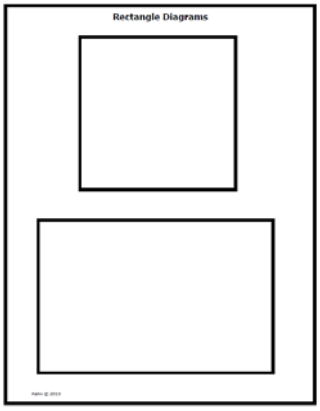
Completing the Square with The Area Model

Numbers like 64 are called perfect squares because they are the squares of integers, in this case 8 or -8. The trinomial x^2+6x+9 is $(x+3)^2$. So it is also called a perfect square.

Identify which of these trinomials are perfect squares?



A. $x^2+16x+64$ B. $x^2-18x+81$

C. $x^2+12x+16$ D. $x^2-12x+36$

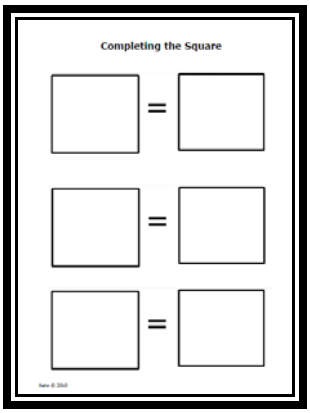
Use the Rectangle Diagrams Template to show the area model for each of the trinomials that are perfect squares.

Study each of the following trinomials. Draw the model for each trinomial. If a number is missing, use your model to figure out the missing number.

A. x^2+2x+1 B. x^2-6x+9 C. $x^2+\_\_\_x+36$

D. $x^2-\_\_\_\_x+121$ E. $x^2+14x+\_\_\_\_$ F. $x^2-10x+\_\_\_\_$

What is the connection between the middle term and the last term that makes it possible to form a square?



Trinomials can also be placed in an equation such as $x^2+6x+9=36$. Use the Completing the Square Template to show this equivalence. Represent the trinomial in the square on the left and the constant in the square on the right. If the two squares have the same area then their dimensions must be equal also. Find the dimensions of each square. Don't forget both the negative and positive values.

x^2+6x+9 = 36

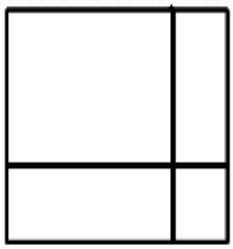
$$(x+3)^2=6^2$$
$$\sqrt{(x+3)^2}=\sqrt{6^2}$$
$$|x+3|=6$$
$$x+3=\pm 6$$
$$x=\pm 6-3$$
$$x=+3 \text{ or } -9$$

Solve each of the following equations using the Completing the Square Template.

A. $x^2 + 12x + 16 = 25$ B. $x^2 - 12x + 36 = 25$

C. $x^2 + 18x + 81 = 100$ D. $x^2 - 14x + 49 = 64$

Study the square on the left in each of the problems. Describe any patterns you notice in the area of each section and the coefficients from the trinomial.

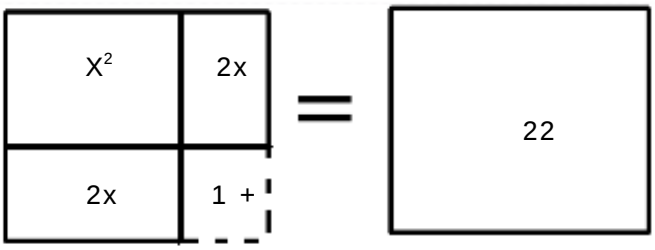


Suppose the trinomial was $x^2 + 16x + 64$. Label this square with the appropriate area and dimensions.

What if the trinomial is not a perfect square? Can a problem like this still be solved?

Suppose $x^2 + 4x + 1 = 22$.

- Begin to set up the two squares, but notice on the left that the area of the small square is too small.
- How much should it be?
- How much can we add to both sides of this equation?
- Since this is an equation, add the amount of area you need to both sides. Now complete the problem.



- After adding 3 square units to both sides, the shape on the left will be a square and you will have:

$$x^2 + 4x + 1 + 3 = 22 + 3$$
$$x^2 + 4x + 4 = 25$$
$$(x + 2)^2 = 25$$
$$x + 2 = \pm 5$$
$$x = -2 \pm 5$$

Solve each of these problems using the Completing the Square Template:

A. $x^2 + 6x + 1 = 28$ B. $x^2 - 16x + 30 = 15$

C. $x^2 + 12x + 6 = 70$ D. $x^2 - 18x + 60 = 43$

D. $x^2 + 4x + 1 = 0$ E. $x^2 + 3x + 2 = 8\frac{3}{4}$