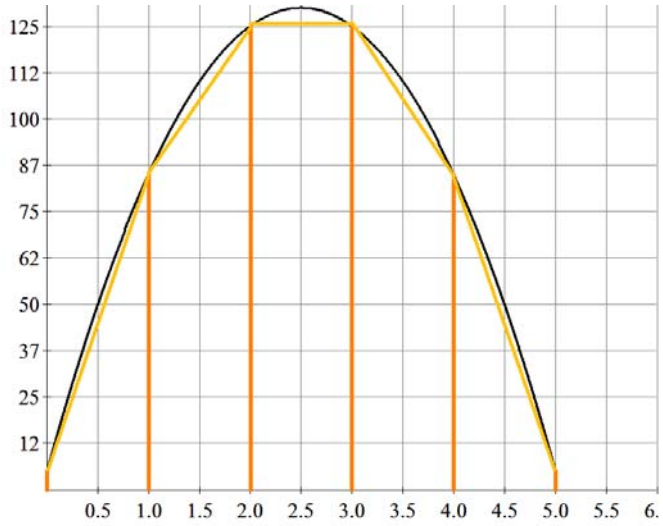
Finding Definite Integrals Another Way

The velocity of a particle, in centimeters per second, is represented by the graph at the right. You are interested in determining how far the particle has traveled during the time interval 0 seconds ≤ t ≤ 5 seconds.

$$v(t) = -20x^2 + 100x + 5$$



Subdivide the interval $0 \le t \le 5$. into 5 equal widths. Draw trapezoids in each interval by connecting two consecutive function values. Find the height of the velocity graph at the endpoints of each interval. Record them in the given table.

Find an approximation for the definite integral of v(t) with respect to t by finding the sum of the area of the four trapezoids.

$$\text{Trapezoids}: \frac{1}{2}(5+85)1 + \frac{1}{2}(85+125)1$$
$$+(125)1 + \frac{1}{2}(85+125)1 + \frac{1}{2}(85+5)1 = 425\ cm$$

What does this definite integral represent? The definite integral represents the distance traveled in centimeters by the particle in 5 seconds.

t	0	1	2	3	4	5
v(t)	5	85	125	125	85	5

Increase the number of intervals to 10. Find a second approximation for the definite integral of v(t) with respect to t by finding the sum of the area of the eight trapezoids.

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
v(t)	5	50	85	110	125	130	125	110	85	50	5

$$\frac{1}{2}(5+50)\frac{1}{2}+\frac{1}{2}(50+85)\frac{1}{2}+\frac{1}{2}(85+110)\frac{1}{2}+\frac{1}{2}(110+125)\frac{1}{2}+\frac{1}{2}(125+130)\frac{1}{2}$$
$$+\frac{1}{2}(130+125)\frac{1}{2}+\frac{1}{2}(125+110)\frac{1}{2}+\frac{1}{2}(110+85)\frac{1}{2}+\frac{1}{2}(85+50)\frac{1}{2}+\frac{1}{2}(50+5)\frac{1}{2}=$$
$437.5\ cm$

What do you notice about your two answers for the definite integral of this velocity from $0 \le t \le 5$.

The answer to the second approximation is closer to the actual definite integral because less area is being left out.

How could you improve your answer for the definite integral of the velocity from $0 \le t \le 5$?

Keep increasing the number of sub-intervals.

Can you approximate the particle's rate of change of velocity when t=5? Was the particle speeding up or slowing down at that time. Explain your reasoning.

Time	Velocity
4	85
5	5

Time	Velocity
4.9	14.8
5	5

Time	Velocity
4.99	5.998
5	5

Time	Velocity
4.999	5.0999
5	5

The rate of change of the velocity based on these charts is changing as the increment is made smaller and smaller: 81 cm/sec/sec, 99 cm/sec/sec, 100.8 cm/sec/sec, and 100.9 cm/sec/sec. Therefore, the approximate rate of change of the velocity is 100.9 cm/sec/sec. Since this rate of change is positive the velocity is speeding up.

At what time is the particle's rate of change in velocity stopped. Explain your reasoning.

This would be at 2.5 seconds. As time intervals near 2.5 seconds are selected and calculated, the rate of change would be zero because the change in velocity would approach zero and the intervals are made smaller and smaller.