A Twist of Lemon

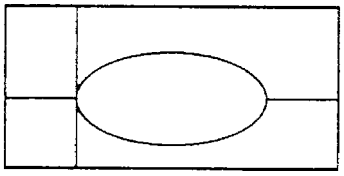
Names_____

Materials Needed:

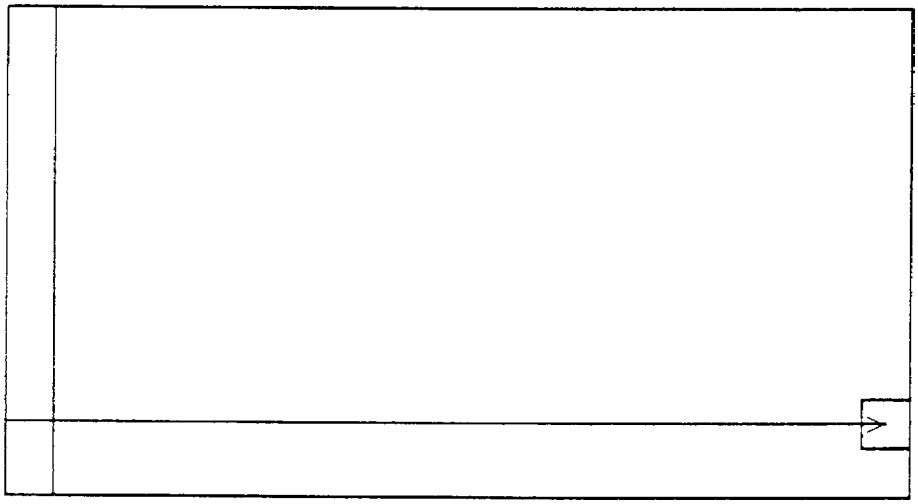
1 lemon for each group of 2 students
1 sharp knife
1 centimeter ruler
1 graduated beaker
water
Graphing calculator with statistical capabilities

Procedures:

1. If your lemon has bulbous ends, trim them neatly with a knife. You need to have a lemon that greatly resembles an egg for the best results.



2. After trimming your lemon, place it on the coordinate system below, with the long axis oriented horizontally, as shown.



3. Carefully trace the top of your lemon on the graph grid above; don't let your pencil slide downward on the lemon or your curve will be inaccurate. Mark the x-intercepts of the lemon clearly. Use your ruler to measure, to the nearest tenth of a centimeter, the x-intercepts and enter these ordered pairs in the chart provided (first and last boxes)

4. On your sketch, subdivide the length of the lemon along the x-axis into six equal subdivisions. Use your ruler and take the coordinates of the lemon's upper curve at each

of these sample points, measuring to the nearest tenth of a centimeter. Enter these coordinates in the chart.

x								
y	0							0

5. Use the data in the chart to find an equation to represent the lemon's upper curve, using your calculator. Write the equation to represent the top of the lemon's curve below.

_____

6. Fill the measuring graduated beaker with enough water to cover the lemon, to the nearest grid line. Record this measurement below. Add the lemon to the beaker and measure the new height of the water; record below.

Height of water before immersion of the lemon: _____

Height of water after immersion of the lemon: _____

7. When this curve is rotated around the x-axis, it represents the shape of the lemon. Cut 6 uniform slices of your lemon, slicing each perpendicular to the horizontal axis at the places where you took measurements between the endpoints.
What does one of these slices look like: _____

8. Write the formula for computing the volume of one slice of the lemon:

dV-= _____

9. Use your ruler to measure each slice of the lemon appropriately, and then use these measurements to find the volume of each slice. Record the volumes below:

dV_1	dV_2	dV_3	dV_4	dV_5	dV_6

10.. Add up the volumes to get an estimate for the lemon's total volume and record below. Compare your results to the volume you got be displacement..

$V \approx \sum_{i=1}^{6} dV_i$	
Volume by Displacement	
Volume by Rotation	
Difference of sum from Displacement	

11. Mark one slice along your curve in drawing in step 2. What calculus expression represents the width of the slice, no matter where your place it? _____
In the volume formula you wrote in step 9, which variable would represent the width of each slice? _____

12. If each of the slices was a disk with a radius r, what expression will give you the radius of each slice? Write the radius expression: f(x) = _____

13. If you could slice the solid generated by rotating your lemon curve about the x-axis, would your slices look about the same as your actual lemon slices? _____ If so, write an expression to approximate the volume of the solid using 10 slices, then 100 slices, then an infinite number of slices. (You may use f(x) in each expression.)

_____ _____ _____

14. Write a general calculus expression used to represent this infinite process.

_____

15. Now write a specific calculus expression for the volume of the solid generated by rotating your lemon about the x-axis.

_____

16. Now find the volume of the lemon using an integration technique. Remember that 1 cubic centimeter - 1 milliliter. Show the work which leads to your volume:

Volume of Lemon _____

Compare this volume for the volume found earlier.

Volume by integration	
$V \approx \sum_{i=1}^{6} dV_i$	
Difference of integration volume from sum	
Difference of integration volume from displacement	

17. Write a summary paragraph describing what the above activity has demonstrated about finding volume. Include in your paragraph

- degrees of accuracy in finding volume
- ways to improve the accuracy of the activity
- what this activity has demonstrated to you about finding volume in general.

(Adapted from Deborah Preston's *A Twist of Lemon*)