Developing an Understanding for Solving a System of Equations Using Matrices

In this activity you will solve systems of equations using matrices. The matrix method is similar to the elimination method, but uses matrices. You may find this quicker because the notation is shorter. The following system has been set up as a matrix.

$$\begin{cases} 5x + 3y = -1 \\ 2x - 6y = 50 \end{cases} \quad \begin{bmatrix} 5 & 3 & -1 \\ 2 & -6 & 50 \end{bmatrix}$$

When writing a matrix the equations must first be placed in general form. Notice that the matrix is made up of three columns and two rows. Each row represents one of the equations. One column shows the coefficients of x, another column the coefficients of y, and the last column the constants.

Find the solution to this system of equations by using the elimination method.

Complete the following two statements to describe the solution of the system of equations:

X = ______ Y=_______

The solution matrix for this system of equations will be: $\begin{cases} x + 0y = 4 \\ 0x + y = -7 \end{cases} \quad \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -7 \end{bmatrix}$

How do we work with a Matrix to get the solution matrix? We perform row operations.

Recall that in the elimination method you combined equations and multiplied them by numbers. In much the same way we can modify the rows of the matrix by performing row operations on each number in those rows.

- *Multiply (or divide) all numbers in a row by a non-zero number*
- *Add all numbers in a row to corresponding numbers in another row*
- *Add a multiple of the numbers in one row to the corresponding numbers in another row*
- *Exchange two rows.*

Our goal will be to change the original matrix into the solution matrix:

$$\begin{bmatrix} 5 & 3 & -1 \\ 2 & -6 & 50 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -7 \end{bmatrix}$$

Let's start:

The original matrix is formed by copying the coefficients and constants from the two equations:

$$\begin{bmatrix} 5 & 3 & -1 \\ 2 & -6 & 50 \end{bmatrix}$$

Notice that the middle column has one negative and one positive integer.
By what number can you multiply the second row (equation) by that will make the middle column opposite numbers? Once you decide, multiply the first row (first equation) by that number and rewrite the matrix.

$$\begin{bmatrix} 5 & 3 & -1 \\ & & \end{bmatrix}$$

Study the new matrix. Notice that if we add the first row to the second row, the middle column will contain a zero in the first row. So replace the second row with the addition of the first and second rows.

$$\begin{bmatrix} 5 & 3 & -1 \\ & & \end{bmatrix}$$

Notice that we can simplify the second row by dividing by a positive number. Rewrite the matrix by dividing the second row by this number.

$$\begin{bmatrix} 5 & 3 & -1 \\ & & \end{bmatrix}$$

You need just one more zero in the first column. By multiplying the second row by a negative number and adding it to the first row you can gain one more zero. So replace the first row with this combination.

$$\begin{bmatrix} & & \\ 1 & 0 & 4 \end{bmatrix}$$

Now we just need to divide the first row by a number to get a 1 in column 2. Complete that operation.

$$\begin{bmatrix} & & \\ 1 & 0 & 4 \end{bmatrix}$$

You can now switch the rows.

$$\begin{bmatrix} & & \\ & & \end{bmatrix}$$

The solution to the system of equations is: x = _____ and y = ______.

Solve this system of equations by using matrices.

$$\begin{cases} x - 2y = 3 \\ 3x + y = 23 \end{cases}$$

$$\begin{cases} 2x + y = 11 \\ 6x - 5y = 9 \end{cases}$$

On Friday, 3247 people attended the county fair. The entrance fee for an adult was $5 and for a child 12 and under the fee was $3. The fair collected $14,273. How many of the total attendees were adults and how many were children. Solve this problem using a system of equations and the matrix method.