Exploring Permutations and Combinations

Do Now

Problem 1: Justin in forming a team of people to work on the upcoming dance. Four students: Abby (A), Bailey (B), Chris (C), and Dallas (D) have volunteered to serve on the committee. Justin has decided that one person will be the chairperson, one the secretary, and finally the third person will be the treasurer. If Justin wants to make up committees of three persons from the four volunteers, how many committees can Justin form? Use the letters A, B, C, D to show all the various committees of three students he can form from these four students.

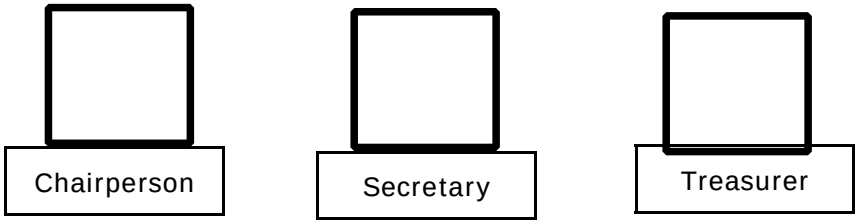
Problem 2: Justin is forming a team of people to work on the upcoming dance. Four students: Abby (A), Bailey (B), Chris (C), and Dallas (D) have volunteered to serve on the committee of three, where everyone will share the duties to see that the dance takes place. If Justin wants to make up committees of three persons from the four volunteers, how many committees can Justin form? Use the letters A, B, C, D to show all the various committees of three students he can form from these four students.

Discussion of Do Now and Lesson

Discuss how students completed each question. Discuss why there are more solutions to the problem 1 and less solutions to the problem 2.

As students talk about the first solution ask students to explain how they systematically listed all the possible ways the committees could be formed.

Ask students if each of these boxes represented a position on the committee, and you had four students to pick from, how many choices would you have for the position of chairperson?



Lead students to saying that there are 4 choices for chairperson, 3 choices for secretary, and 2 choices for treasurer. Ask students if they can determine how these number of choices can lead to the total number of ways to forming these committees of chairperson, secretary, and treasurer from these four persons.

Using the systematic lists produced by the students lead students to seeing that every answer in problem 2 matches with one of the columns from the answers to problem 1. If each column in problem 1 matches with one column in problem 2 ask students how they

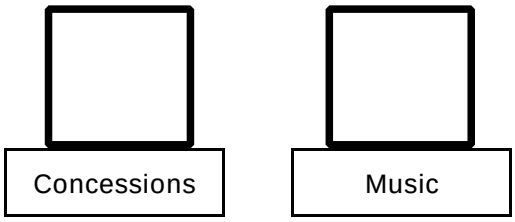
could determine the answer to problem 2 from problem 1. Lead students to say they must divide by 6 since every six in each column of problem 1 end up being the same as one of the columns in problem 2. This should help them see why there are less answers to the second problem.

Inform students that we call the first problem an example of a permutation and the second problem an example of a combination. Show students that we write the first problem as the ${}_4P_3$ or the permutation of 4 students, 3 at a time. In addition show students that we write the second problem as the ${}_4C_3$ or the combination of 4 students, 3 at a time.

Use these next two problems to see if students can see the difference between a permutation and a combination. In problem 1 lead students to see that each position on the committee has a name so this makes switching the order of the letters count as another solution. We say that this means order matters. In problem 2 lead students to see that as long as the same two people are on the committee it is the same committee. This means order does not matter. Lead students to see that in a permutation the order of the letters matters and in a combination the order does not matter.

Justin in forming a team of people to work on the upcoming dance. Four students: Abby (A), Bailey (B), Chris (C), and Dallas (D) have volunteered to serve on the committee. Justin has decided that one person will be the Concessions Chair and second person will be Music Chair. If Justin wants to make up committees of two persons from the four volunteers, how many committees can Justin form? Use the letters A, B, C, D to show all the various committees of two students he can form from these four students.

Lead students to see that you have 4 choices for concessions chair and then 3 choices for music chair. How can these two numbers lead to the total number of ways 4 things can be placed into groups of 2?



Justin is forming a team of people to work on the upcoming dance. Four students: Abby (A), Bailey (B), Chris (C), and Dallas (D) have volunteered to serve on the committee of two, where everyone will share the duties to see that the dance takes place. If Justin wants to make up committees of two persons from the four volunteers, how many committees can Justin form? Use the letters A, B, C, D to show all the various committees of three students he can form from these four students.

Students should see that every answer to problem 2 matches with 2 answer in problem 1 so the answer to problem 2 is less than problem 1 or $\frac{12}{2} = 6$.

- Ask students to read the next question and decide if it is a permutation or combination.
- For each problem have students write the permutation or combination for each. Do not calculate the permutations or combinations.
- Review finding the some of the permutations by use the fill in the box method.
- Model for students how they can find permutation and combinations on the graphing calculator. Ask students to find the answers to all the questions using the graphing calculator.

1. How many ways are there for Alice, Bob, and Carol, to line up at the box office at the movies? (A, B, C), (A, C, B), (C, A, B), (C, B, A), (B, C, A), (B, C, A), P, ${}_3P_3 = 6$

2. How many different committees of 8 people can be formed from a freshman class of 25 students? C, ${}_{25}C_8$

3. How many different ice-cream cones of three flavors can be formed at Baskin Robbins, where they claim to have 28 flavors? A cone that was made with vanilla, then chocolate and finally strawberry is different from a cone that was made in the reverse order. P, ${}_{28}P_3 = 19,656$

4. How many different ice-cream cones of three flavors can be formed at Baskin Robbins, where they claim to have 28 flavors? A cone that was made with vanilla, then chocolate and finally strawberry is different from a cone that was made in the reverse order. P, ${}_{28}C_3$

5. How many different ice-cream cones of four flavors can be formed at Baskin Robbins, where they claim to have 28 flavors? A cone that was made with vanilla, then chocolate, then black walnut and finally strawberry is same no matter what order the flavors are placed on the cone. C, ${}_{28}C_4$

6. How many ways are there to arrange the four letters in the word MATH? P, ${}_4P_4 = 24$

7. If 6 people are running in a race, how many possible ways can they come in if there are no ties and everyone finishes the race? P, ${}_6P_6 = 720$

8. At a restaurant, how many ways can you select three different side dishes from eight possibilities? C, ${}_8C_3$

9. Some states have license plates with five numbers. If those states do not want to use the number 0 on their license plates because it is confused with the letter O, how many different plates are possible? P, ${}_9P_5 = 15,120$

10. You are packing a suitcase for vacation. You have 12 shirts to choose from. How many different grouping of 4 shirts can you make from the 12 shirts? C, ${}_{12}C_4$

11. A five-digit number of the form 5*abc*6 has a thousands digit *a*, hundreds digit *b*, and

tens digit *c*. How many different numbers can be made if no duplicate digits in the number? P, ${}_{10}P_3 = 720$

12. Patti has one copy of each of the six Harry Potter books. How many different ways can Patti place these six books on her book shelf? P, ${}_6P_2 = 30$

13. Patti has one copy of each of the six Harry Potter books. How many different ways can Patti package these book two at a time? C, ${}_6C_2$

14. A prize of two different CD's has been announced. In a class of 25 students how many different pairs of students can be receive these two CD's? P, ${}_{25}C_2 = 300$