The Painted Cube Problem

Jose bought a 1 x 1 inch cube, a 2 x 2 inch cube, and a 3 x 3 inch unpainted cube. He painted each cube green and then cut the cube into one-inch cubes. In each cube how many of these cubes had the following:

0 faces painted?
1 face painted?
2 faces painted?
3 faces painted?
4 faces painted?
5 faces painted?
6 faces painted?

Build each of the cubes: 1 x 1 inch cube, a 2 x 2 inch cube, and a 3 x 3 inch.

Use various colors to represent how many sides will be painted. Use one color if the cube has no faces painted, another color if 1 face painted, another color if 2 faces are painted, etc.

Collect the data on the chart on the next page for the first three cubes: 1 x 1 x 1, 2 x 2 x 2, and 3 x 3 x 3.

Build a 4 x 4 x 4 cube using the same color scheme you chose to build the first three cubes. Complete the table for number of painted sides for the 4 x 4 x 4 cube.

Begin to look for any patterns in the table.

Describe any patterns you see in the table.

Try to extend the table to 5 x 5 x 5 and 6 x 6 x 6 without building the cubes.

	Size of cube						
	1 x 1 x 1	2 x 2 x 2	3 x 3 x 3	4 x 4 x 4	5 x 5 x 5	6 x 6 x 6	n x n x n
Total Cubes →							
Painted Faces ↓	Number of Cubes						
0							
1							
2							
3							
4							
5							
6							

Let's analyze the chart. Start with the rows that indicate the number of cubes with 0 painted faces.

- What do you notice about each of the numbers in the zero row?
- Where are these cubes located? Does this explain their number?
- How could each be expressed as a cube?
- What relationship do you notice between the number being cubed and the dimension of the corresponding cube?
- Describe how you would determine the number of unpainted, 1-inch cubes in a 10x10x10 inch cube.
- Write an expression to represent the number of unpainted cubes in a cube whose face is n units.

Move to the row representing the number of cubes that have 1 painted face.

- What do you notice about each of the numbers in this row?
- Where are these cubes located? Does this explain their number?
- How could each be expressed as a factor of 6?
- What do you notice about each of the other factors in each term?
- Write these factors as a perfect square.
- What relationship do you notice between the number being squared and the dimension of the corresponding cube?
- Describe how you would determine the number of 1-inch cubes with one painted face in a 10x10x10 inch cube.
- Write an expression to represent the number of cubes that have exactly one face painted in a in a cube whose face is n units.

Move to the row representing the number of cubes that have 2 painted faces.

- What do you notice about each of the numbers in this row?
- Where are these cubes located? Does this explain their number?
- How could each be expressed as a factor of 12?
- What relationship do you notice between the second factor and the dimension of the corresponding cube?
- Describe how you would determine the number of 1-inch cubes with two painted faces in a 10x10x10 inch cube.
- Write an expression to represent the number of cubes that have exactly two one faces painted in a cube whose face is n units.

Move to the row representing the number of cubes that have 3 painted faces.

- What do you notice about each of the numbers in this row?
- Where are these cubes located? Does this explain their number?
- Why do you think all the numbers are eight?
- Are there any cubes that have more or less than 8 vertices?
- How many cubes with 3 painted faces would be in a cube with dimensions n?.

What do you notice about these rows? Why is this?

What do you notice about the column of numbers?

Show that this also works with the n x n x n column?