

AP Calculus – Final Review Sheet

When you see the words	This is what you think of doing
1. Find the zeros of a function.	Set the function equal to zero and solve for x.
2. Find equation of the line tangent to $f(x)$ at $(a, f(a))$.	Find $f'(x)$, the derivative of $f(x)$. Evaluate $f'(a)$. Use the point and the slope to write the equation: $y = f'(a)(x-a) + f(a)$
3. Find equation of the line normal to $f(x)$ at $(a, f(a))$.	Find $f'(x)$, the derivative of $f(x)$. Evaluate $f'(a)$. The slope of the normal line is $-\frac{1}{f'(a)}$. Use the point and the slope to write the equation:
4. Show that $f(x)$ is even.	Evaluate f at $x = -a$ and $x = a$ and show they are equal.
5. Show that $f(x)$ is odd.	Evaluate f at $x = -a$ and $x = a$ and show they are opposite.
6. Find the interval where $f(x)$ is increasing.	Find $f'(x)$ and find all intervals in the domain of f and f' where $f'(x) > 0$.
7. Find the interval where the slope of $f(x)$ is increasing.	Find $f''(x)$ and find all intervals in the domain of f , f' , and f'' where $f''(x) > 0$.

8. Find the relative minimum value of a function $f(x)$.	Find all the critical points for f , where $f'(x)=0$ or $f'(x)$ does not exist. Find all locations where f' changes from negative to positive or where f changes from decreasing to increasing.
9. Find the absolute minimum slope of a function $f(x)$ on $[a,b]$.	Find all critical points of f' , where $f''(x)=0$ or $f''(x)$ does not exist. Evaluate $f'(x)$ at all critical points of f' and the endpoints. From these values find where f' is minimum.
10. Find critical values for a function $f(x)$.	Find $f'(x)$ and then locate all points where $f'(x)=0$ or $f'(x)$ does not exist.
11. Find inflection points of a function $f(x)$.	Find $f''(x)$ and then find all locations where $f''(x)$ changes sign.
12. Show that $\lim_{x \rightarrow a} f(x)$ exists.	Find $\lim_{x \rightarrow a^+} f(x)$ and $\lim_{x \rightarrow a^-} f(x)$ and show they are equal.
13. Show that $f(x)$ is continuous.	Show that $\lim_{x \rightarrow a} f(x)$ exists and that $\lim_{x \rightarrow a} f(x) = f(a)$
14. Show that a piecewise function is differentiable at the point a where the function rule splits such as $h(x) = \begin{cases} f(x) & \text{for } x \leq a \\ g(x) & \text{for } x > a \end{cases}$	Find $f'(x)$ and $g'(x)$. Then show that $\lim_{x \rightarrow a^-} f'(x) = \lim_{x \rightarrow a^+} g'(x)$.
15. Find vertical asymptotes of a function $f(x)$.	Look at the definition of the function $f(x)$. If f is written in a ratio, first check that the function cannot be simplified. Then locate all places where the denominator of the function equals zero.

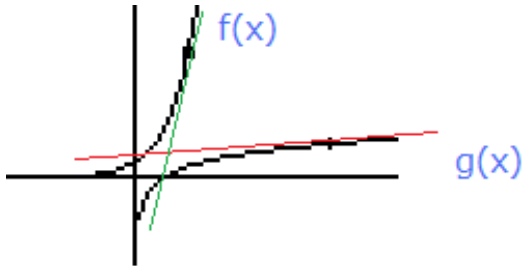
16. Find horizontal asymptotes of function $f(x)$.	Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$. If either of these limits exists then the function has at least one or two horizontal asymptotes. Their form would be $y = k$ where k is the limit.
17. Find the average rate of change of $f(x)$ on $[a,b]$.	This is the slope of the secant line between $(a, f(a))$ and $(b, f(b))$ or $\frac{f(b) - f(a)}{b - a}$.
18. Find instantaneous rate of change of $f(x)$ on $[a,b]$.	This is another name for $f'(a)$, or the derivative the function evaluated at $x = a$.
19. Find the average value of $f(x)$ on $[a,b]$.	This means to find the average value that f takes on between $(a, f(a))$ and $(b, f(b))$. It is found by find the area of the function bounded by $x=a$, $x=b$, $x=0$, and $y=f(x)$. Then divide this by the width of the interval $b-a$. It is written as $\frac{\int_a^b f(x) dx}{b - a}$.
20. Find the absolute maximum of $f(x)$ on $[a,b]$.	Find all the critical points for f , where $f'(x)=0$ or $f'(x)$ does not exist. Evaluate the function at all critical points of f and endpoints. From these values find where f is maximum.
21. Show that a piecewise function is differentiable at the point a where the function rule splits	Find the derivative of each piece of the function. Show that the $\lim_{x \rightarrow a} f'(x)$ exists or is equal from the left and the right.
22. Given $s(t)$, the position function, find $v(t)$, the velocity function.	Find the derivative of $s(t)$.

<p>23. Given $v(t)$, the velocity function, find how far a particle travels on $[a,b]$.</p>	<p>Evaluate $\int_a^b v(t) dt$. Remember that $\int_a^b v(t)dt$ only finds the net distance traveled.</p>
<p>24. Find the average velocity of a particle on $[a,b]$ given $s(t)$, the position function.</p> <p>Find the average velocity of a particle on $[a,b]$ given $v(t)$, the velocity function.</p>	<p>The average velocity of a particle, given $s(t)$, is the slope of the secant line: $\frac{s(b) - s(a)}{b - a}$.</p> <p>The average velocity of a particle, given $v(t)$, is the same as finding the average value of a</p>
<p>25. Given $v(t)$, the velocity function, determine the intervals where a particle is speeding up.</p>	<p>Evaluate $v(t)$ for its sign. Find the derivative of $v(t)$ to determine $a(t)$. Determine when the particle is stationary ($v(t)=0$). Determine when $a(t)=0$. Study the intervals where the particle is initially at rest and then shows positive or negative velocity, which means it will move left or right. The particle will have to speed up until it reaches point where $a(t)=0$. Locate the point where the particle will have an $a(t)=0$. (Now it will begin to slow down and eventually come to rest again.)</p>
<p>26. Given $v(t)$, the velocity function, and $s(0)$, the initial position, find $s(t)$, the position function as a function of t.</p>	<p>To write $s(t)$ you will need to write a function using an integral: $s(t) = s(0) + \int_0^t v(x)dx$</p>
<p>27. Show that Rolle's Theorem holds for a function $f(x)$ on $[a,b]$.</p>	<p>Verify that $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b). Verify that $f(a)=0$ and $f(b)=0$. Then you are guaranteed that there exists a point c ($a < c < b$) where $f'(c)=0$.</p>

<p>28. Show that the Mean Value Theorem holds for a function $f(x)$ on $[a,b]$.</p>	<p>Verify that $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b). Then you are guaranteed that there exists a point c ($a < c < b$) where</p> $f'(c) = \frac{f(b) - f(a)}{b - a}$
<p>29. Find domain of $f(x)$.</p>	<p>Analyze the function f. Look for radical expressions in the description. Determine values of x that cannot be used within the radical. Exclude these from the domain. Look at the denominator. If the denominator contains a polynomial, find the zeros for this polynomial and exclude these x values from the domain.</p>
<p>30. Find range of $f(x)$ on $[a,b]$.</p>	<p>If f is continuous on $[a,b]$, then the range of f will be between [minimum value of f, maximum value of f].</p>
<p>31. Find range of $f(x)$ on $(-\infty, \infty)$.</p>	<p>If f is continuous on $(-\infty, \infty)$ then you will need to consider $\lim_{x \rightarrow +\infty} f(x) = k_1$ and $\lim_{x \rightarrow -\infty} f(x) = k_2$. If these limits are above the local maximum or below the local minimum the range will be $[k_1, k_2]$. Otherwise you will have to adjust the range. If the limits go to infinity then the range is $(-\infty, \infty)$.</p>
<p>32. Find $f'(x)$, the derivative of $f(x)$, by definition</p>	<p>Use $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$</p>

33. Given two functions f and f^{-1} are inverse functions ($f(a)=b$ and $f^{-1}(b)=a$) and $f'(a)$, find derivative of inverse function f^{-1} at $x=b$.

Suppose that $g^{-1}(x)=f(x)$ and $g(x)=f^{-1}(x)$. Suppose a tangent line is drawn at (a,b) on the function f . Find the slope of the function g at the point (b,a) .



$$(f^{-1})'(b) = \frac{1}{f'(a)}$$

The slope of the function g at the point (b,a) is simply the reciprocal of the slope of the f function at (a,b) or $g'(b) = \frac{1}{f'(a)}$.

34. Given $\frac{dy}{dt}$ is increasing proportionally to y , find a family of functions that describe the population as a function of time.

First begin with $\frac{dy}{dt} = ky$, then separate the variables, integrate each side and add a constant of integration to one side. Continue to solve for the population as a function of t . If you are told that the graph will pass through a point, substitute this point into the equation to solve for the constant. After replacing the constant continue to write the equation as a function, by considering any restrictions placed on the problem by the point used in the substitution step.

$$\frac{dy}{dt} = ky \Rightarrow \frac{dy}{y} = k dt$$

$$\int \frac{dy}{y} = \int k dt$$

$$\ln|y| = kt + C$$

$$y = e^{kt+C}$$

$$y = Ce^{kt}$$

<p>35. Find the line $x=c$ that divides the area under $f(x)$ on $[a,b]$ to two equal areas.</p>	<p>Find a point c such that $\int_a^c f(x)dx = \frac{\int_a^b f(x)dx}{2}$</p>										
<p>36. $\frac{d}{dx} \int_a^x f(t) dt =$</p>	<p>$f(x)$</p>										
<p>37. Given that u is some function of x find $\frac{d}{dx} \int_a^u f(u) dt =$</p>	<p>$f(u) \frac{du}{dx}$</p>										
<p>38. Find the area bounded by $f(x)$, the x-axis, $x=1$ and $x = 10$ using 3 trapezoids, where $\Delta x=3$.</p>	<p>Find $f(1)$, $f(4)$, $f(7)$, and $f(10)$. Use these for the bases in finding the area of three trapezoids with heights of 3: $\frac{1}{2}(3)(f(1) + f(4)) + \frac{1}{2}(3)(f(4) + f(7)) + \frac{1}{2}(3)(f(7) + f(10))$</p>										
<p>39. Approximate the area bounded by $f(x)$, the x-axis, $x=0$ and $x = 7$ using left Reimann sums from information about $f(x)$ given in tabular data.</p> <table border="1" data-bbox="94 1304 792 1373"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>5</td> <td>7</td> </tr> <tr> <td>y</td> <td>1</td> <td>13</td> <td>16</td> <td>5</td> </tr> </table>	x	0	1	5	7	y	1	13	16	5	<p>Find the base, difference between x values, and height (at left hand end) of the three rectangles. $(1)(1) + (4)(13) + (2)(16)$</p>
x	0	1	5	7							
y	1	13	16	5							
<p>40. Approximate the area bounded by $f(x)$, the x-axis, $x=0$ and $x = 7$ using right Reimann sums from information about $f(x)$ given in tabular data.</p> <table border="1" data-bbox="126 1566 748 1629"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>6</td> <td>7</td> </tr> <tr> <td>y</td> <td>-1</td> <td>-13</td> <td>-16</td> <td>-5</td> </tr> </table>	x	0	1	6	7	y	-1	-13	-16	-5	<p>Find the base, difference between x values, and height (at right hand end) of the three rectangles. $(1)(-13) + (5)(-16) + (1)(-5)$</p>
x	0	1	6	7							
y	-1	-13	-16	-5							

41. Approximate the area bounded by $f(x)$, the x-axis, $x = 0$, and $x = 14$ using two subintervals and midpoint rectangles from information about $f(x)$

x	0	3	6	10	14
y	1	7	12	11	3

Find the intervals for the two rectangles: $(0,6)$ and $(6,14)$. The midpoints are 3 and 10. Find the height of the rectangles: 7 and 11 respectively.

Find the area: $(6)(7) + (8)(11)$

42. Approximate the area bounded by $f(x)$, the x-axis, $x = 0$, and $x = 10$ using three trapezoids from information about $f(x)$ given in tabular data.

x	1	5	6	10
y	2	7	12	15

Find the height of the three trapezoids: 4, 1, and 4. Find the bases: 2 and 7, 7 and 12, and 12 and 15. Find the areas:

$$\frac{1}{2}(4)(2+7) + \frac{1}{2}(1)(7+12) + \frac{1}{2}(4)(12+15)$$

43. Given the graph of $f'(x) > 0$ between $x=0$ and $x = a$ and $f(0) = 8$, find $f(a)$.

$f(a) = f(0) + \int_0^a f'(x) dx$ To find the integral you can find the area under the f' graph between $x=0$ and $x=a$.

44. Find the function $y=f(x)$ whose differential equation is $\frac{dy}{dx} = \frac{1+x}{y}$ if the function passes through the point $(1, -2)$.

Separate the variables and then integrate each side. Remember to include a constant of integration. Replace x and y with the values of the point to find the constant. Solve for y as a function of x keeping in mind the point that the function passes through. (Answer: $y = -\sqrt{x^2 + 2x + 1}$)

45. Describe the meaning of $\int_a^x f(t) dt$

Suppose $f(x)$ is a rate equation for $F(t)$. Then this integral represent the net change in $F(t)$ from time a to time x .

46. Given a base is bounded by $x = a$, $x = b$, $f(x)$ and $g(x)$, where $f(x) < g(x)$ for all $a < x < b$, find the volume of the solid whose cross section, perpendicular to the x-axis are squares.

$$\text{Volume of the solid} = \int_a^b (g(x) - f(x))^2 dx$$

47. Find where the tangent line to $f(x)$ is horizontal.	Find $f'(x)$ and then set $f'(x) = 0$ and solve for x .
48. Find where the tangent line to $f(x)$ is vertical.	Find $f'(x)$ and then analyze $f'(x)$ to determine where $f'(x)$ is undefined because of a denominator.
49. Find the minimum acceleration given $v(t)$, the velocity function.	Find $a(t)$ or the derivative of $v(t)$ and $a'(t)$. Find the critical points for $a(t)$ from $a'(t)$. Find where $a'(t)$ is changing from negative to positive ($a(t)$ changing from decreasing to increasing). These are locations for the local minimum accelerations.
50. Approximate the value of $f(1.1)$ by using the tangent line to f at $x=1$.	Write the tangent line at $x=1$. $y = f'(1)(x - 1) + f(1)$. Use $x = 1.1$ in this tangent line to find the approximate value of $f(1.1)$.
51. Given the value of $F(a)$ and the fact that the anti-derivative of f is F , find $F(b)$.	$F(b) = F(a) + \int_a^b f(x) dx$
52. Find the derivative of $f(g(x))$.	$(f(g(x)))' = f'(g(x)) \cdot g'(x)$
52. Given $\int_a^b f(x) dx$, find $\int_a^b [f(x) + k] dx$	$\int_a^b [f(x) + k] dx = \int_a^b f(x) dx + \int_a^b k dx =$ $\int_a^b f(x) dx + k(b - a)$

<p>53. Given a graph of $f'(x)$, find where $f(x)$ is decreasing.</p>	<p>From the graph of $f'(x)$ find where the graph is below the x-axis. This means $f'(x)$ is negative. Describe these intervals.</p>
<p>54. Given $v(t)$, the velocity function, and $s(0)$, the initial position, find the greatest distance from the origin of a particle on $[0, b]$.</p>	<p>Find when $v(t)$ is zero. This means the function is at rest at these values. Write $s(t)$.</p> $s(t) = s(0) + \int_0^t v(x) dx$ <p>Evaluate $s(t)$ at each place $v(t)$ is zero. Pick out the greatest distance from the origin.</p>
<p>55. Given a water tank with g gallons initially, is being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$, find the amount of water in the tank at m minutes where $t_1 < m < t_2$.</p>	$\int_{t_1}^m (F(t) - E(t)) dt$
<p>56. Given a water tank with g gallons initially, is being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$, find the rate the water amount is changing at m.</p>	$F(t) - E(t)$
<p>57. Given a water tank with g gallons initially, is being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$, find the time when the water is at a minimum.</p>	<p>Differentiate the integral in question 55 with respect to t. This will give you a rate equation or the equation in question 56. Find the zeros for $F(t) - E(t)$. Evaluate the integral from question 55 at these zero's and the endpoints. Pick out the minimum value.</p>
<p>58. Given a chart of x and $f(x)$ on selected values between a and b, estimate $f'(c)$ where c is between a and b.</p>	<p>Use two sets of points $(a, f(a))$ and $(b, f(b))$ near c to evaluate $f'(c) \approx \frac{f(b) - f(a)}{b - a}$</p>

<p>59. Given $\frac{dy}{dx}$, draw a slope field</p>	<p>Identify points on the graph. Name the coordinates of these points. Evaluate $\frac{dy}{dx}$ at these points. Draw a short line that represents the given slope at that point. The slope field should model the slope of a family of functions whose derivative</p>
<p>60. Given that $f(x) < g(x)$. find the area between curves $f(x)$ and $g(x)$ between $x = a$ and $x = b$ on $[a,b]$.</p>	$\int_a^b (f(x) - g(x)) dx$
<p>61. Given that $f(x) > g(x)$. Find the volume of the solid created if the region between curves $f(x)$ and $g(x)$ between $x = a$ and $x = b$ on $[a,b]$. is revolved about the x-axis.</p>	$\pi \int_a^b ((f(x))^2 - (g(x))^2) dx$
<p>62. Find a limit in the form $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$.</p> <p>Find the limit: $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x - 1}$</p>	<p>Determine the value of a and the function f. Differentiate f and evaluate at a.</p> <p>Notice that $f(x) = \cos x$ and that x is approaching 0 so this expression is of the form</p> $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ <p>The limit can be found by taking the derivative of $\cos x$ at the point $x = 0$.</p>
<p>63. Given information about $f(x)$ for x in $[a,b]$, show that there exists a c in the interval $[a,b]$, where</p> $f'(c) = \frac{f(b)-f(a)}{b-a}$	<p>Check to see that $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b). Then the Mean Value Theorem guarantees that there exists a c such that</p> $f'(c) = \frac{f(b)-f(a)}{b-a}$
<p>64. Given $f''(x)$ and all critical values of x in (a,b) where $f'(x)=0$, determine the location of all relative extrema for f.</p>	<p>Check the concavity of f at each critical value where $f'(x) = 0$. If $f''(x) > 0$ you have found the location of a minimum. If $f''(x) < 0$ you have found the location of a maximum.</p>

<p>65. Given $f'(x)$ in graphical form on a domain (a,b), determine the location of all relative extrema for f.</p>	<p>Find locations where the graph of f' is changing from being below the x-axis to being above the x-axis. This is a location of a relative minimum. Find locations where the graph of f' is changing from being above the x-axis to being below the x-axis. This is a location of a relative maximum.</p>
<p>66. Given that functions f and g are twice differentiable, find $h'(x)$ if $h(x) = f(x)g(x) + k$.</p>	<p>$h'(x) = f(x)g'(x) + g(x)f'(x)$</p>
<p>67. Given a differential equation $\frac{dy}{dx} = F(x,y)$ and a lattice of points on a coordinate grid, draw a slope field for the differential equation.</p>	<p>Set up a table for each of the lattice points and calculate the slope at each lattice point. Draw a line segment at each lattice point that has the approximate slope of the function $F(x,y)$ at that point.</p>
<p>68. Given a slope field that represents $\frac{dy}{dx} = F(x,y)$ on the coordinate grid.</p> <ol style="list-style-type: none"> Determine if the slope field is a function of x alone. Determine if the slope field is a function of y alone. Determine the conditions that must be true for $F(x,y)$ to have a slope of zero. Determine if the slope field illustrates special patterns along the x- or y-axes. Determine if the slope field illustrate special patterns in any of the quadrants. Determine if the slope field illustrates special patterns at x gets very large or very small. Determine if the slope field illustrates special patterns at y gets very large. Determine if the slope field illustrates any symmetry. 	<ol style="list-style-type: none"> Look to see if the slopes are the same for each column of x. Look to see if the slopes are the same for each row of y. Look to see where the slope fields are horizontal segments. See if you can describe a pattern in these locations, such as along the line $y=x$. Study the x-axis and y-axis to see if something special is happening along these two lines Study each quadrant of the slope field to see if you see any patterns within each quadrant or between pairs of quadrants. Study the slope field to see if the slopes illustrate a pattern as you move from left to right from the y axis or as you move from right to left from the y-axis. Study the slope field to see if the slopes illustrate a pattern as you move from up from the x-axis or as you move from down from the x-axis. Study the slope field to see if there is any symmetry about either axis or about a special line.

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