Discovering Pick's Theorem for Area

Build a simple polygon on your geoboard. How much area is contained in your polygon?

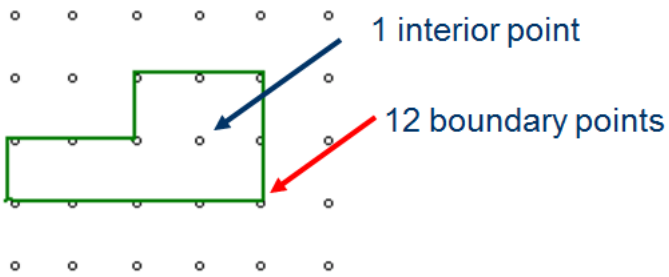
Is there a way to determine the area other than just counting the squares?

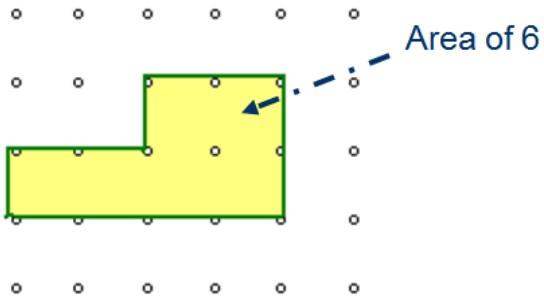
Is there a relationship between the number of interior points, boundary points and the area?

There is a formula that relates the area of the polygon to the number of interior points and boundary points. Georg Pick discovered that the area of a polygon was dependent on the number of lattice points on the interior of the polygon and the number of lattice points on the boundary of the polygon.

As you work through this activity you will discover it.

You will need to know the difference between a interior point and boundary point. You will need to be able to count the area.





PART I

- On the geoboard create several polygons with an area of 6 with a different number of interior points.
- Count the number of lattice points on the interior.
- Count the number of lattice points on the boundary.
- Record the data in a chart.

Area =6	Interior (I)				
	Boundary (B)				

- You may want to rearrange the data in your table to reflect the pattern you observe.
- Can you describe the pattern?
- Can you describe a few more numbers that should appear in your table?
- Create a graph of interior pts. vs. boundary pts. (interior, boundary)
- Can you predict an equation that relates the number of boundary points from the number of interior points?

Part II

- Each group pick an integral area other than 6.
- On the geoboard create several polygons with the new area
- Count the number of lattice points on the interior.
- Count the number of lattice points on the boundary.
- Record the data in the chart.
- Write an equation that can be used to find the number of boundary points from the number of interior points.

Area =	Interior(I)				
	Boundary(B)				

Area =	Interior(I)				
	Boundary(B)				

Area =	Interior(I)				
	Boundary(B)				

Area =	Interior(I)				
	Boundary(B)				

Part III

- Gather data from other groups.
- Create a graph of the new data on the same piece of graph paper.
- How are the lines related?
- If the lines are all parallel, what would be in common between all the lines?

Part IV

- Compare the equations for (interior, boundary).
- What type of number do you think the y-intercept can be?
- Let Yint represent the y-intercept in each equation. Write a formula to find the y-intercept using B and I.

PART V

- Each group select an odd number for the y-intercept.
- Return to your geoboard and build at least two polygons with this area.
- Collect the number of boundary and interior lattice points on each polygon.
- Graph these points on the same graph to see if they make sense.
- Does your equation Yint=B+2I make sense?

Pulling it together - observation

- Study each of the lines you wrote. Notice that the y-intercept changes as the area changes.
- Collect some new data from all the lines.
- Write an equation that relates the area (A) and the y-intercept (Y).

Area						
Y-intercept						

- What equation did you write for the relationship between the y-intercept and the boundary and interior lattice points?
- What equation did you write for the relationship between the area and the y-intercept?
- Combine these two equations to write an equation for the relationship between the area, boundary points, and the interior points.

- Solve this equation for A
- When you think about the number of boundary and interior lattice points, does this equation make sense?
- If the area of the polygon was 16 can you describe the number of boundary and interior lattice points that could be possible?
- If the number of boundary points are 12 on a polygons, can you describe the relationship between the area and the number of interior lattice points?
- Can you have a polygon with 11 boundary lattice points? Why or why not?
- Describe at least two polygons with 11 boundary lattice points.
- Can you have a polygon with 5 interior lattice points? Why or why not?
- Describe at least two polygons with 5 interior lattice points.

You did it!!

You found Pick's Formula for Area