

BC Calculus –Review Sheet

When you see the words	This is what you think of doing...
1. Find the area of the unbounded region represented by the integral $\int_1^{\infty} f(x)dx$ (sometimes called a horizontal improper integral).	Set up $\lim_{b \rightarrow \infty} \int_1^b f(x) dx$ to see if the area diverges or converges.
2. Find the area of a different unbounded region under $f(x)$ from $(a,b]$, where $\lim_{x \rightarrow a^+} f(x) = \infty$ or $-\infty$, where the area is represented by $\int_a^b f(x)dx$, (sometimes called a vertical improper)	Set up $\lim_{x \rightarrow a^+} \int_a^b f(x) dx$ to see if the area diverges or converges.
3. Given a $f(x)$, find arc length of the function on the interval $(a, f(a))$ and $(b,f(b))$.	Use the integral: $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$.
4. Given a curve in parametric form where $x = f(t)$, $y = g(t)$, find the arc length of the curve on the interval $[t_1, t_2]$.	Use the integral: $L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
5. Given $\frac{dy}{dx} = F(x, y) = xy$ and an initial point $(x_0, y_0) = (1, 1)$, find an approximate value for $f(1.2)$ and $\Delta x = 0.1$	Think about Euler's Method to draw tangent lines and approximate along the tangent lines. First calculate the slope at $(1, 1)$ and write an equation of a tangent line to f at $(1, 1)$. Use this line to approximate a new point at $x=1.1$ using $\Delta x = 0.1$. This gives you a second point to repeat the procedure again. Write another tangent line with a new slope and approximate the value of $f(1.2)$ by moving along this second tangent line to the point $x = 1.2$.
6. Given the differential equation of the form $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$ for P as a function of t , where k and L are constants.	Separate the differentials, use partial fractions, integrate, use an initial condition to solve for the constant and end up with an equation of the form: $P = \frac{L}{1 + Ae^{-Mkt}}$

<p>7. Given the differential equation $\frac{dP}{dt} = 12P - 4P^2$ where P is measuring the number an animal present on day 0. Find the value of P when the number of these animals is increasing the fastest.</p>	<p>7. First notice that $\frac{dP}{dt} = 12P - 4P^2$ is a parabola, so rewriting it in the form $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$ or $\frac{dP}{dt} = 12P\left(1 - \frac{P}{3}\right)$ tells us that the $\frac{dP}{dt} = 0$ v when P=0 or P=3. The number of animals is increasing fastest at the midpoint of 0 and 3 or 1.5.</p>
<p>8. Given the differential equation $\frac{dP}{dt} = 1200P - 400P^2$ where P is measuring the number an animal present on day 0. Determine the $\lim_{n \rightarrow \infty} P(t)$.</p>	<p>Factoring $\frac{dP}{dt} = 1200P - 4P^2 = 1200P\left(1 - \frac{P}{300}\right)$ we can see that $\frac{dP}{dt} = 0$ when P=0 and P=300. Therefore, P=300 is the $\lim_{n \rightarrow \infty} P(t)$ since the grow increases between P = 0 and P = 300 but stops at P = 300.</p>
<p>9. Given that a line segment has endpoints of (1,2) and (5,10), write a set of parametric equations for the line that passes through these two points.</p>	<p>Determine the slope (m) of the line segment (m=2), write an equation for the line segment using point slope form ($y=2(x-1)+2$), and then rewrite this equation as parametric equations where $x(t)=t$ and $y(t)=2(t-1)+2$ or $y(t)=2t$. Select values for t from knowing that x or t starts at 1 and goes to 5 so $1 \leq t \leq 5$</p>
<p>10. Given the position function of two particles in parametric form, $x_1(t) = f(t)$, $y_1(t) = g(t)$ and $x_2(t) = h(t)$, $y_2(t) = k(t)$, determine if the particles intersect or collide.</p>	<p>For the paths to intersect $x_1(t_1) = x_2(t_2)$ and $y_1(t_1) = y_2(t_2)$. Solve these equations simultaneously to find the time when the paths intersect. For the particles to collide they must be at a point at the same time. Determine the times when each particle is at the given point. If the times match, the particles collide, otherwise their paths only cross.</p>
<p>11. Given a set of parametric equations where $x = f(t)$, $y = g(t)$, find $\frac{dy}{dx}$ or the slope of the tangent line.</p>	<p>Recall that $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$</p>

<p>12. A path of a particle is described with a set of parametric equations $x = f(t), y = g(t)$. Find the equation of the tangent line when $t = t_0$.</p>	<p>Determine the point where the particle is $(x(t_0), y(t_0))$. Then find the slope of the graph at the time $t = t_0$ by calculating $\left. \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \right _{t=t_0}$. Then write the equation of the line in point-slope form.</p>
<p>13. A path of a particle is described with a set of parametric equations $x = f(t), y = g(t)$.</p> <p>a. Find all values of t where the particle's path is vertical.</p> <p>b. Find all values of t where the particle's path is horizontal.</p>	<p>a. Determine the times when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$.</p> <p>b. Determine the times when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$.</p>
<p>14. Given a set of parametric equations where $x = f(t), y = g(t)$, find $\frac{d^2y}{dx^2}$</p>	<p>First find $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ then calculate $\frac{d}{dx} \left[\frac{dy}{dx} \right] = \frac{\frac{d}{dt} \left[\frac{dy}{dx} \right]}{\frac{dx}{dt}}$.</p>
<p>15. Given the position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$. Find the velocity vector.</p>	<p>Recall that the velocity vector is $v(t) = \langle x'(t), y'(t) \rangle$ which means that you must differentiate $x(t)$ and $y(t)$ respect to t and then write a vector.</p>
<p>16. The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$. Find the acceleration vector.</p>	<p>Recall that the acceleration vector is $a(t) = \langle x''(t), y''(t) \rangle$ which means that you must differentiate $x'(t)$ and $y'(t)$ respect to t and then write a vector.</p>
<p>17. The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$. Find the speed of the particle at a moment at time $t = a$.</p>	<p>Recall that speed is the magnitude of the velocity vector and is found by calculating $v(a) = \langle x'(a), y'(a) \rangle = \sqrt{(x'(a))^2 + (y'(a))^2}$</p>
<p>18. Given the velocity vector $v(t) = \langle x'(t), y'(t) \rangle$ and position vector at $t = 0$ as $\langle x(0), y(0) \rangle$, find the position vector at time $t = a$.</p>	<p>Recall that the position vector is $\left\langle x(0) + \int_0^a x'(t) dt, y(0) + \int_0^a y'(t) dt \right\rangle$</p>

<p>19. Given $v(t) = \langle x'(t), y'(t) \rangle$ determine when the particle is stopped.</p>	<p>You must consider both $x'(t)$ and $y'(t)$. You need to determine when both $x'(t)$ and $y'(t)$ equal zero.</p>
<p>20. Given $v(t) = \langle x'(t), y'(t) \rangle$ find the slope of the tangent line to the vector at t_1.</p>	<p>You must calculate $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$ and evaluate this expression at t_1.</p>
<p>21. Given a particle moves along a function $y = 3x^2 + 1$, the rate of change of x or $\frac{dx}{dt} = 3t$ for $t > 0$ and $x(0) = 1$. Find the particle's position at time $t = 3$.</p>	<p>Find the change in the x direction or $x(3) = x_0 + \int_0^3 x'(t) dt = 1 + \int_0^3 (3x^2 + 1) dt = 31$. Determine the y coordinate using the function $y = f(x) = 3(31^2) + 1 = 2884$. Write the coordinate: (31, 2884)</p>
<p>22. Find the slope of the tangent line to the polar curve $r = f(\theta)$.</p>	<p>Recall that $x = r \cos \theta, y = r \sin \theta \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$</p>
<p>23. Given a polar curve $r = f(\theta)$, find horizontal tangents to curve.</p>	<p>Recall that $x = r \cos \theta, y = r \sin \theta$ and then find where $r \sin \theta = 0$ and where $r \cos \theta \neq 0$</p>
<p>24. Find vertical tangents to a polar curve $r = f(\theta)$.</p>	<p>Recall that $x = r \cos \theta, y = r \sin \theta$ and then find where $r \cos \theta = 0$ and where $r \sin \theta \neq 0$</p>
<p>25. Find the area inside one of the petals on the flower described by $r = 2 \cos(3\theta)$.</p>	<p>Recall that one petal can be traced by $-\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}$ and the area can be found by calculating the integral</p>

<p>26. Find the area outside $r_1(\theta)$ but inside $r_2(\theta)$.</p>	<p>First find the points of intersection $\theta = a$ and $\theta = b$ and then integrate $\int_a^b \frac{1}{2}(r_2(\theta) - r_1(\theta))^2 d\theta$</p>
<p>27. Find the arc length of a function $r_1(\theta)$ from $\theta = a$ and $\theta = b$.</p>	<p>Recall the $x = r \cos \theta$ and $y = r \sin \theta$ to convert from polar form to parametric form. Then use the integral for arc length with parametric equations. Perform the integral $\int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta$</p>
<p>28. Find the sum $\frac{3}{2} + \frac{1}{2} + \frac{1}{6} + \frac{1}{216} + \dots$.</p>	<p>Notice that the sum is a geometric series where $a = \frac{3}{2}$ and $r = \frac{1}{3}$ so the sum is given by $\frac{a}{1-r} = \frac{\frac{3}{2}}{1-\frac{1}{3}} = 1\frac{1}{2}$</p>
<p>29. Determine if the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{n}{n+1} + \dots$ converges or diverges</p>	<p>Think about the nth term in this series: $\frac{n}{n+1}$. By the nth term test, since $\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1 \neq 0$ the series diverges.</p>
<p>30. Determine if the series $\sum_{n=1}^{\infty} \frac{3}{n+1}$ converges or diverges</p>	<p>Think about using the integral test. $\int_1^{\infty} \frac{3}{x+1} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{3}{x+1} dx = \lim_{b \rightarrow \infty} \ln \frac{b+1}{2} = \infty$ Therefore, the series diverges since the integral diverges.</p>

<p>31. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{3x^2}$ converges or diverges.</p>	<p>One test you might think of using is the p-series test. Since $p = \frac{3}{2} > 0$ converges.</p>
<p>32. Determine if the series $\sum_{n=1}^{\infty} \frac{1 + \sin x}{x^2}$ converges or diverges.</p>	<p>Since the series $\sum_{n=1}^{\infty} \frac{2}{x^2}$ converges and $0 < \frac{1 + \sin x}{x^2} \leq \frac{2}{x^2}$ the series converges by the comparison test.</p>
<p>33. Determine if the series $\sum_{n=1}^{\infty} \frac{3^n}{4^n + 1}$ converges or diverges.</p>	<p>Using the ratio test, $\lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}}{4^{n+1} + 1}}{\frac{3^n}{4^n + 1}} = \frac{3}{4^{n+1} + 1} (4^n + 1) < \frac{3}{4^{n+1}} (4^n) = \frac{3}{4} < 1$ So the series converges.</p>
<p>34. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ converges or diverges.</p>	<p>Using the alternating series test, since each term $\frac{1}{n}$ decreases as n approaches infinity and converges to 0, then the alternating series converges.</p>
<p>35. Write a series for $x \cos x$ where n is an integer</p>	<p>Recall that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ and then multiply through by x.</p>
<p>36. Write a series for $\ln(1 + 3x)$ centered at $x = 0$.</p>	<p>Recall that $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1} \frac{x^n}{n} + \dots$ Substitute $3x$ for x.</p>

<p>37. If $f(x) = 2 + x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \dots$ represents a Taylor Polynomial about $x = 0$, find $f(-2)$</p>	<p>Notice that $f(x)$ is a geometric series with $a = 2$ and $r = \frac{1}{2}x$ so the sum of the series can be written as $\frac{a}{1-r}$ or</p> $\left. \frac{2}{1 - \frac{1}{2}x} \right _{x=-2} = \frac{2}{1 - \frac{1}{2}(-2)} = 1$
<p>38. Write the nth degree Taylor Polynomial for $f(x)$ at $x = c$.</p>	$T_n(x) = f(c) + f'(c)(x-c) + f''(c)\frac{(x-c)^2}{2!} + \dots + f^{(n)}(c)\frac{(x-c)^n}{n!} + \dots$
<p>39. Given a Taylor series, find the Lagrange form of the remainder for the 4th term.</p>	<p>This error is no greater than the value of the 5th term at some value of a between x and c. $R_n(x) = \frac{f^{(n+1)}(a)}{(n+1)!}(x-c)^{n+1}$</p>
<p>40. Let S_4 be the sum of the first 4 terms of an alternating series for $f(x)$. Approximate $f(x) - S_4$.</p>	<p>You should recognize this as the error for the 4th term of an alternating series which is no greater than the absolute value of the 5th term.</p>
<p>41. Given the polynomials $f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, what is $f(x)$?</p>	$f(x) = e^x$
<p>42. Given the polynomial $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$, what is $f(x)$?</p>	$f(x) = \sin(x)$
<p>43. Given the polynomial $f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$, what is $f(x)$?</p>	$f(x) = \cos(x)$

44. Find the interval of convergence of a series.	Apply the ratio test to find the interval and then test convergence at the endpoints.
45. Find $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$	Check to see if you can use L'Hopital's Rule. Check to see if $f(a) = g(a) = 0$ or $f(a) = g(a) = \infty$. If this is true, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$. You may have to repeat these steps.
46. Find $\int \frac{dx}{x^2 + x - 12}$	Use partial fraction to set up two integrals: $\int \frac{A}{x+4} dx + \int \frac{B}{x-3} dx$. Solve for A and B and then complete the integration. $\int \frac{-1}{x+4} dx + \int \frac{1}{x-3} dx = \frac{1}{7} \ln \left \frac{x-3}{x+4} \right + C$