions of the graphs for the following three functions and explain how the positions are related to the equations: $f(x) = 5x$, $g(x) = 5x + 2$, and $h(x) = 5x - 2$. • Apply direct proportions, as a special linear relationship, and analyze their graphs in a context.
c. Graph the absolute value of a linear function and determine and analyze its key characteristics.	• Key characteristics include vertex, slope of each branch, intercepts, domain and range, maximum, minimum, transformations, and opening direction. *Example*: Graph each of the following absolute value equations and compare and contrast the graphs with the graph of $p(x) = \lvert x \rvert$: $q(x) = -\lvert x \rvert$, $r(x) = \lvert 2x \rvert$, $s(x) = \lvert x + 2 \rvert$, and $t(x) = \lvert x \rvert + 2$ • For items where a student is required to graph the equation or function, axes and scales should be labeled.

d. Recognize, express and solve problems that can be modeled using linear functions. Interpret their solutions in terms of the context of the problem.	• Interpret slope and y-intercept in the context of a problem. When equations are presented, any form of a linear equation can be used. *Example*: The linear function $40t = d$ can be used to describe the motion of a certain car, where t represents the time in hours and d represents distance traveled, in miles. What does the coefficient, 40, represent in the equation? Include units with the answer. • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision. *Assessment Limitation*: Absolute value items will not be used in context.
L2. Linear Equations and Inequalities	
a. Solve single-variable linear equations and inequalities with rational coefficients.	• Solve multi-step equations and inequalities. *Example*: Solve the equation $\frac{x}{2} - \frac{x+1}{3} = 2.$ *Sample Solution*: $\frac{x}{2} - \frac{x+1}{3} = 2$ $6\left[\frac{x}{2} - \frac{x+1}{3}\right] = 6[2]$ $3x - 2(x+1) = 12$ $3x - 2x - 2 = 12$ $x = 14$ • Represent solution sets for inequalities symbolically as intervals or graphically on a number line.

	Example: Solve $3 - x < 5$ $3 - x < 5$ *Sample Solution:* $-x < 2$ $x > -2$ *Example*: Solve $-3 \le 5x + 4 \le 24$ • Linear equations may have no solution (empty set), an infinite number of solutions (identity) or a unique solution. *Example*: Determine and explain the solutions for each of the following three equations: $A)\ x + 0 = x + 2$ $B)\ x + 0 = x$ $C)\ x + 0 = 2x$ *Sample Solution*: $A)\ x + 0 = x + 2$ $0 = 2$ $\therefore$ no solution $B)\ x + 0 = x$ $0 = 0$ $\therefore$ all real numbers $C)\ x + 0 = 2x$ $x = 0$ *Assessment Limitation*: Limited to single variable, first degree for both equations and inequalities.
b. Solve equations involving the absolute value of a linear expression.	• Determine all possible values in the solution. *Example: Solve:* $\lvert x + 3 \rvert = 7.$ *Sample Solutions*: $\lvert x + 3 \rvert = 7$ $x + 3 = 7$ or $x + 3 = -7$ $x = 4$ or -10 OR

	Since $\lvert x-b\rvert$ can be interpreted as the distance from x to b, the solutions of the above absolute value equation may be interpreted as the numbers, x, that are 7 units from –3. (i.e. $x=-3+7=4$ or $x=-3-7=-10$). *Assessment Limitation:* Equations will include only one absolute value expression and will be one of the following forms: $\lvert ax+b\rvert=c,\quad a\lvert x+b\rvert=c,\quad \lvert ax\rvert+b=c,\quad \lvert ax+b_1\rvert+b_2=c.$
c. Graph and analyze the graph of the solution set of a two-variable linear inequality.	• Represent algebraic solutions graphically on a coordinate plane. • For graphs of two-variable inequalities use shaded half-plane with solid or open boundary. *Example*: Graph $5x-y\geq 3$ *Solution*:  *Example*: Graph $2x+4y<7$ *Solution*:  • Provide examples of ordered pairs that are included in the solution set of a two-variable linear inequality. *Example*: Determine a point in the solution set for $3x+2y<6$.

	• For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).
d. Solve systems of linear equations in two variables using algebraic and graphic procedures.	• Systems of equations may include intersecting, parallel or coincident lines, some of which may be equations of horizontal or vertical lines. • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).
e. Recognize, express and solve problems that can be modeled using single-variable linear equations; one- or two-variable inequalities; or two-variable systems of linear equations. Interpret their solutions in terms of the context of the problem.	*Example*: Jim spent $200 on gifts for his family. He spent the money on toys, clothes and a $15 DVD. He spent 4 times as much on clothes as he did on toys. Write an equation in one variable that can be used to determine how much money Jim spent on toys. Solve the equation to determine how much Jim spent on toys. *Example*: A triangle is formed by the intersections of the x-axis, the y-axis and the line $2x + 3y = 6$. What is the area of the triangle? *Example*: One angle of an acute triangle is twice the first angle while the third angle is 40º more than the first angle. Determine the degree measure of each of the three angles. *Example:* Quick Trip rental car agency charges a flat weekly rate of $193.00 and $0.19 per mile. Drive Easy rental car agency charges a flat weekly rate of $219.00 and $0.15 per mile for an identical car. For a one-week rental, how many miles does the car need to be driven so that the charges for a rental at Quick Trip are the same as a rental at Drive Easy? • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision.

Achieve ADP Algebra I End-of-Course Exam Content Standards with Comments & Examples

October 2008

N: Non-linear Relationships

Priority: 20%

Successful students will be able to recognize, represent, analyze, graph, solve equations and apply non-linear functions, including quadratic and exponential. There is a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems students will be required to graph and interpret their solutions in terms of the context. They should be able to apply such problem solving heuristics as: identifying missing or irrelevant information; testing ideas; considering analogous or special cases; making appropriate estimates; using inductive or deductive reasoning; analyzing situations using symbols, tables, graphs or diagrams; evaluating progress regularly; checking for reasonableness of results; using technology appropriately; deriving independent methods to verify results; and using the symbols and terms of mathematics correctly and precisely. On the Algebra I End-of-Course test function notation may be used.

Content Benchmarks	Explanatory Comments and Examples
N1. Non-linear Functions (In this section, all coefficients will be integers.)	
a. Recognize, describe, represent and analyze a quadratic function using words, tables, graphs or equations.	• Use correct terminology and notation for functions (e.g. *f(x)*, independent and dependent variables, etc). • Determine and analyze key characteristics of quadratic functions and their graphs (e.g. axis of symmetry, vertex, zeros, *y*-intercept, domain, range, maximum, minimum, opening direction, etc.). • Sketch a graph of a quadratic equation using the zeros and vertex when given the equation. *Example*: Determine the vertex of the function $f(x) = 4x^2 - 8x - 5$ *Sample Solutions*: $f(x) = 4x^2 - 8x - 5$ $f(x) = (2x - 5)(2x + 1)$ $0 = (2x - 5)(2x + 1)$ $x = \left\{-\frac{1}{2}, \frac{5}{2}\right\}$

	To find the x-value of the vertex, average the zeros: $x = \frac{\left(-\frac{1}{2} + \frac{5}{2}\right)}{2} = 1$ $f(1) = 4(1)^2 - 8(1) - 5 = -9$ $(1,\ -9)$ OR Substitute 4 and -8 into $x = \frac{-b}{2a}$, and then solve for $f(x)$. $x = \frac{-(-8)}{2(4)} = 1$ $f(1) = 4(1)^2 - 8(1) - 5 = -9$ $(1,\ -9)$ • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). *Assessment Limitations*: In constructed response items, students will not be required to derive quadratic equations from tables, graphs or words. Completing the square will not be required. Quadratic functions will have integral coefficients. When the vertex must be determined, the vertex of a quadratic function must have integral values. When zeros are to be determined or used, the zeros of the quadratic function must be rational. Quadratic functions may be represented in the following forms polynomial $\left(f(x) = ax^2 + bx + c\right)$ or factored $\left(f(x) = a(x - r)(x - s)\right)$.
b. Analyze a table, numerical pattern, graph, equation or context to determine whether a linear, quadratic or exponential relationship could be represented. Or, given the type of relationship, determine elements of the table, numerical pattern or graph.	• Distinguish between types of functions, including linear, quadratic, and exponential. • Recognize when an exponential model is appropriate (growth or decay). • Determine if an exponential function is increasing or decreasing.

	• Students may be required to explain their reasoning. *Example*: Given the following increasing numerical pattern, determine the type of relationship that exists (linear, quadratic or exponential) and justify your conclusion.. 3, 6, 12, 24, 48, … • Extend a table, numerical pattern or graph given the type of relationship (quadratic or exponential). • Use first and second differences to determine the type of function represented. *Assessment Limitation*: Exponential functions in the form $y = ab^x$ will include rational non-zero values for both a and b, where $b > 0$. When exponents are specifically named for exponential functions, the exponents will be integers.
c. Recognize and solve problems that can be modeled using a quadratic function. Interpret the solution in terms of the context of the original problem.	• For physics applications, formulas will be provided (e.g. $s = -16t^2 + 48t + 64$). • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision. • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). *Assessment Limitation*: Contexts will be accessible for students working at this level (e.g. area, Pythagorean relationships or motion). No formal physics notation will be used (e.g. v_0, s_0, etc). Quadratic functions will have integral coefficients. When the vertex must be determined, the vertex of the quadratic function must have integral values. When zeros are to be determined, or used, the zeros of the quadratic function must be rational.

N2. Non-linear Equations (In this section, all coefficients will be integers.)	
a. Solve equations involving several variables for one variable in terms of the others.	*Example*: Solve for *r:* $V = \pi r^2 h$ *Example*: Solve for *y:* $z = 3x^2 y + 4y$ *Assumption*: All algebraic functions are defined. All radical expressions represent real numbers. *Assessment Limitation*: Equations may contain variables to a power higher than the second degree, but students will not be asked to solve for any variable that is higher than the second degree.
b. Solve single-variable quadratic equations.	*Example*: Solve the following for x: $x(2x+5) = 0$ *Example*: Solve the following for x: $3x^2 - x - 10 = -8$ *Assessment Limitation*: Quadratic equations will have integral coefficients and rational solutions. Students may use any valid method to determine solutions for a quadratic equation.

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D: Data, Statistics and Probability
Priority: 20%

Successful students will be able to apply algebraic knowledge to the interpretation and analysis of data, statistics and probability. Analysis and interpretation of univariate and bivariate data includes the use of summary statistics for sets of data and estimation of lines of best fit. While some important components in the study of data and statistics, such as misleading uses of data, sampling techniques, bias, question formulation and experiment design are addressed when possible in this Algebra I End-of-Course Exam, those topics will be expected to be assessed in more depth in the classroom. These benchmarks are intended to support and reinforce algebra concepts. For this reason, several sample algebraic solutions are provided for examples. There is a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems, students will be required to graph and interpret their solutions in terms of the context.

Content Benchmarks	Explanatory Comments and Examples
D1: Data and Statistical Analysis	
a. Interpret and compare linear models for data that exhibit a linear trend including contextual problems.	• Create scatter plots and estimate a line of best fit. • Describe the correlation of data. • Interpret the slope and *y*-intercept of the line of best fit (regression line) in the context of the model • Use lines of best fit to extrapolate or interpolate within the range of the data and within the context of the problem. Determine when, within the context of a problem, it may be unreasonable to extrapolate beyond a certain point. *Example*: If a linear trend describes population growth in a small town over 5 years, explain why it would not be best to use the same linear trend to predict population in the town after 100 years. • For items where a student is required to model data with a graph, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). *Assessment Limitation*: Students will not be required to calculate the correlation coefficient. Students will not be required to use regression to calculate a line of best fit. In items, it may be helpful for students to sketch a line of best fit to interpret the behavior of the data, however, students will not be required to draw the line of best fit for credit.

b. Use measures of center and spread to compare and analyze data sets.	• Analyze data sets and use summary statistics to compare the data sets and to answer questions regarding the data. *Example*: A student has scores of 78, 82, 91, 84 and 67 on the first five tests in a semester. What score must she earn on the sixth test in order to raise her average to 82? *Sample Solution*: $\frac{78+82+91+84+67+x}{6}=82$ $\Rightarrow 402+x=6(82)$ $\Rightarrow x=492-402=90$ • Determine the effects of outliers on statistics. *Example*: Explain what happens to the mean, median and mode when the same value, x, is added to each data point. *Example*: Given the following data set: 55, 55, 57, 58, 60 and 63. Describe how the measures of center or spread will or will not change if an additional data point of 57.5 is included with the set. *Assessment Limitation*: No item will assess only the calculations of mean, median or mode. Items will require the use of these concepts and/or calculations and will be at an appropriate cognitive level and difficulty for Algebra I. Measures of spread are limited to range.
c. Evaluate the reliability of reports based on data published in the media.	• Explain the impact of bias and the phrasing of questions asked during data collection. • Identify and explain misleading uses of data and data displays. • Analyze the appropriateness of a data display and the reasonableness of conclusions based on statistical studies. • Explain the difference between randomized experiments and observational data.

D2: Probability	
a. Use counting principles to determine the number of ways an event can occur. Interpret and justify solutions.	• Use understanding of permutations and combinations to solve problems with and without replacement. *Example:* Compare the number of ways the letters of the words FROG and DEER can be arranged to form unique password configurations. Explain your answer. *Example*: If a person has twice as many shirts as pairs of pants, how many different combinations can be made of a shirt and pair of pants, based on the number of pants? Sample Solution: $p =$ number of pairs of pants $2p =$ number of shirts $p(2p) = 2p^2 =$ number of combinations of pants and shirts *Assumption:* All spinners, number cubes and coins are fair unless otherwise noted. *Assessment Limitation*: Neither factorial notation nor factorial forms of formulas for combination $\left({}_nC_r\right)$ or permutation $\left({}_nP_r\right)$ will be used in items or be required to solve items on the test, however, students may use any valid method to solve the problem. Numbers involved will be manageable without formulas.
b. Apply probability concepts to determine the likelihood an event will occur in practical situations.	• Determine, exactly or approximately, the probability that an event will occur based on simple experiments (e.g., tossing number cubes, flipping coins, spinning spinners), counting principles or data. *Example*: If there are four brown, four black and four blue socks in a drawer, what is the probability that a matched pair will be selected when drawing out first one and then another, without replacing the first sock or being able to see the socks as they are drawn? • Make predictions based on experimental and theoretical probabilities and compare results. *Example:* In a sample of 100 randomly selected students, 37 of them could identify the difference in two brands of soft drinks. Based on these data, what is the best estimate of how many of the 2352 students in the school could distinguish between the soft drinks?

	Sample Solution: $\frac{37}{100} = \frac{x}{2352}$ $100x = 37(2352)$ $x = 870.24$ $\therefore$ 870 students would be expected to distinguish between the soft drinks *Assumption*: All events are equally likely and samples are representative of the population, unless otherwise stated. All spinners, number cubes and coins are fair unless otherwise noted.