

Discovering Angle Relationships in Star Polygons

Star Polygons are figures formed by arranging a set of points roughly around a circle or oval and then connecting each point to the next with segments. Different shape figures are formed when the points are connect in different ways.

- Figure 1 shows the type of figure which is formed when each point is connected to the next point.
- Figure 2 shows the type of figure formed if every second point is connected.

The degree measure for each of these stars is different. You can find the total for each star by measuring each angle at the point and then finding their sum.

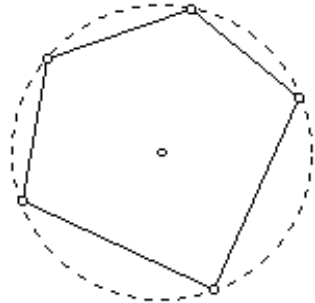


Figure 1

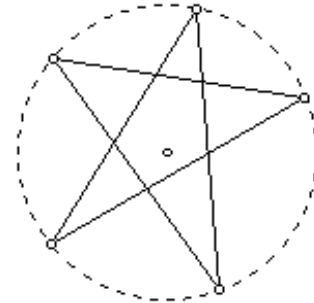


Figure 2

Sometimes the same pattern can appear when you connect every 3rd, or every 4th point. Don't be surprised when this happens. (See Figure 3) Sometimes another figure appears. Then other times it is necessary to create two overlapping figures when connecting the points. Figure 3 shows what happens when starting with 6 points around a circle, and then you connect every 2nd point. (Note the same figure appears when you start with 6 points and connect every fourth point.) Then other times you can get just a series of intersecting line segments such as Figure 4 when every sixth point is connected with twelve points placed in a counter clockwise direction. The total of the star points in this case would be zero degrees.

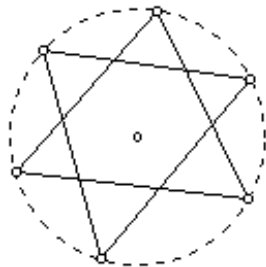


Figure 3

This shows 6 points with every second point connected. It is also 6 points with every fourth point connected.

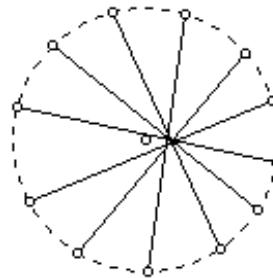


Figure 4

This is 12 points with every six point connected. This would be considered a total of zero degrees.

You should follow these steps in your investigation:

Step 1: Start by drawing with 5 points A through E arranged in clockwise order so that they roughly lie on a circle. Connect each point to the next point. Find the sum of the 5 angles formed.

Step 2: Redraw another star with 5 points A through E arranged in clockwise order so they roughly lie on a circle. Connect every 2nd point with segments \overline{AC} , \overline{CE} , \overline{EB} , \overline{BD} , and \overline{DA} . Measure the five angles A, through E at the star points.

Step 3: Redraw another star with 5 points A through E arranged in clockwise order so they roughly lie on a circle. Connect every 3rd point with segments. Does this remind you of an earlier star polygon? If so, you probably already know the total for the 5 angles. If not you will have to measure the 5 angles and find their total.

Step 4: Redraw another star with 5 points A through E arranged in clockwise order so they roughly lie on a circle. Connect every 4th point with segments. Does this remind you of an earlier star polygon? If so, you probably already know the total for the 5 angles. If not you will have to measure the 5 angles and find their total.

Step 5: Since you started with 5 points you do not have to draw another circle to connect every 5th point. This would not be possible to do. You must now move to drawing a star with 6 points A through F arranged in clockwise order so they roughly lie on a circle. Begin by connect every 2nd point with segments, measuring the angles and recording it. Then build another 6 pointed star by connecting every 3rd point, measure the angles and record it. Continue to do this until you have drawn each picture and connected every 4th point, and every 5th point. Again you do not have to draw a 6 pointed start by connected every 6th point.

Step 6: Continue to draw stars with more points (you do not have to do more than 12 points around a circular shape), connecting the points to form new shaped stars, measuring the angles, and recording the total.

Step 7: Complete a table showing the network, whether it can be traveled, and the number of odd and even points in each network. You should see a pattern develop as you build the table.

In completing the above steps your tasks include

- Collecting as much data as necessary to complete the chart and begin to see a pattern develop.
- Describing several patterns you see in your chart.
- Writing a rule for the number of degrees in an n-point star. See page 318 for a hint.
- Predict the number of degrees in each of the 15 point star polygons without actually drawing and measuring the angles.
- Making several conjectures about the sum of the angles or any other relationships you observe from your chart.

To complete this project you may either draw the figures with a ruler or you may use the Sketchpad program. If you draw the figures, make sure they each fill an entire page. Use the protractor accurately and then look at the final sum to see if it might need to be rounded off slightly. Your total degree measure should always end in a zero. If you use the sketchpad program you will be able to have the program measure the angles and also add the angles. Just be careful to name the angle carefully so the total actually represents all the angles in the picture. You can also use your own drawing program at home and then measure the angles with a protractor. BE CAREFUL MEASURING.

Place all your materials in a booklet. Design the booklet so that it shows an organized investigation which includes the pictures, the chart, and the conclusions. Place your name, period and date on the front cover of the booklet.