

## Understanding the Composition of Functions

Do Now: Distribute copies of the time vs. radius graph and radius vs. area graph to students. Ask students to read various information off the two graphs.

Using the time vs. radius graph, what is the radius when the time is 2 seconds, 4 seconds, and 6 seconds?

Using the same graph, at what time will the radius be  $1\frac{1}{4}$  km,  $1\frac{3}{4}$  km, and 3 km?

Using the radius vs. area graph, what is the area when 1 km,  $1\frac{1}{2}$  km, and 3 km?

Using the radius vs area graph, about how big is the radius when the area is 1 sq km and 2 sq km?

Lesson: Ask students to enter two equations in their graphing calculator:  $y = 2.5x + 1$  and  $y = \pi x^2$ . The first equation describes the radius as a function of time. The second equation describes the area of the circle as a function of radius. Again ask students to find the radius at various time by using the 2<sup>nd</sup> Calc 1. Value Command. ( 2 seconds, 4 seconds, and 6 seconds). Ask students to record the radius for each time.

Time	Radius
2 sec	
4 sec	
6 sec	

Ask students to use the 2<sup>nd</sup> Calc 1. Value command to find the area at each of these radii. Record their values in the chart.

Radius	Area

Using substitution we can find a function that describes the area of the circle as a function of time.

If  $y_1 = f(x)$  describes the radius of the circle as a function of time and  $y_2 = g(x)$  describes the area as a function of radius, then  $y_3 = g(f(x))$  describes the area of the circle as function of time.

Enter  $y_3 = \pi (.25x + 1)^2$ ,  $y_4 = y_2(.25x + 1)$  or  $y_5 = y_2(y_1)$ . Turn off  $y_1$  and  $y_2$ . Press zoom 0 fit. Have students observe that all three graphs are the same graph. Read values for time 2, 4 and 6 seconds. Notice that the values that result are the area at the various times. (Previously calculated.)

Summarize what  $y_1$ ,  $y_2$  and  $y_3$  express.

Introduce two new functions:  $y_1 = f(x) = 3x - 2$  and  $y_2 = g(x) = |x|$ .

Ask students to look at each graph on their graphing calculator. Ask students to first place  $x = 2$  in  $f(x)$  by calculating the value of  $f$  at  $x = 2$ . Then place the result in  $g$ . Do this with several values to see what the results will be.

Now let's form the new function that you have been working with:  $g(f(x))$ .

Enter  $y_3 = y_2(y_1)$ ,  $y_4 = y_2(3x - 2)$  and  $y_5 = |3x - 2|$ .

Explore placing a number in  $y_2$  first and then taking the result and placing it in  $y_1$ . Can you describe what is happening. Can you predict the graph of  $y_1(y_2(x))$ ?