Performance Assessment Questions on Quadratics
Solutions

1. Solve the given equation using

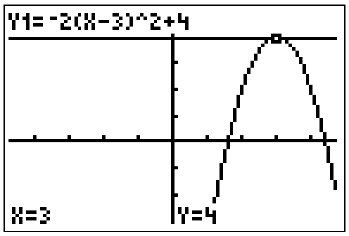
   a. A graph.
   b. A table.
   c. A symbolic method

Possible Equations: $4=-2(x-3)^2+4$

$-12=-2.5(x-3)^2-2$

$-36.5=-2(x-1.5)^2+4$

$4=-2(x-3)^2+4$



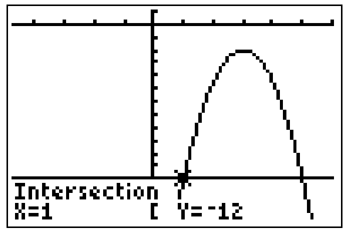
X	Y1	Y2
0	-14	4
1	-4	4
2	2	4
3	[illegible]	4
4	2	4
5	-4	4
6	-14	4

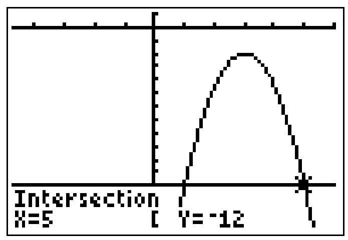
Y1=4

$$4=-2(x-3)^2+4$$
$$4=-2(x^2-6x+9)+4$$
$$4=-2x^2+12x-18+4$$
$$0=-2x^2+12x-18$$
$$0=-2(x^2-6x+9)$$
$$0=-2(x-3)(x-3)$$
$$x=3$$

At x = 3, the parabola has a value of y = 4 which is the solution to the given equation. This is illustrated in both the graph and the table.

$-12=-2.5(x-3)^2-2$





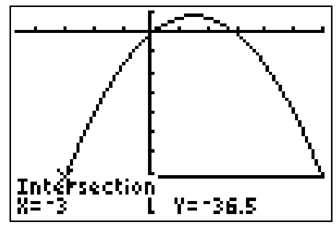
X	Y1	Y2
0	-24.5	-12
[illegible]	-12	-12
2	-4.5	-12
3	-2	-12
4	-4.5	-12
5	-12	-12
6	-24.5	-12

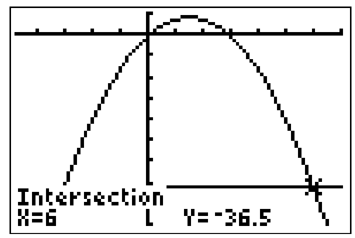
X=1

$$-12=-2.5(x-3)^2-2$$
$$-12=-2.5(x^2-6x+9)-2$$
$$-12=-2.5x^2+15x-22.5-2$$
$$0=-2.5x^2+15x-12.5$$
$$0=-2.5(x^2-6x+5)$$
$$0=-2.5(x-5)(x-1)$$
$$x=1 \text{ or } 5$$

At x = 1 and 5, the parabola has a value of y = -12 which is the solution to the given equation. This is illustrated in both the graph and the table.

$-36.5=-2(x-1.5)^2+4$





X	Y1	Y2
-3	-36.5	-36.5
0	-.5	-36.5
3	-.5	-36.5
6	-36.5	-36.5
9	-108.5	-36.5
12	-216.5	-36.5
15	-360.5	-36.5

Y1=-36.5

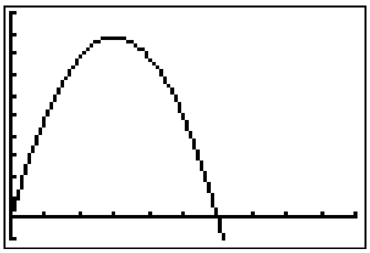
$$-36.5=-2(x-1.5)^2+4$$
$$-36.5=-2(x^2-3x+2.25)+4$$
$$-36.5=-2x^2+6x-4.5+4$$
$$0=-2x^2+6x+36$$
$$0=-2(x^2-3x-18)$$
$$0=-2(x-6)(x+3)$$
$$x=-3 \text{ or } 6$$

At x = -3 and 6, the parabola has a value of y = -36.5 which is the solution to the given equation. This is illustrated in both the graph and the table.

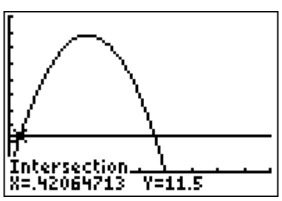
2. Create a graph of $h(t)=-4.9t^2+29.4t$ in an appropriate window. The variable t represents time in seconds, and h(t) represents the height in meters of a projectile.

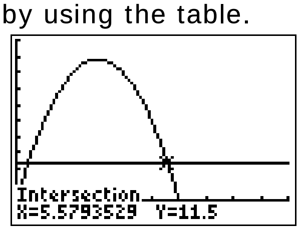
a. Create a sketch of your graph and describe window used.
b. What is a real-world meaning for the x-intercepts in the graph?
c. Find the x-intercepts to the nearest second.
d. Use your answer to part c to find the vertex of this parabola.
e. Describe the real-world meaning for the vertex in the graph.
f. Explain the meaning of h(2.3).
g. Describe when the projectile is 11.5 m high. Explain how to find these solutions on a graph.

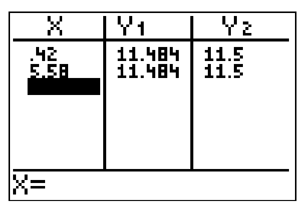
a. [0,10,1,-5,50,5]



b. The x-intercepts represent the times that the projectile was on the ground.
c. The x-intercepts are t=0 and t = 6.
d. The vertex is at (3,h(3)) or (3 seconds, 44.1 feet).
e. The meaning of the vertex is that at 3 seconds the projectile reaches its greatest height of 44.1 feet.
f. h(2.3) = 41.699 feet. The particle at 2.3 seconds is at the height of 41.600 feet and is on its way up to its maximum height.
g. There are two times that the projectile is at a height of 11.5 feet. The first time on its way up at about t = 0.42 seconds and the second time on its way down at about t = 5.58 seconds. This is when the parabola has a y value of 11.5.
This can also be found by using the table.





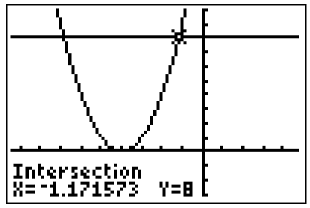


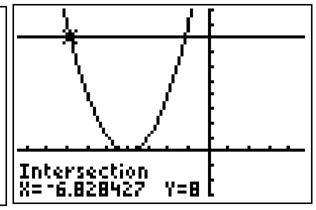
3. Let $(x+4)^2 = 8$.

a. Solve the equation symbolically and show that your answer is a solution to the equation.
b. Graph $y=(x+4)^2$ and y = 8. Describe the relationship between your solution to part a and these graphs.

$$(x+4)^2 = 8$$
$$|x+4| = \sqrt{8}$$
$$x+4 = \pm\sqrt{8}$$
$$x = -4 \pm \sqrt{8}$$

$$(-4 \pm \sqrt{8}+4)^2 = 8$$
$$(\pm\sqrt{8})^2 = 8$$
$$8 = 8$$



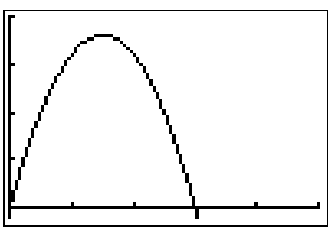
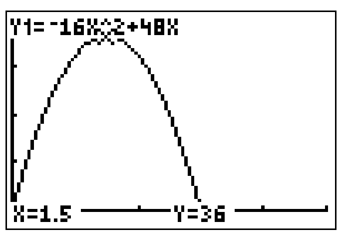


The solution in part a identifies the x values where the left side of the equation equals 8. This is illustrated by the graph of the parabola, the left side, intersecting with the horizontal line y = 8. The $x = -4 \pm \sqrt{8}$ are approximately -1.17 and -6.82.

4. The height of a golf ball is given by $h(t)=-16t^2+48t$, where t is in seconds and h is in feet.

a. Describe the times the golf ball on the ground and how your found the times.
b. Describe the time and height when the golf ball at its highest point. Show work that leads to your answer.
c. Describe the domain and range values that make sense in this situation.
d. If another golf ball had a height that was given by $h=-16t^2+64t$, how would its behavior change from the first golf ball?

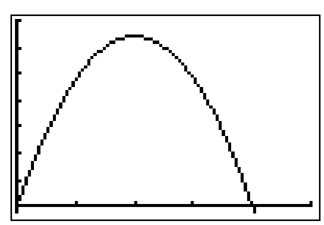
The graph shows the height of the golf ball over the first 3 seconds on the flight. According to the graph the ball in on the ground at t = 0 and t = 3 seconds because this is where the graph of h(t) equals zero or the t-axis intercepts.

The golf ball is at its highest point at 1.5 seconds. The height of the golf ball at 1.5 seconds = $h(1.5) = -16(3)^2 + 48(3) = 36$ feet.

The domain for this problem would be the time the ball is in flight or $0 \le t \le 3$ and the range is the height of the ball during that time period or $0 \le h \le 36$.

If the equation were changed, the graph would be altered. The ball would go higher and would be in the air longer. It would travel for 4 seconds and reach a height of 64 feet at time 2 seconds. This is the location of the vertex on the graph: (2,64) or (2, h(2)).



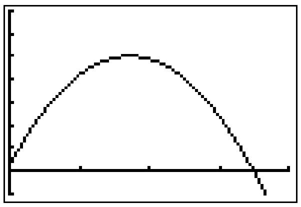
5. Juan hits a baseball, and its height in the air at time x is given by the equation $y = -16x^2 + 55x + 3$, where x is in seconds and y is in feet.

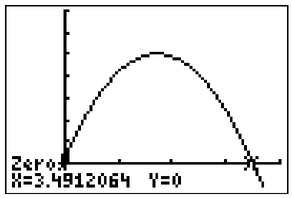
a. Create a graph for the 4 seconds after the ball is hit. Make a sketch of your window. Use the graph and tables to help you answer these questions. Give explanations for your answer.

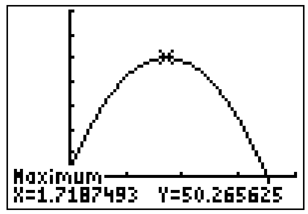
b. Find the initial height of the ball when it is hit by Juan. Label this point on your graph.

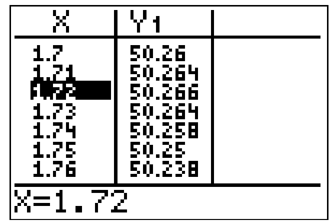
c. When does the ball hit the ground? Use your calculator table to find the answer to two decimal places. Label this point on your sketch.

d. Use the graph and table to determine the time, to one decimal place, that the ball the ball at its highest points. Give the time and the height. Label this point on your graph.









$y(0) = -16(0)^2 + 55(0) + 3 = 3$ So the baseball started its flight at the height of 3 feet. This is where the parabola crosses the y axis.

The ball hits the ground at time t = 3.49 seconds.

The ball reaches it maximum height of 50.265 feet at t = 1.719 seconds

6. Consider the parabola illustrated at the right. Feet

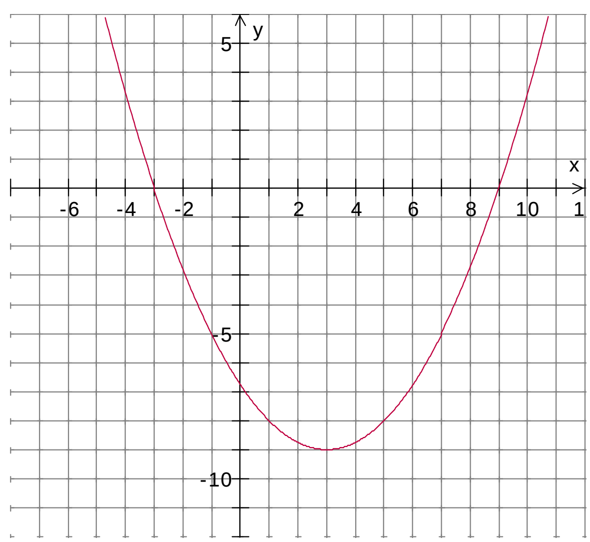
a. Determine the coordinates for the vertex.

b. Write the equation for the parabola in factored form. Explain how this equation relates to the graph of the parabola.

c. Write the equation for the parabola in vertex form. Explain how this equation relates to the graph of the parabola.

d. Write the equation of the parabola in general form.

e. Select 6 integral values for x. Create a table for these x-values for each equations from part b, c, and d. What does your table illustrate?



The vertex would be at (3, 9).
The factored form for the equation of the parabola uses the zeros of x = -3 and x = 9: $y = (x+3)(x-9)$, but this equation would have a vertex at (3,-36). Therefore, the graph must be shrunk by a factor of 1/3. Therefore the equation is $y = \frac{1}{4}(x+3)(x-9)$.

The equation in vertex form would be $y = \frac{1}{4}(x-3)^2 - 9$. This form uses the coordinates of the vertex and uses the shrinking factor from the factored form.

The general form for the equation of the parabola can be found by expanding either the factored form or the vertex form:

$$y = \frac{1}{4}(x^2 - 6x - 27)$$
$$y = \frac{1}{4}x^2 - \frac{3}{2}x - \frac{27}{4}$$

or

$$y = \frac{1}{4}(x-3)^2 - 9$$
$$y = \frac{1}{4}(x^2 - 6x + 9) - 9$$
$$y = \frac{1}{4}x^2 - \frac{3}{2}x + \frac{9}{4} - \frac{36}{4}$$
$$y = \frac{1}{4}x^2 - \frac{3}{2}x - \frac{27}{4}$$

X	Y1	Y2
-3	0	0
-2	-11/4	-11/4
-1	-5	-5
0	-27/4	-27/4
1	-8	-8
2	-35/4	-35/4
3	-9	-9

Y1=0

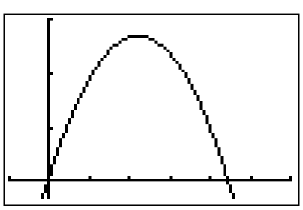
X	Y2	Y3
-3	0	0
-2	-11/4	-11/4
-1	-5	-5
0	-27/4	-27/4
1	-8	-8
2	-35/4	-35/4
3	-9	-9

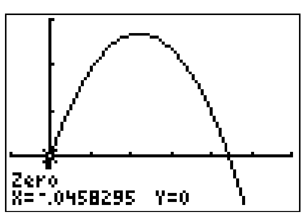
Y3=0

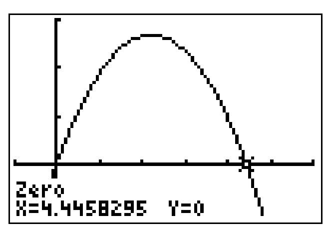
The three columns of table values shows that all three equations have the same sets of values for the integers from -3 to 3.

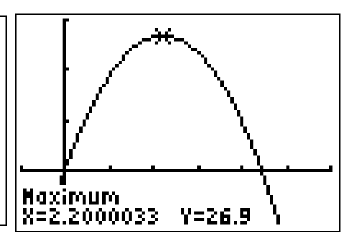
7. A professional football team uses computers to describe the projectile motion of a football when punted. After compiling data from several games, the computer models the height of an average punt with the equation $h(t) = -\frac{16}{3}(t-2.2)^2 + 26.9$, where t is the time in seconds and h(t) is the height in yards. The punter's foot makes contact with the ball when t = 0. Use the graph, table, or equation to answer each question below. Show or explain your reason for each question.

   a. When does the punt reach its highest point? How high does the football go?
   b. Find the zeros of the height function h(t). Which solution is the hang time—that is, the time it takes until the ball hits the ground?
   c. How high is the ball when the punter kicks it?
   d. Graph the equation. Label the vertex, y-intercepts, and the x-intercepts. Then describe the real-world meanings of the vertex, the y-intercept, and the x-intercepts.
   e. Name the form that the height function was given. Find one additional form for the height function and explain what you can read from this form.









The football reaches it maximum height at the vertex of the parabola. This can be read from the equation because it is in vertex form. The vertex is (2.2, 26.9). So at 2.2 seconds the football is at a height of 26.9 feet.

There are two times when the ball is on the ground. These are the zeros of the equation or t = -0.045 seconds or 4.445 seconds. So this indicates that the ball did not start on the ground. If we calculate h(0) we can determine the initial height.

$$h(0) = -\frac{16}{3}(0-2.2)^2 + 26.9$$

$$h(0) = -\frac{16}{3}(2.2)^2 + 26.9 = 1.086 \text{ yards}$$

The height formula was given in vertex form. We can change this to general form by expanding the equation.

$$h(t) = -\frac{16}{3}(t^2 - 4.4t + 4.84) + 26.9$$

$$h(t) = -\frac{16}{3}t^2 + \frac{352}{15}t - \frac{1936}{75} + \frac{269}{10}$$

$$h(t) = -\frac{16}{3}t^2 + \frac{352}{15}t + \frac{163}{150}$$

or

$$h(t) = -5.3t^2 + 23.46t + 1.08$$

8. The rate at which a bear population **grows** in a park is given by the equation $P(b) = \frac{1}{1000}b(100-b)$. The function value P(b) represents the rate at which the population is growing in bears per year, and b represents the number of bears.

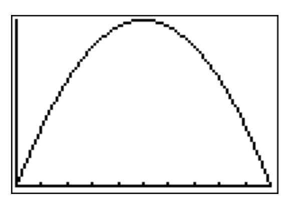
   a. Find P(20) and describe the real-world meaning for this value.
   b. Find the zero of P(b) and describe the real-world meanings for these solutions.
   c. For what size bear population would the population grow fastest? Explain your reasoning.
   d. What is the maximum number of bears the park can support?
   e. Evaluate P(110). Explain the meaning of this value.

P(20) = 1.6 bears per year or the rate at which bears are increasing when there are 20 bears.

$$P(b) = 0 = \frac{1}{1000}b(100-b)$$

$$b = 0 \text{ or } b = 100$$

This means that when the bears of zero or the bears are 100 in number the rate of grow of the bears will be 0 bears per year.



The graph show the rate of change of the bears. It is greatest at the vertex or at the time when there are 50 bears. The vertex is (50, 2.5) or the bears are growing at a rate of 2.5 bears per year when there are 50 bears.

The greatest number of bears that park can support is 100 because when the number of bears is 199 there is not growth rate.

P(110) = -1.1 bears per year, so the number of bears is decreasing when there are more than 100 bears.

9. A shopkeeper is redesigning the rectangular sign on her store's rooftop. She wants the largest area possible for the sign. When she considers adding an amount to the length, she subtracts that same amount from the width. Her original sign has width 8 m and length 3 m.

Increase (x) (m)	**Length (m)**	**Width (m)**	**Area (m^2)**	**Perimeter (m)**
0	3	8	24	22
.5	3.5	7.5	26.25	22
1.0	4	7	28	22
1.5	4.5	6.5	29.25	22
2.0	5	6	30	22
2.5	5.5	5.5	30.25	22
3	6	5	30	22

a. Complete the table.
b. How do the changes in width and length affect the perimeter?
c. How do the changes in width and length affect the area?
d. Write an equation in factored form for the area A(x) of the rectangle in terms of x, the amount she adds to the width.
e. Explain the meaning of the zeros for the equation you wrote in part d.
f. What are the dimensions of the rectangle with the largest area? Explain how you determined these dimensions.
g. Rewrite the area A(x) in vertex form.
g. Rewrite the equation for A(x) from part d in general form. Explain the meaning of A(0).

There perimeter is unchanged because what we are subtracting from the length we are adding to the width so the perimeter remains 22 meters.

The changes in the length and width cause the area to begin increasing, reach a maximum area, and then begin decreasing.

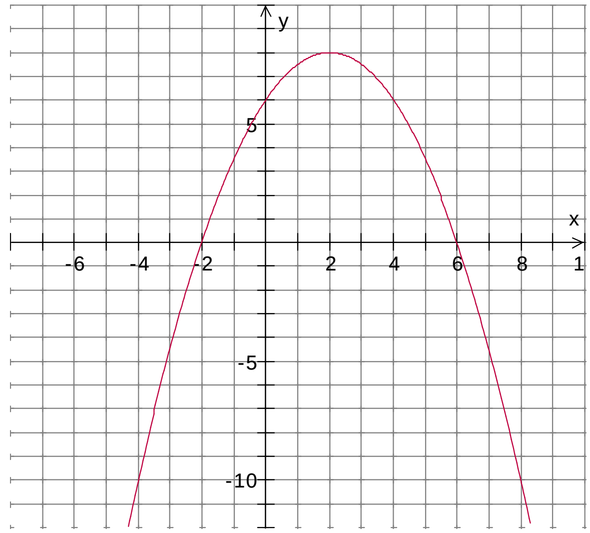
$A(x) = (3 + x)(8 - x)$ The zeros of this equation are x = 8 and x = -3. Since we are not subtracting from the length (x=-3) the only zero that makes sense in this problem would be x = 8. This would create a rectangle that measure 11 x 8. It would have a perimeter of 22 and an area of zero.

The vertex of the parabola is at (2.5, A(2.5)) or when 2.5 is added to the length the area will be 30.25 square meters. $A(x) = -(x - 2.5)^2 + 30.25$

$$A(x) = -(x - 2.5)^2 + 30.25$$
$$A(x) = -(x^2 - 5x + 6.25) + 30.25$$
$$A(x) = -x^2 + 5x + 24$$

A(0)=24 which is the area of the original sign.

10. Demonstrate, using the graph at the right, that you can write the equation of the parabola in three forms: factored form, vertex form, and general form. Explain how various points from the parabola corresponding values in each equation. Use the calculator to demonstrate that all three forms are equivalent.



Using the zeros of the parabola, it might be thought that the equation of the parabola is $y = -(x+2)(x-6)$, but this equation has a vertex at at (2,16) and not (2,8). So the parabola must be shrunk by a factor of ½. So the equation would be $y = -\frac{1}{2}(x+2)(x-6)$.

Expanding this form gives us the general form:

$$y = -\frac{1}{2}(x+2)(x-6)$$
$$y = -\frac{1}{2}(x^2 - 4x - 12)$$
$$y = -\frac{1}{2}x^2 + 2x - 6$$

Using the vertex we can write the equation for the parabola as $y = -\frac{1}{2}(x-2)^2 + 8$

This form can also be expanded to create the general form:

$$y = -\frac{1}{2}(x-2)^2 + 8$$
$$y = -\frac{1}{2}(x^2 - 4x + 4) + 8$$
$$y = -\frac{1}{2}x^2 + 2x - 2 + 8$$
$$y = -\frac{1}{2}x^2 + 2x + 6$$

X	Y1	Y2
-3	-4.5	-4.5
-2	0	0
-1	3.5	3.5
0	6	6
1	7.5	7.5
2	8	8
3	7.5	7.5

Y1=-4.5

X	Y2	Y3
-3	-4.5	-4.5
-2	0	0
-1	3.5	3.5
0	6	6
1	7.5	7.5
2	8	8
3	7.5	7.5

Y3=-4.5

The tables show that all three equations have the same sets of values for the integer x values from -3 to 3.