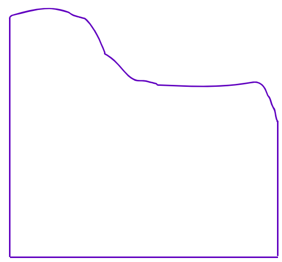
Area of an Irregular Shaped Region

Both the NCTM Curriculum and Evaluation Standards for School Mathematics and the New Jersey Curriculum Framework suggestion that students be introduced to the underpinnings of calculus in each grade cluster: k-4, 5-8, and 9-12.   Standard 13 in the 9-12 cluster of the Curriculum and Evaluation Standards states that students should be given experience studying both graphically and numerically the idea of a limit.   Standard 15 in the 9-12 cluster of the New Jersey Mathematics Curriculum Frameworks calls for students to have the opportunity to informally investigate the central ideas of calculus such as the limit, the rate of change, the area under a curve, and the slope of a tangent line.

Developing a technique for finding the area of an irregular shaped region can be investigated in a high school geometry class through an activity which encourages some of these ideas suggested by these two standards.

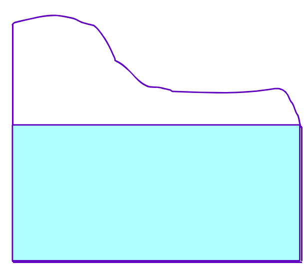
On centimeter grid graph paper have students draw a region with the following restrictions:

Three sides of the figure are straight lines and the fourth side is made up of a curve or several curved lines.

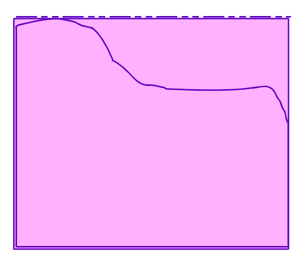


How can we find the area of this region. Let’s explore this idea by looking at several rectangles which approximate the area.

- First find the lowest value your curved line takes on along its path. Draw a rectangle with this height and the width of your region. What is the area of this rectangle? How does this area compare with the area of your region?
- Next find the highest value your curved line takes on along its path. Draw a rectangle with this height and the width of your region. What is the area of this rectangle? How does this area compare with the area of your region? How does this area compare to the first approximation?
- Can you predict the area of your region using the first two approximations?
  (See figures at the right)



Rectangle drawn with same width and height equal to lowest value of curved line



Rectangle drawn with same width and height equal to highest value of curved line

Is there a way we can get a better approximation?

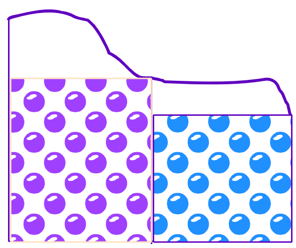
This time subdivide the width of your region into two equal parts and repeat the process used above again for each of the two regions.

For each region

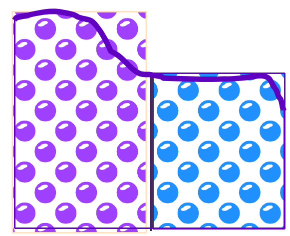
- First find the lowest value your curved line takes on along its path. Draw a rectangle with this height and the width ½ of your region. What is the area of each rectangle? What is the total area? How does this area compare with the area of your region?

- Next find the highest value your curved line takes on along its path. Draw a rectangle with this height and the width ½ of your region. What is the area of each rectangle? What is the total area? How does this area compare with the area of your region? How does this area compare to the first approximation?
- Can you predict the area of your region using these two approximations?

(See figures below)



2 Rectangles drawn, each with width equal to ½ of the original width and height equal to lowest value of curved line



2 Rectangles drawn, each with width equal to ½ of the original width and height equal to lowest value of curved line

Record the information you have collected in the chart below. Continue to subdivide the original width in half and calculate the two areas (lowest value of curve and highest value of curve in the region). Record your estimates of the area using these subdivisions.

Number of Rectangles	1	2	4	8	. . .	Infinite Number of Subdivisions
Total Area using Lowest Value on Curve						
Total Area using Highest Value on Curve						
Estimated Area						

Predict the exact area of the region after you complete the entire chart.

My prediction for the area of my region is ______________________________

Describe another way this area could be approximated. ______________________________

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