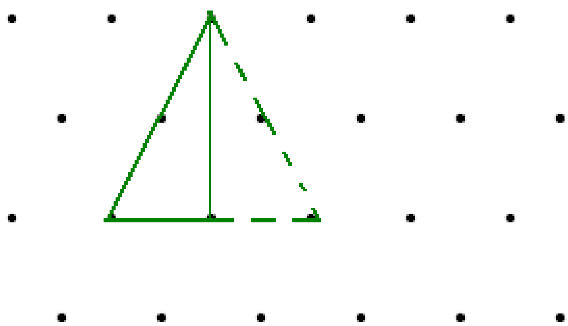
Developing Special Right Triangles
30-60-90 Triangles

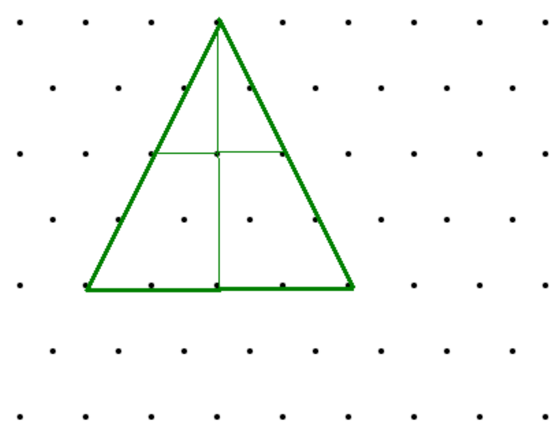
Start students with a equilateral triangle on isometric dot paper. Ask students to draw an equilateral triangle whose sides are 2.
Have students add the angle bisector to the angle at the top. Ask students if this line has any other properties in an equilateral triangle. (Altitude and median)
Students should find the measure of each angle.



To help the students concentrate on the special triangle have them dot one of the right triangles.
Be sure that students see that the two triangles formed are congruent. The angles should be 30, 60 and 90 degrees.

Have students label the hypotenuse as 2 unit. Based on the properties of the angle bisector also being a median the students should be able to find the measure of the short side of the right triangle. (1) Using the Pythagorean Theorem have students find the length of the altitude. ($\sqrt{3}$).

Now have the students draw a equilateral triangle whose sides are 4 units on the side. Students should add the angle bisector to the top angle as they did before. Students should note the size of the angles.

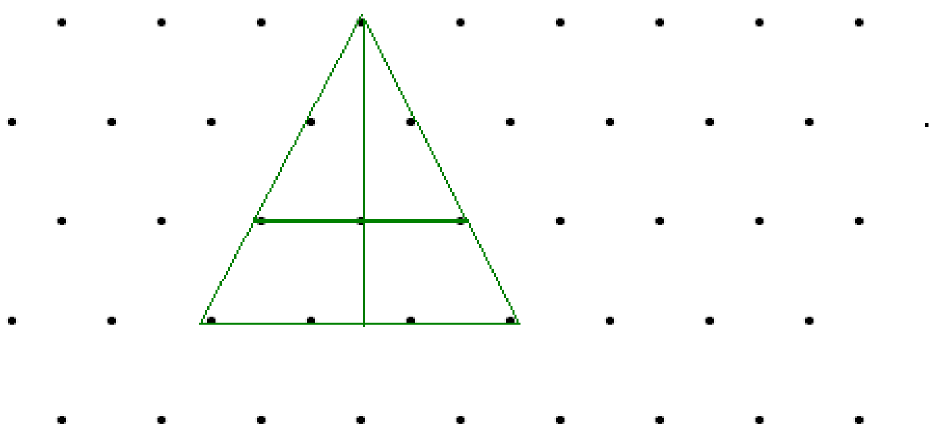


Students should be able to find the measure of the short side and again use the Pythagorean Theorem to find the length of the altitude. The short side should be 2 and the altitude should be $\sqrt{12}$. But by further observations they should also notice that the diagonal or hypotenuse is equal to $2\sqrt{3}$.

Ask students to draw an equilateral triangle that is 6 units on a side. Ask students to draw in the angle bisector again and note the measure of the angles. Using the Pythagorean Theorem to find the length of the hypotenuse. Some students may find it equals $\sqrt{27}$. But by further observations they should also notice that the altitude is equal to $3\sqrt{3}$.

If student continue to draw larger and larger equilateral triangles whose sides are even integers, they will observe that the short side is half the length of the side of the equilateral triangle, and the length of the altitude is equal to the short side times the square root of 3. Or in other words, if the hypotenuse is ***2a*** then the short side is ***a*** and the altitude is equal to $a\sqrt{3}$.

Students can be challenged to work with an equilateral triangle whose side is an odd number. They should notice the same relationship to be true.



Or in other words, if the hypotenuse is ***a*** then the short side is $\frac{a}{2}$ and the altitude is equal to $\frac{a}{2}\sqrt{3}$.