Exploring Permutations and Combinations with the Graphing Calculator

Permutations:

Situation 1:

Warren along with three of his classmates Rajan , Gage and Baron recently won a writing essay. The prize is a pair of tickets for the LoudOne Concert. The pair of tickets are scattered throughout the venue. Each person will get to choose a pair of tickets, so the order in which they choose the tickets is very important. In this investigation you will be exploring the possible ways these tickets can be chosen.

If Warren gets to choose first choose his pair of tickets first, what order can the other students select their pair of tickets?

Warren	Rajan	Gage	Baron
Warren	Rajan	Baron	Gage
Warren	Gage	Rajan	Baron
Warren	Gage	Baron	Rajan
Warren	Baron	Gage	Rajan
Warren	Baron	Rajan	Gage

Who else could be first to choose their tickets? Using the letters W for Warren, R for Rajan, G for Gage, and B for Baron, systematically list all the other possible orders for the students to pick their pair of tickets.

RWGB	GWRB	BWGR
RWBG	GWBR	BWRG
RGWB	GBWR	BGWR
RGBW	GBRW	BGRW
RBGW	GRWB	BRGW
RBWG	GRBW	BRWG

How many total ways are their for these four students to pick their pair of tickets? 24

This type of selection is called a permutation of 4 students, using 4 at a time. Let's study the permutations a little more closely.

Since there are four students choosing tickets and all four will get a pair of tickets. How many different students could be selected to choose their pair of tickets first? 4

Once the first person was chosen, study how many choices did you have left to be second to choose their tickets? 3

Once the first two persons were chosen, how many choices did you have left to be third to choose their tickets? 2

Once the first two persons were chosen, how many choices did you have left to be last to choose their tickets? 1

Using the counting principal we would say that the total number of ways the students could be selected would be P = (# of ways to select the first person)(# of ways to select the second person)(# of ways to

select the third person)(# of ways to select the last person). What would that be in this case?

P = (4)(3)(2)(1) or 24 This permutation is written ${}_4P_4$.

Suppose that on the day the pair of tickets was to be chosen it was discovered that only three pairs of tickets had been delivered to the principal's office. How many ways are their to choose 4 students, 3 at a time? Use the letters to see what happens? Systematically make a list of ways 4 students can be chosen using three at a time.

WRG	RWG	GWR	BWG
WRB	RWB	GWB	BWR
WGR	RGW	GBW	BGW
WGB	RGB	GBR	BGR
WBG	RBG	GRW	BRG
WBR	RBWG	GRB	BRW

What multiplication problem matches up to get this answer.

P= (4)(3)(2) =24 or ${}_4P_3$ This is the permutation of 4 students, 3 at a time.

Suppose we want to find out how many ways 4 students could be chosen taking only 2 at a time. Systematically write out the ways these students can be chosen.

WR	RW	GW	BW
WG	RG	GR	BR
WB	RB	GB	BG

What multiplication problem matches up to get his answer?

P= (4)(3) = 12 or ${}_4P_2$ This is the permutation of 4 students, 2 at a time.

Can you predict how many ways you can use four students and pick 1 at a time? 4

What multiplication problem matches up to get this answer?

P= 4 or ${}_4P_1$. This is the permutation of 4 students, 1 at a time.

In mathematics we have a symbol to tell us to multiply 4 x 3 x 2 x 1. We write it as 4! (Four factorial)

The permutation formula is defined as follows: ${}_nP_r = \frac{n!}{(n-r)!}$ where n is the total number of students, r is how many you are choosing at a time and $n! = (n)\cdot(n-2)\cdot(n-3)\cdot\cdots\cdot 1$. Remember 0!=1.

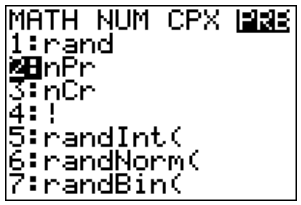
$${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4}{1} = 4$$

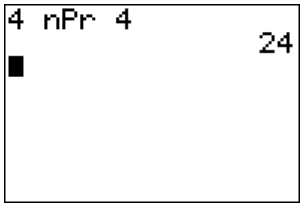
So Use this formula to calculate ${}_4P_3$, ${}_4P_2$, and ${}_4P_1$.

$${}_4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = \frac{4}{1} = 4,\ {}_4P_2 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4}{2} = 2,\ {}_4P_1 = \frac{4!}{(4-1)!} = \frac{4!}{1!} = \frac{4}{1} = 4$$

The graphing calculator can be use to calculate these permutation .

Press MATH and cursor to the column that say PRB. You should have the screen illustrated at the right. Notice that #2 says nPr. To calculate ${}_4P_4$ first start on the home screen, type in the first 4, then press MATH, PRB, 2. nPr, and then enter the second 4. Press enter.





Calculate ${}_4P_4$, ${}_4P_3$, ${}_4P_2$, and ${}_4P_1$. Same as above.

Combinations:

Warren along with three of his classmates Rajan , Gage and Baron are the winners of the writing essay. The prize is a pair of tickets for the LoudOne Concert. Each person will get to choose a pair of tickets for the concert, but all the tickets are in the same row of seats, so the order in which the students pick the tickets is not important. In this investigation you will be exploring the number of groups of different sizes that can be formed from these four students if changing the order does not product a new group.

This time the principal is going to call the student to her office to let them pick their tickets. Using the spaces below, list all the different groups she can form to call the students to her office if she is going to choose a group of four students from the four students.

W R G B

Why is there only one group of students in this case? This is called the number of combinations that can be formed from 4 people if four people are chosen at a time.

Suppose the principal want to offer tickets to three students. How many different groups of three students can she make from the four students? Systematically list all different groups of three students that can be selected from four students.

W R G W R B R G B G W B

This is the number of combinations that be formed from 4 students being chosen three at a time.

If the principal wants to offer tickets to only two students, how many different groups of two students can she make from the four students? Systematically list all different groups of two students that can be selected from four students.

WR WG WB RG RB GB

This is the number of combinations that be formed from 4 students being chosen two at a time.

If the principal wants to offer tickets to only 1 student, how many different groups of one student can she make from the four students? Systematically list all different groups of one student that can be selected from four students.

W R G B

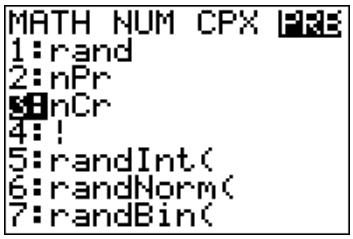
This is the number of combinations that be formed from 4 students being chosen one at a time.

It is also possible to determine the number of combinations that can be formed by beginning with 4 students and taking 0 at a time. How many ways do you think this can happen?
This is the number of combinations that be formed from 4 students being chosen one at a time.

The combination formula is defined as follows: $_nC_r = \frac{n!}{(n-r)!(r!)}$ where n is the total number of students,

r is how many you are choosing at a time and $n! = (n)\cdot(n-2)\cdot(n-3)\cdot\cdots\cdot 1$. Remember 0!=1.

So $_4C_4 = \frac{4!}{(4-4)!(4!)} = \frac{4!}{(0)!(4!)} = \frac{4!}{1(4!)} = 1$. Use this formula to calculate $_4C_3$, $_4C_2$, $_4C_1$ and $_4C_0$.

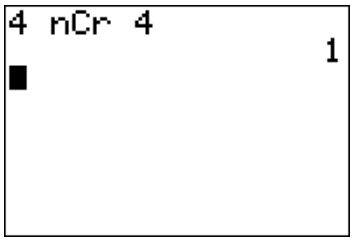


$_4C_3 = \frac{4!}{(4-3)!(3!)} = \frac{4!}{(1)!(3!)} = 4$, $_4C_0 = \frac{4!}{(4-0)!(0!)} = \frac{4!}{(4)!(1)} = 1$,

$_4C_1 = \frac{4!}{(4-1)!(1!)} = \frac{4!}{(3)!(1!)} = 4$, $_4C_2 = \frac{4!}{(4-2)!(2!)} = \frac{4!}{(2)!(2!)} = \frac{4\cdot 3}{2} = 6$

The graphing calculator can be used to calculate these combinations.

Press MATH and cursor to the column that say PRB. You should have the screen illustrated at the right. Notice that #3 says nCr. To calculate ${}_4C_4$ first start on the home screen, type in the first 4, then press MATH, PRB, 2. nCr, and then enter the second 4. Press enter.



Calculate ${}_4C_4$, ${}_4C_3$, ${}_4C_2$, and ${}_4C_1$. See above

Various Problems

1. Let's look at some patterns. Notice from your work above that some of the permutations are equal to each other such as ${}_4P_4$ and ${}_4P_3$.

$${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4}{1} = 4 \quad {}_4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = \frac{4}{1} = 4$$

Use the permutation formula to see why these two are equal. Use the calculator to find the following permutation ${}_5P_5$ and ${}_5P_4$. Use the permutation formula to show why these two are equal.

$${}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5}{1} = 5 \quad {}_5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{1!} = \frac{5}{1} = 5$$

If you started with 7 objects, what permutations would be equal. Verify this using the graphing calculator.

$${}_7P_7 = {}_7P_6 = 5040$$

2. Look at ${}_4P_4$ and ${}_5P_5$. Can you predict any other probabilities from these two permutations?

Support your answer by using the graphing calculator. ${}_4P_4 = {}_4P_3 \; {}_5P_5 = {}_5P_4$

3. Here's another pattern. Notice from your work above that some of the combinations are equal to each other such as ${}_4C_3$ and ${}_4C_1$. Calculate the following combinations using the graphing calculator first to see which are equal to each other. ${}_5C_5$, ${}_5C_4$, ${}_5C_3$, ${}_5C_2$, ${}_5C_1$ and ${}_5C_0$.

Explain why one of these pairs is equal by using the combination formula. ${}_5C_5 = {}_5C_0$,

${}_5C_4 = {}_5C_1$, ${}_5C_3 = {}_5C_2$

4. Use your graphing calculator to find the following combinations: ${}_6C_6$, ${}_6C_5$, ${}_6C_4$, ${}_6C_3$,

${}_6C_2$, ${}_6C_1$ and ${}_6C_0$. Which are equal to each other. Explain why one of these pairs is equal by using the combination formula. ${}_6C_6 = {}_6C_0$, ${}_6C_5 = {}_6C_1$, ${}_6C_4 = {}_6C_2$

5. Can you predict which combinations will be equal to each other in this set? ${}_7C_7$, ${}_7C_6$, ${}_7C_5$, ${}_7C_4$, ${}_7C_3$, ${}_7C_2$, ${}_7C_1$, and ${}_7C_0$. Support why this is for one pair by using the combination formula. ${}_7C_7 = {}_7C_0$, ${}_7C_6 = {}_7C_1$, ${}_7C_5 = {}_7C_2$, ${}_7C_4 = {}_7C_3$

6. A coach must choose five starters from a team of 11 players. Each player can play any position. How many different ways can the coach choose the starters? ${}_{11}C_5 = 462$

7. How many ways can 8 colored vases be arranged in a straight line on the windowsill if 4 vases are used at a time? ${}_8P_4 = 1680$

8. There are twelve juniors and twenty seniors in the Service Club. The club is to send four representatives to the State Conference. How many different ways are there to select a group of five students to attend the conference? ${}_{22}P_5 = 3,160,080$

9. A teacher is making a multiple choice quiz. She wants to give each student the same questions, but have each student's questions appear in a different order. If there are twenty-five students in the class, what is the least number of questions the quiz must contain?If there were two questions: ${}_2P_2 = 2$ quizzes. If there were 3 questions: ${}_3P_3 = 6$ quizzes. If there were 4 questions: ${}_4P_4 = 24$ quizzes. So the teacher would have to make a quiz with 5 questions.

10. Determine whether the following situations would require calculating a permutation or a combination:
    a.) Selecting three students to attend a conference in New York City? Combination
    b.) Assigning students to their seats on the first day of school Permutation
    c.) Selecting a lead and an understudy for a school musical. Permutation