2007 AB6

A.

$f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$

$= \frac{k\sqrt{x} - 2}{2x}$

and

$f''(x) = -\frac{k}{4}x^{-3/2} + x^{-2}$

$= -\frac{k}{4x^{3/2}} + \frac{1}{x^2} = \frac{-kx^{1/2} + 4}{4x^2}$

B.

First Derivative Test

A critical point occurs when f ' (x)=0 or is undefined. f '(x) is never undefined.

f '(x)=0 when $k\sqrt{x} - 2 = 0$

$k = \frac{2}{\sqrt{x}}$

So if a critical value occurs when x = 1 then k = 2. To the left of x = 1 f '(x)<0 (so f is decreasing)and to the right of x = 1 f '(x) >0 (so f is increasing). Therefore these is a minimum at x = 1.

Second Derivative Test

Since f'(1)=0 and $f''(1) = \frac{1}{2} > 0$ so f has a minimum at x = 1.

C. The function f has a zero when f(x)=0 or $k\sqrt{x} - \ln x = 0$. This means $\ln x = k\sqrt{x}$ or $k = \frac{\ln x}{\sqrt{x}}$. f "(x) changes sign when changes sign. This means $\frac{-kx^{1/2} + 4}{4x^2}$ must change sign.

Combining the two yields $\frac{-\left(\frac{\ln x}{\sqrt{x}}\right)x^{1/2} + 4}{4x^2}$. The numerator changes sign when the numerator equals zero. $-\left(\frac{\ln x}{\sqrt{x}}\right)x^{1/2} + 4 = 0$ or lnx = 4 or $x = e^4$. So $k = \frac{\ln e^4}{\sqrt{e^4}} = \frac{4}{e^2}$