2007 Form B AB6

A. $f_{avg} = \dfrac{f(5) - f(2)}{5-2} = \dfrac{2-5}{5-2} = -1$

Since the average rate of change of f between x = 2 and x = 5 is -1, by the MVT there must be a location c such that 2<c<5 where $f_{avg} = f'(c) = -1$.

B.

$$g'(x) = f'(f(x)) \bullet f'(x) \qquad g'(x) = f'(f(x)) \bullet f'(x)$$
$$g'(2) = f'(f(2)) \bullet f'(2) \quad \text{and} \quad g'(5) = f'(f(5)) \bullet f'(5)$$
$$= f'(5) \bullet f'(2) \qquad = f'(2) \bullet f'(5)$$

Since g'(2)=g'(5), then $\dfrac{g'(2) - g'(5)}{2-5} = g'_{avg} = 0$. By the MVT there must exist a 2<c<5 such that $g'_{avg} = g''(c) = 0$.

C.

$$g''(x) = f'(f(x)) \bullet f''(x) + f'(x) \bullet f''(f(x)) f'(x)$$
$$= f'(f(x) \bullet f''(x) + (f'(x))^2 \bullet f''(f(x))$$
$$= f'(f(x) \bullet 0 + (f'(x))^2 \bullet 0$$
$$= 0$$

For g to have a point of inflection g" must change sign. But since we know that g" equals zero it cannot change sign. Therefore these is no point of inflection.

D.
$$h(5) = f(5) - 5 = 2 - 5 = -3$$
$$h(2) = f(2) - 2 = 5 - 2 = 3$$

Therefore, by the IVT we know that there exists a 2<r<5 such that h(r)=0.

