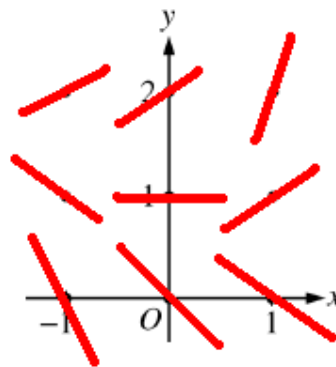


2007 Form B AB5

A.

coordinate	slope
(1,0)	$-\frac{1}{2}$
(1,1)	$\frac{1}{2}$
(1,2)	-1
(0,1)	0
(-1,0)	$-\frac{3}{2}$
(-1,1)	$\frac{1}{2}$
(-1,2)	$\frac{1}{2}$



B.

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{1}{2} + \frac{dy}{dx} \\ &= \frac{1}{2} + \frac{1}{2}x + y - 1 \\ &= -\frac{1}{2} + \frac{1}{2}x + y \end{aligned}$$

For f to be concave up the second derivative of f must be positive therefore

$$\begin{aligned} \frac{d^2y}{dx^2} &= -\frac{1}{2} + \frac{1}{2}x + y > 0 \\ y &> \frac{1}{2} - \frac{1}{2}x \end{aligned}$$

This describes a half plane above the line

$$y = \frac{1}{2} - \frac{1}{2}x$$

C. We know that $f(0)=1$. We know to the left of this point the function is decreasing and to the right of this point the function is increasing (based upon the slopefield.) Therefore the function f has a relative minimum at the point $(0,1)$.

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Alternatively: Since we know that the point $(0,1)$ is in this half plane so the second derivative at $(0,1)$ is positive. We can also see that at this point $\frac{dy}{dx} = 0$. So this tells us that there is a minimum at $(0,1)$.

D. If $y = mx + b$ then $\frac{dy}{dx} = m$.

Substituting these in the differential equation yields:

$$m = \frac{1}{2}x + (mx + b) - 1$$

$$\left(m + \frac{1}{2}\right)x + (b - m - 1) = 0$$

For this to be true

$$m + \frac{1}{2} = 0 \quad \text{and} \quad b - m - 1 = 0$$

$$m = -\frac{1}{2} \quad \text{and} \quad b + \frac{1}{2} - 1 = 0 \quad \text{or} \quad b = \frac{1}{2}$$

Alternatively:

$$\frac{d^2y}{dx^2} = -\frac{1}{2} + \frac{1}{2}x + y = 0$$

$$y = \frac{1}{2} - \frac{1}{2}x$$

This says $m = -\frac{1}{2}$ and $b = \frac{1}{2}$.

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