

Exploring Permutations and Combinations with the Graphing Calculator

Permutations: Warren, Rajan, Gage, and Baron

Situation 1:

Warren _____
Warren _____
Warren _____
Warren _____
Warren _____
Warren _____

W for Warren, R for Rajan, G for Gage, and B for Baron

_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

Total ways four students can be picked _____ This is called a permutation of 4 students, using 4 at a time.

P = _____ or _____ This permutation is written ${}_4P_4$.

Situation 2: W for Warren, R for Rajan, G for Gage, and B for Baron
Systematically write out the ways four students can be chosen three at a time.

_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____

What multiplication problem matches up to get this answer.

P = _____ or ${}_4P_3$ This is the permutation of 4 students, 3 at a time.

Situation 3: W for Warren, R for Rajan, G for Gage, and B for Baron
 Systematically write out the ways four students can be chosen two at a time.

_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____

What multiplication problem matches up to get his answer?

P= _____ or ${}_4P_2$ This is the permutation of 4 students, 2 at a time.

Can you predict how many ways you can use four students and pick 1 at a time?

What multiplication problem matches up to get this answer?

P= _____ or ${}_4P_1$. This is the permutation of 4 students, 1 at a time.

In mathematics we have a symbol to tell us to multiply $4 \times 3 \times 2 \times 1$. We write it as $4!$ (Four factorial)

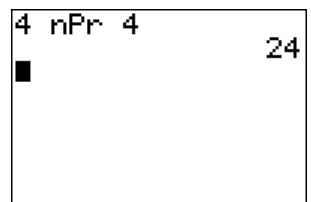
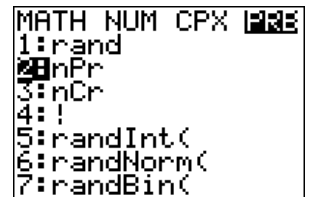
The permutation formula is defined as follows: ${}_nP_r = \frac{n!}{(n-r)!}$ where n is the total number of students, r is

how many you are choosing at a time and $n! = (n) \cdot (n-2) \cdot (n-3) \cdot \dots \cdot 1$. Remember $0! = 1$.

So ${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4}{1} = 4$ Use this formula to calculate ${}_4P_3$, ${}_4P_2$, and ${}_4P_1$.

The graphing calculator can be use to calculate these permutations.

Press MATH and cursor to the column that say PRB. You should have the screen illustrated at the right. Notice that #2 says nPr. To calculate ${}_4P_4$ first start on the home screen, type in the first 4, then press MATH, PRB, 2. nPr, and then enter the second 4. Press enter.



Calculate ${}_4P_4$, ${}_4P_3$, ${}_4P_2$, and ${}_4P_1$.

Combinations: Warren (W), Rajan(R), Gage(G), and Baron(B)

This is called the number of combinations that can be formed from 4 people if four people are chosen at a time.

Systematically list all different groups of three students that can be selected from four students.

This is the number of combinations that be formed from 4 students being chosen three at a time.

Systematically list all different groups of two students that can be selected from four students.

This is the number of combinations that be formed from 4 students being chosen two at a time.

Systematically list all different groups of one student that can be selected from four students.

This is the number of combinations that be formed from 4 students being chosen one at a time.

The combination formula is defined as follows: ${}_n C_r = \frac{n!}{(n-r)!(r!)}$ where n is the total number of students,

r is how many you are choosing at a time and $n! = (n) \cdot (n-2) \cdot (n-3) \cdot \dots \cdot 1$. Remember $0! = 1$.

So ${}_4 C_4 = \frac{4!}{(4-4)!(4!)} = \frac{4!}{(0)!(4!)} = \frac{4!}{1(4!)} = 1$. Use this formula to calculate

${}_4 C_3$, ${}_4 C_2$, ${}_4 C_1$ and

${}_4 C_0$.

Press MATH and cursor to the column that say PRB. You should have the screen illustrated at the right. Notice that #3 says nCr. To calculate ${}_4 C_4$ first start on the home screen, type in the first 4, then press MATH, PRB, 2. nCr, and then enter the second 4. Press enter.

Calculate ${}_4 C_4$, ${}_4 C_3$, ${}_4 C_2$, and ${}_4 C_1$.

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

```
4 nCr 4
1
```

Various Problems

- Let's look at some patterns. Notice from your work above that some of the permutations are equal to each other such as ${}_4P_4$ and ${}_4P_3$. Use the permutation formula to see why these two are equal. Use the calculator to find the following permutation ${}_5P_5$ and ${}_5P_4$. Use the permutation formula to show why these two are equal. If you started with 7 objects, what permutations would be equal. Verify this using the graphing calculator.
- Look at ${}_4P_4$ and ${}_5P_5$. Can you predict any other probabilities from these two permutations? Support your answer by using the graphing calculator.
- Here's another pattern. Notice from your work above that some of the combinations are equal to each other such as ${}_4C_3$ and ${}_4C_1$. Calculate the following combinations using the graphing calculator first to see which are equal to each other. ${}_5C_5$, ${}_5C_4$, ${}_5C_3$, ${}_5C_2$, ${}_5C_1$ and ${}_5C_0$. Explain why one of these pairs is equal by using the combination formula.
- Use your graphing calculator to find the following combinations: ${}_6C_6$, ${}_6C_5$, ${}_6C_4$, ${}_6C_3$, ${}_6C_2$, ${}_6C_1$ and ${}_6C_0$. Which are equal to each other. Explain why one of these pairs is equal by using the combination formula.
- Can you predict which combinations will be equal to each other in this set? ${}_7C_7$, ${}_7C_6$, ${}_7C_5$, ${}_7C_4$, ${}_7C_3$, ${}_7C_2$, ${}_7C_1$, and ${}_7C_0$. Support why this is for one pair by using the combination formula.
- A coach must choose five starters from a team of 11 players. Each player can play any position. How many different ways can the coach choose the starters?
- How many ways can 8 colored vases be arranged in a straight line on the windowsill if 4 vases are used at a time?
- There are twelve juniors and twenty seniors in the Service Club. The club is to send four representatives to the State Conference. How many different ways are there to select a group of five students to attend the conference?
- A teacher is making a multiple choice quiz. She wants to give each student the same questions, but have each student's questions appear in a different order. If there are twenty-seven students in the class, what is the least number of questions the quiz must contain?
- Determine whether the following situations would require calculating a permutation or a combination:
 - Selecting three students to attend a conference in New York City? _____
 - Assigning students to their seats on the first day of school _____

c.) Selecting a lead and an understudy for a school musical. _____