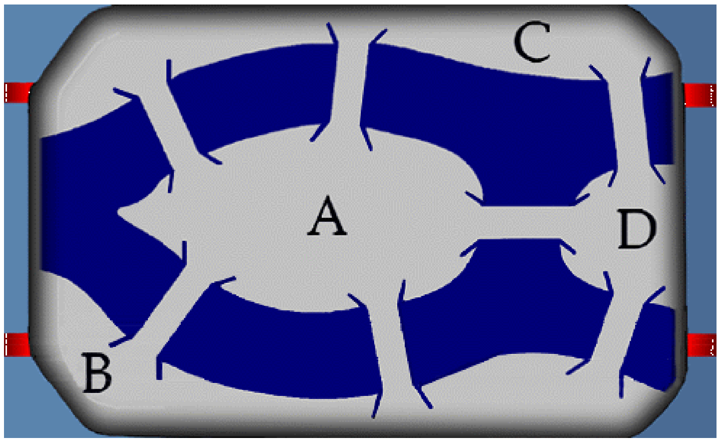
The Konigsberg Bridge Problem

The river Pregel divides the town of Konigsberg into four separate land masses, A, B, C, and D. Seven bridges connect the various parts of town, and some of the town's curious citizens wondered if it were possible to take a journey across all seven bridges without having to cross any bridge more than once.



Try to travel all seven bridges exactly once and visit all four land masses.

Create a network or Vertex-Edge Graph to represent the Konigsberg Bridge Problem.

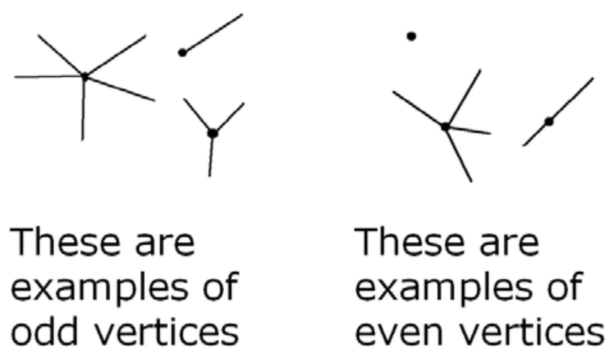
Let each land mass be represented by a point.
Let each bridge be represented by a line segment or arc.
Draw a line segment or arc between each pair of points to represent a bridge between the land masses.

Let’s try something.
Suppose we start at land mass C and want to end up at land mass B. Try adding one bridge so you travel this network.

Does it matter where you place the bridge?
Why does this make the network travelable?
Suppose we start at land mass C and want to end up at land mass B. Try removing one bridge so you travel this network.
Does it matter where you remove the bridge?
Why does this make the network travelable?

Study the networks on the next page.
Some networks can be traveled and others cannot.
Try traveling each of these networks to see which are possible and which are impossible.

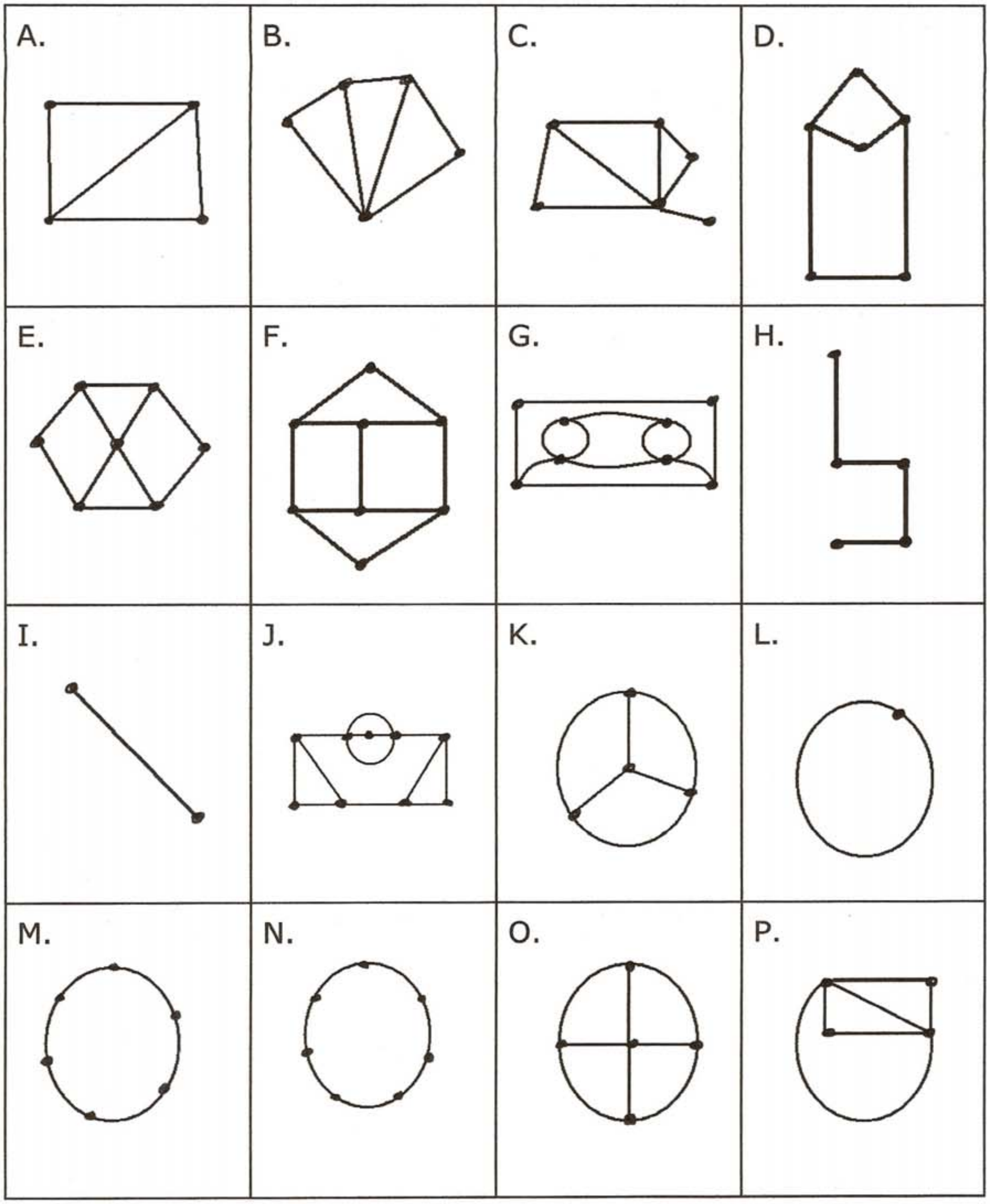
After determining which networks can be traveled and which cannot, Euler realized that is was the points of intersection that determined if a network could be traveled. He defined points of intersection as odd or even points.



What is an odd vertex?

What is an even vertex?

Chart from Discovering Geometry by Michael Serra



Make a chart of the number of odd and even vertices in each network and whether the network could be traveled.

NETWORK	A	B	C	D	E	F	G	H
NUMBER OF ODD POINTS								
NUMBER OF EVEN POINTS								
TRAVELED?								

NETWORK	I	J	K	L	M	N	O	P
NUMBER OF ODD POINTS								
NUMBER OF EVEN POINTS								
TRAVELED?								

Did you notice that all the networks that could be traveled had zero or two odd vertices? What is true about the number of odd points when the network can't be traveled?

Did you find any networks that had only one odd point?

Can you draw a network with only one odd point?

Can you draw a network that has an odd number of odd points? Explain why or why not.

Make a conjecture about the condition that must be true for a network to be traveled.

Make two new networks up:

one that cannot be traveled

one that can be traveled.

Use your conjecture to explain why each can or cannot be traveled.