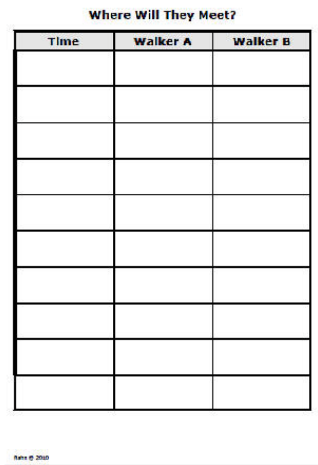
What Does It Mean to Solve a System of Equations?

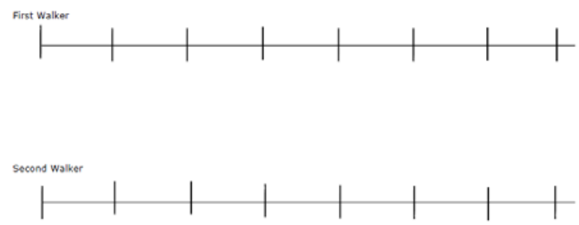
In this activity you will solve a system of simultaneous equations to find the time and distance at which two walkers meet. Suppose that two people begin walking in the same direction at different average speeds. The 1st walker starts behind the slower 2nd walker. When and where will the faster walker overtake the slower walker?

- Mark a 6 m segment at 1 m intervals.
- Have the 1st Walker start at 0.5 meters and walks toward the 6 m mark at a speed of 1 m/s. Have the 2nd walker start at the 2 m mark and walk toward the 6 m mark at 0.5 m/s.
- Have a timekeeper count 1 second counts out loud. (This does not have to be in real time.)
- Have a recorder record both the time and position of the first walker and second walker on a communicator using the "Where Will They Meet?" template.


Where Will They Meet?

Time	Walker A	Walker B

- In addition you can distribute copies of the Path for Two Walkers template and have students keep tract of the position of the walkers on their own template.



Path for Two Walkers

- What does the data in the chart illustrate? Is this same data illustrated by the walkers?
- Enter the data in the lists of the graphing calculator.
  - L1 = time
  - L2 = position of 1st walker
  - L3 = position of 2nd walker

  Create two scatter plots: one for walker 1 (L1 vs. L2) and one for walker 2(L1 vs. L3).
- Trace along scatter plot 1. What do the values along this graph represent?
- Trace along scatter plot 2. What do the values along this graph represent?
- Trace along the plots to see where the two sets of data intersect. What is the meaning of this point?

- When you collect a data chart on the time and distance for two walkers and observe that for some time, both distance columns contain the same number, what does this mean?

- When you create two scatter plots that represent the distance traveled by two walkers and the graphs intersect at one point what does it mean?

- If y = distance the walker is from the starting line and x = time walked, write an equation that fits each walkers data.

  $y_{walker1}$ = ____________________

  $y_{walker2}$ = ____________________

  We call these two equations a system of equations because they are describing two different situations that relate the same two variables.

- Enter the equations into the graphing calculator. Enter $y_{walker1}$ in y1 and $y_{walker2}$ in y2. Create the graph of these equations with the data. How does the data and the lines compare? How is the line different from the data?
- Turn off the scatter plots. Re-graph the lines.
- Trace along line y1. What do these points on the graph represent?
- Trace along line y2. What do these points on the graph represent?
- Find the approximate point where the lines intersect. Explain the real-world meaning of the intersection point.
- Check that the coordinates of the point of intersection satisfy both of your equations.
- When you create two graphs that represent the distance traveled by two walkers and the graphs intersect at one point, what does it mean?
- Create a set of table values for y1 and y2 using the table feature of the calculator. Set table start = 0 and Δtbl=1. Does this table of value look familiar? Find the point of intersection using the table.
- How is the point of intersection found using a table of values?
- How is the point of intersection found using a scatter plot for the collected data?
- How is the point of intersection found using the graph of two equations?
- Explain in your own words what the meaning of the point of intersection is for set of table values, scatter plot, or graph of two equations.

Questions to think about:

- Suppose that walker 1 walks faster than 1 m/s. Select a new rate of change that is faster and change the equation for walker 1 to reflect this new rate of change. Describe how the graph is different. Locate the point of intersection for this new situation using the graph and table. What happens to the point of intersection?

- Suppose that two people walk at the same speed and direction from different starting marks. Change the equations to reflect the rate of change of both walkers so they are equal. Check both the graph and table for this new situation. What does the table tell you? What does the graph tell you? What happens to the point of intersection?

- Suppose that two people walk at the same speed in the same direction from the same starting mark. Change the equations to reflect that each walker is walking at the same rate of change and starting at the same location. Change the type of line for y2 to a bold line. What does this graph of this situation look like? How many points satisfy this system of equations?

Creating tables and graphs can often take a lot of time and produce solutions that are only approximate points of intersection. Let's explore one more way to solve a system of equations. Return to the original system of equations.

y_{walker1} = ____________________

y_{walker2} = ____________________

Notice that each equation describes the distance of each walker from the starting point.

- At the point of intersection, what will be true about the distance of each walker from the starting point?
- Since the two distances are equal then the expressions that represent those two distances must also be equal. Equate the two expressions for the distance of each walker from the starting point.

- By creating this new equation you have eliminated one of the variables in these equations. Complete the solving of this new equation for the value of x.

- What is the significance of this value of x?
- How far has each walker walked for this value of x?
- You have just found the solution to the system of equations by working with the equations analytically or algebraically rather than using tables and graphs.