# AP CALCULUS EXAM TIPS

1. **Show all work**. Remember that the grader is not really interested in finding out the answer to the problem. The grader is interested in seeing if *you* know how to *solve* the problem.
2. **Do not round partial answers.** Store them in your calculator so that you can use them *unrounded* in further calculations.
3. **Do not let the points at the beginning keep you from getting the points at the end.** If you can do part (c) without doing (a) and (b), *do it.* If you need to import an answer from part (a), *make a credible attempt at part (a)* so that you can import the (possibly wrong) answer and get your part (c) points.
4. **If you use your calculator to solve an equation, write the equation first.** An answer without an equation might not get full credit, even if it is correct.
5. **If you use your calculator to find a definite integral, write the integral first.** An answer without an integral will not get full credit, even if it is correct.
6. **Do not waste time erasing bad solutions.** If you change your mind, simply cross out the bad solution after you have written the good one. *Crossed-out work will not be graded.* If you have no better solution, leave the old one there. It might be worth a point or two.
7. **Do not use your calculator to anything except:** (a) graph functions, (b) compute numerical derivatives, (c) compute definite integrals, and (d) solve equations. In particular, do not use it to determine max/min points, concavity, inflection points, increasing/decreasing, domain, and range. (You can *explore* all these with your calculator, but your *solution* must stand alone.)
8. **Be sure you have answered the problem.** For example, if it asks for the maximum value of a function, do not stop after finding the $x$ at which the maximum value occurs. Be sure to express your answer in *correct units* if units are given.
9. **If you can eliminate some incorrect answers in the multiple-choice section, it is advantageous to guess.** Otherwise it is not. Wrong answers can often be eliminated by estimation, or by thinking graphically.
10. **If they ask you to justify your answer, think about what needs justification.** They are asking you to say more. If you can figure out why, your chances are better of telling them what they want to hear. For example, if they ask you to justify a point of inflection, they are looking to see if you realize that a sign change of the second derivative must occur.

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1. $f''(x)=0 \Leftrightarrow (x,f(x))$ **is a point of inflection.**

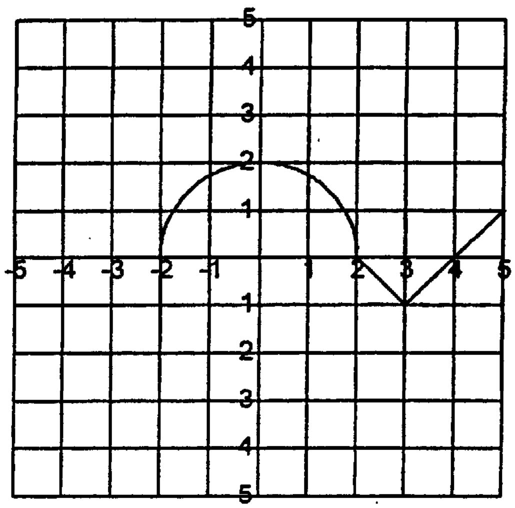
   Example: 1969 AB/BC 17

   The graph of $y=5x^4-x^5$ has a point of inflection at
   (A) (0,0) only (B) (3,162) only (C ) (4,256) only
   (D) (0,0) and (3,162) (E) (0,0) and (4,256)

   In 1969 62% of the AB students chose choice (D) and 18% chose choice (B).

   Example: 1997 AB/BC 5

   $f(x)$




The graph of a function $f$ consists of a semicircle and 2 line segments as shown above. Let $g$ be the function given by

$$g(x)=\int_0^x f(t)dt.$$

(d) Find the $x$ coordinate of each point of inflection of the graph of $g$ on the open interval (-2,5). Justify your answer.

2. **$f(x)$ is a maximum (minimum) $\Leftrightarrow$ $f'(x)=0$.**

Example: 1993 AB 19

Let $f$ be the function defined by $f(x)=\begin{cases} x^3 \text{ for } x\le 0 \\ x \text{ for } x>0 \end{cases}$

(A) $f$ is an odd function.
(B) $f$ is discontinuous at $x=0$.
(C) $f$ has a relative maximum.
(D) $f'(0)=0$
(E) $f'(x)>0$ for $x\neq 0$.

22% of the students chose (A), 24% chose (D) and 33% chose (E). 11% of the students omitted the answer.

Example: 1998 AB 89

If $g$ is a differentiable function such that $g(x)<0$ for all real numbers $x$ and if $f'(x)=(x^2-4)g(x)$, which of the following is true?

(A) $f$ has a relative maximum at $x=-2$ and a relative minimum at $x=2$.
(B) $f$ has a relative minimum at $x=-2$ and a relative maximum at $x=2$.
(C) $f$ has relative minima at $x=-2$ and at $x=2$.
(D) $f$ has relative maxima at $x=-2$ and at $x=2$.
(E) It cannot be determined if $f$ has any relative extrema.

38% of the AB students answered the question correctly in 1998.

**3. Average rate of change of $f$ on [a, b] is $\frac{f'(a)+f'(b)}{2}$.**

**This, of course, is the wrong definition of ARC. The ARC of $f$ on [a, b] is**

$\frac{f(b)-f(a)}{b-a}$.

Example: 1998 AB 3 (b)

A table for values for the velocity, *v(t), in ft/sec* of a car traveling on a straight road, for $0 \le t \le 50$ is shown below. Find the average acceleration of the car in $ft/\sec^2$ over the interval $0 \le t \le 50$.

| *t (seconds)* | *v(t) (feet per second)* |
|---|---|
| 0 | 0 |
| 5 | 12 |
| 10 | 20 |
| 15 | 30 |
| 20 | 55 |
| 25 | 70 |
| 30 | 78 |
| 35 | 81 |
| 40 | 75 |
| 45 | 60 |
| 50 | 72 |

Answer: $\frac{f(50)-f(0)}{50-0} = \frac{72-0}{50} = \frac{72}{50} ft/\sec^2$

**4. Volume by washers is $\int_a^b \pi(R-r)^2 dx$.**

**Example: Let $R$ be the region between the graphs of $y$=1 and $y$=sin $x$ from $x = 0$ *to* $x = \frac{\pi}{2}$. The volume obtained by revolving $R$ about the *x-axis* is given by**

(A) $2\pi\int_0^{\frac{\pi}{2}} x\sin x\,dx$ (B) $2\pi\int_0^{\frac{\pi}{2}} x\cos x\,dx$ (C) $\pi\int_0^{\frac{\pi}{2}} (1-\sin x)^2 dx$

(D) $\pi\int_0^{\frac{\pi}{2}} \sin^2 x dx$ (E) $\pi\int_0^{\frac{\pi}{2}} (1-\sin^2 x)dx$

5. **Separable differential equations can be solved without separating the variables.**

**Example: 1997 AB 6**
**Let $v(t)$ be the velocity, in feet per second, of a skydiver at time $t$ seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$. Use separation of variables to find an expression for $v$ in terms of $t$. where $t$ is measured in seconds.**

6. $\frac{d}{dx} f(y) = f'(y)$ **and other chain rule errors.**

**Example: 1997 AB 7**

$$\frac{d}{dx}\cos^2(x^3) =$$

**(A)** $6x^2 \sin(x^3)\cos(x^3)$ **(B)** $6x^2 \cos(x^3)$ **(C)** $\sin^2(x^3)$
**(B)** $-6x^2 \sin(x^3)\cos(x^3)$ **(E)** $-2\sin(x^3)\cos(x^3)$

**70% of the students got the right answer in 1997.**

7. **Graders will assume the correct antecedents for all pronouns used in justification. [My students were never permitted to use the word "it" in justification. They had to talk about the function, the first derivative and the second derivative.]**

8. **Only use the calculator for 4 operations:**
   a) **graphing in an arbitrary viewing window**
   b) **evaluating a definite integral**
   c) **evaluating a derivative at a point (the numerical derivative in a calculator that is not a CAS)**
   d) **finding the zeros of a function**

**The student must always show the correct setup in correct mathematical notation. Calculator syntax is never accepted.**

9. **Universal logarithmic antidifferentiation:** $\int \frac{1}{f(x)} dx = \ln|f(x)| + C$

**Example:** $\int \frac{4x}{1+4x^4} dx$

**10. Bad algebra**

**And here are several errors that I have found to be very common:**

**11. Forgetting the "C" in an analytic solution of a differential equation. I would always tell my students that if they forgot the "C" they should do another problem.**

**Example 2000 AB 6**

**Consider the differential equation** $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

**(a) Find a solution** $y = f(x)$ **to the differential equation satisfying** $f(0) = \frac{1}{2}$.

**(b) Find the domain and range of the function** $f$ **found in part (a).**

**Solution:**

| | | |
|---|---|---|
| **1 point** | $e^{2y}dy = 3x^2dx$ | **separation of variables** |
| **1 point** | $\frac{1}{2}e^{2y}$ | |
| **1 point** | $x^3$ | |
| **1 point** | $\frac{1}{2}e^{2y} = x^3 + C$ | **If the student omitted the C the student obtained a maximum 3 of 6 points.** |
| **1 point** | $\frac{1}{2}e^{\frac{1}{2}2} = C = \frac{1}{2}e$ | |
| **1 point** | $\frac{1}{2}e^{2y} = \frac{2x^3 + e}{2} \Rightarrow e^{2y} = 2x^3 + e$ | |
| | $y = \frac{1}{2}\ln(2x^3 + e)$ | |
| **1 point** | $2x^3 + e > 0$ | |
| **1 point** | $x > -\left(\frac{1}{2}e\right)^{\frac{1}{3}}$ | |
| **1 point** | **range** $-\infty < y < \infty$ | |

12. Omission of units in the solution of a "real life" calculus problem.

13. Rounding too early when solving a problem.

Example: 2000 AB/BC 1

Let $R$ be the region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$ and $y = 1 - \cos x$. Find the area of $R$, etc.

The curves intersect at x=.941944. Store .941944 in your calculator as B. Define B for the reader. Then express your answer as a definite integral

$$\text{Area} = \int_0^B \left(e^{-x^2} - (1 - cox\, x)\right)dx.$$