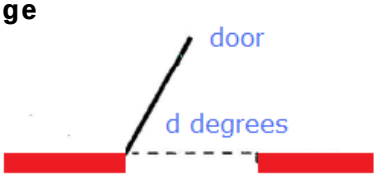
Introducing Instantaneous Rate of Change



The diagram shows a door with an automatic closer.

At time t = 0 seconds someone pushes the door. It swings open, slows down, stops, starts closing, then closes quietly at time t = 7 seconds. As the door is in motion the number of degrees, d, it is from its closed position depends on t.

Sketch a reasonable graph of d versus t.

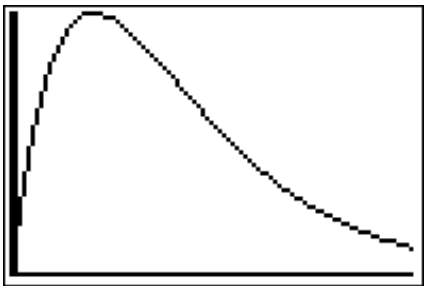
Graphs will vary. But all graphs should first show an increasing graph and then a decreasing graph. Graphs should show a smooth change in direction at the top of the graph.

Compare your graph with a graph drawn by a neighbor. Answers will vary.

What characteristics are possessed by both graphs. Answers will vary.

How do your graphs differ? Answers will vary.

Suppose that d is given by the equation $d = 200t\,(2^{-t})$. Use your calculator to graph this funct ion for $0 \le t \le 7$.



What common characteristics does this graph have to your graph? Answers will vary.

Does your range make sense? Answers will vary. Many students may not have gone above 90 degrees. But most doors open beyond 90 degrees.

Make a table of values of d for each second from t = 0 through t = 10. Round to the nearest 0.10 degree.

Time	Degrees
0	0
1	100
2	100
3	75
4	50
5	31.25
6	18.75
7	10.94
8	6.25
9	3.52
10	1.96

Looking at only the chart, does the door appear to be opening, closing, or standing still at t = 1 second? Explain your reasoning. It appears to be open for 1 second.

Using the graph, does the door appear to be opening, closing, or standing still at t = 1 second? Explain your reasoning. With the graph you can see at 1 second it was in the process of opening since the graph is increasing.

During the time interval [0, 1] seconds, what is the average rate of change of the door's angle? $\frac{100-0}{1-0} = 100$ degrees per second

Create a table of values for the angle of the door's opening over the interval [0, 1] seconds with steps of 0.1, what is the average rate of change of the angle of door.

Time	Degrees
0	0
0.1	18.68
0.2	34.82
0.3	48.74
0.4	60.63
0.5	70.71

Time	Degrees
0.6	79.17
0.7	86.18
0.8	91.90
0.9	96.46
1.0	100

During the time interval [0.9, 1] seconds, what is the average rate of change of the door's angle? $\frac{100-96.46}{1-0.9} = 35.4$ degrees per second

How does this average rate of change compare with the average rate of change on the interval [0,1]? Does this make sense? Yes, the segment appears to have less slope than the last interval, therefore, the slope is less.

Are you surprised by the amount of difference? Explain your reasoning. The time period has decreased so the number of degrees changed has also decreased.

Create a table of values for the angle of the door's opening over the interval [0.9, 1] seconds with steps of 0.01, what is the average rate of change of the angle of door.

Time	Degrees
0.9	96.46
0.91	96.76
0.92	97.25
0.93	97.62
0.94	97.99
0.95	98.35
0.96	98.70
0.97	99.04
0.98	99.37
0.99	99.68
1.00	100

During the time interval [0.99, 1] seconds, what is the average rate of change of the door's angle? $\frac{100-99.68}{1-0.99} = 32$ degrees per second

How does this average rate of change compare with the average rate of change on the interval [0.9,1]? Does this make sense? The change in the rate of change of the angle appears to be slowing down. It appears that it might be close to 32-32 degrees per second.

Are you surprised by the amount of difference? Explain your reasoning. The rate of change

is approaching the instantaneous rate of change of the door's angle at 1 second.

Can you predict the rate of change of the opening of the door's angle at time t = 1?
Between 30 and 31 degrees per second.

How could you get a better approximation? Keep decreasing the time period.

Continue until you can predict the door's rate of change of the angle to a hundredths of a second. 30.68 degrees per second

Describe in your own words how you can find the instantaneous rate of change of the door at exactly time 1 second.

Descriptions will vary. But they should incorporate the idea of selecting two time periods closer and closer to the time t = 1 and calculating the slope of the line segment. These slopes approach the rate of change of the door at 1 second.