How is the Box Changing?

Materials Needed:

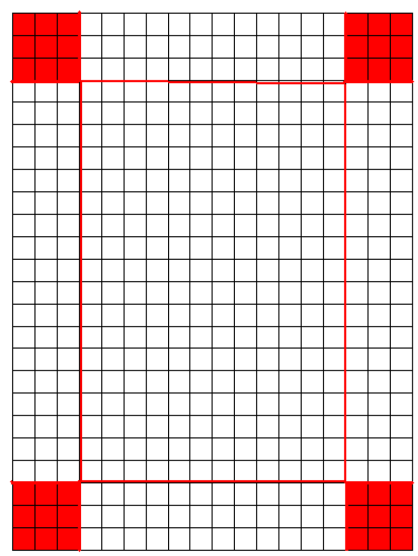
- Centimeter Grid Paper
- Communicators and Student Activity Sheet
- Class Record Sheet
- Scissors
- Tape
- Graphing Calculators

Objective:

The students will design various rectangular boxes without lids to determine the rectangular box that has the largest possible volume.
Students, in groups, will be given centimeter grid paper and asked to remove a square corner from each of the four corners and form a box. Students will determine the volume of their open rectangular box. Then all groups will determine the volume and surface area of their box. Using the class data, students will determine th box that has the largest volume and the largest surface area.

Procedure:

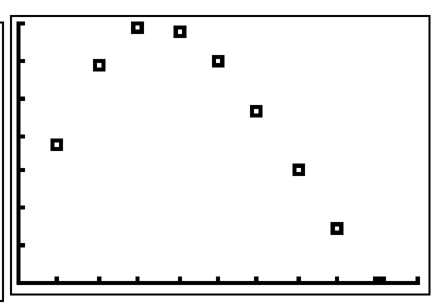
- Groups students into groups of four. Give each groups 2 pairs of scissors, some tape, and two sheets of centimeter grid paper.
- Students should first trim the centimeter grid paper so that the margins are removed.
- Assign, randomly, cards that tell the students the size of the square they are to cut from each corner of the 18 cm x 25 cm grid paper.
- After students have cut the corners they should form an open box out of the left over grid by folding up the four sides.
- Students should calculate the surface area of their box and the volume of their box.
- After students have collected the data the data should be shared on a class chart. Collect the data in numerical order using the height of the box.
- Look at the data to observe any possible errors. Ask students to make an observation about the volume of the rectangular box.(Volume is increasing and then decreasing and the size of the corner increases.)
- Have students enter the data in the graphing calculator by placing the height in L1, the volume in L2, and the surface area in L3.
- Set up an appropriate window to view L1 vs. L2.



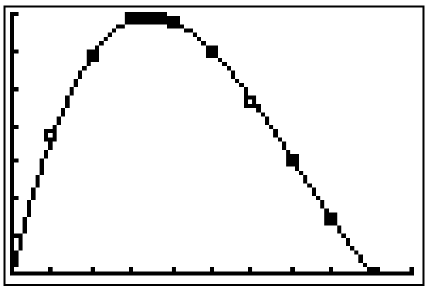
Students cut out the four red squares and fold on the red lines.

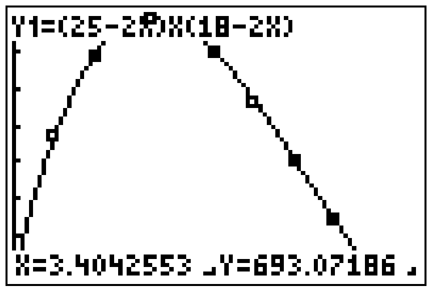
L1	L2	L3 1
1	368	446
2	588	434
3	684	414
4	680	386
5	600	350
6	468	306
7	308	254

L1(1)=1

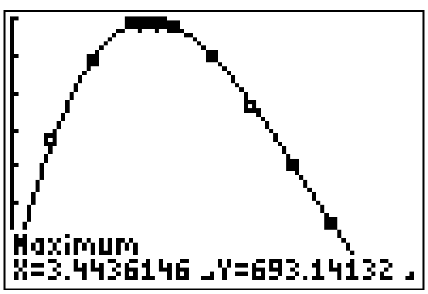


- Set up a scatterplot to view L1 vs. L2.
- Ask students to make an observation about the scatterplot. (The volume is increasing and decreasing, but not symmetrically.)
- Using tracing ask the students to predict where the maximum volume will take place. (Somewhere between a corner of 3 and a corner of 4 cm.)
- Have students add several additional points of data to their chart for corners that measure between 3 and 4 cm. (3.25, 3.5, and 3.75 cm) Calculate the volume of each new box. (New volumes are about 691, 693 and 689 cubic centimeters.)
- Develop with students a way to write an equation that would fit the scatterplot. Ask students to use x for the size of each corner. What would the height of the box be? (x) Look at a cut out piece of grid paper and describe the length using x. (25-2x). Look at a cut out piece of grid paper and describe the width using x. (18-2x)
- What would the volume of the box be? (x)(25-2x)(18-2x). Enter this expression in y1 and graph against the scatterplot.





- Trace along the equation to find the size of the square the produces the maximum volume. (About 693.07 cubic centimeters.
- Use the built in feature of the calculator to find the maximum volume.(about 693.14 cu.cm.)



- Ask students to study their surface areas and make an observation about what his happening with the surface area as the size the removed corner is changing. (Students should notice that the surface area is always decreasing.)
- As the volume is changing by increasing and decreasing, what is the surface area doing? (Always decreasing.)

If time permits:

- Ask students to build a scatterplot for height (L1) vs. surface area (L3).
- Ask students to write an equation for the surface area: 2x(25-2x)+2x(18-2x)+(25-2x)(18-2x)
- Ask students to use the distributive property and re-write the expression:
  $50x - 4x^2 - 36x - 4x^2 + 450 - 86x + 4x^2$ or $-4x^2 + 450$
- Enter this equation in the calculator to see that it passes through the data also.
- When x=0, what is special about the rectangular box? (It is flat. All the surface area is on the bottom.