

Lesson on Using Vertex Edge Drawings

Objective: Students will be able to use a vertex edge drawing to solve the common handshake problems and other problems similar to it.

Materials:

- Communicators
- Dry Erase Markers
- Erasing Cloths

Do Now: Distribute the Do Now and ask students to study the five patterns and find the next term in each sequence. Students should be ready to explain their answer when the group is brought back together.

Lesson:

- Ask for four volunteers and have them come to the front of the room. Ask these students to handshake with each other.
- Ask students to use the back side of their communicators to record their guess as to the number of handshakes that needs to take place so each person shakes hands with every other person. On the count of 3 ask students to reveal their answers by holding up the communicators.
- After students have revealed their answers ask a few students to support their answers.
- Ask students to predict the number of handshakes that will take place if we brought the entire class up to the front of the classroom. On the count of 3 ask students to reveal their answers by holding up the communicators. Survey the answer and make a record of their responses on the classroom white board.
- Tell students that we are going to develop an approach that will help them reveal the true answer.
- Ask students to clear their communicators off and make a small circle that represents four students: Andrew, Ben, Chloe, and Destiny. Each of the circles should contain one of the letters A, B, C, or D to represent one of the students.
- The teacher asks Andrew to shake hands with Ben. Ask the students to draw one line segment between circle A and B and make one tally mark on their communicator.
- The teacher asks Andrew to shake hands with Chloe. Ask the students to draw one line segment between circle A and C and make a second

- tally mark on their communicator.
- The teacher asks Andrew to shake hands with Destiny. Ask the students to draw one line segment between circle A and D and make a third tally mark on their communicator.
- Ask students if Andrew has shaken everyone's hand in the group? (Yes.)
- Turn to Ben and ask the class if Ben needs to shake Andrew's hand to complete the task outlined in the problem. (No.) Why not? (When Andrew shook Ben's hand, Ben completed shaking hands with Andrew. He does not have to do it again because each student only shakes every other student's hand exactly once.)
- Have students understand that Ben does, however, have to shake Chloe's hand. Ask students to add the line segment to their drawing and make one more tally mark. (There are now a total of four tally marks.)
- Ask students who else Ben also has to shake hands with? (Destiny). Have students add the line segment to their drawing and add one more tally mark. (There are now a total of five tally marks.)
- Ask students if completes all the handshakes for Ben. (Yes)
- Turn the students' attention to Chloe. Ask the class if Chloe has to shake either Andrew or Ben. (No.) Why not? (The task was completed when Andrew shook Chloe's hand and Ben shook Chloe's hand.)
- Does Chloe have to shake Destiny's hand? (Yes.) Have students add the line segment and one tally mark. (There are a total of six tally marks.)
- Does Destiny have to shake anyone's hand? (No.) Why not? (Everyone already shook her hand.)
- How many handshakes occurred in all when a group of four people all shook everyone else's hand in the group exactly once? (6)
- What would happen if there were five people in the group instead of four, and everyone had to shake everyone else's hand exactly once? Give students a moment to discuss it with their partners and then take conjectures from the class, and then bring up one more student and act it out.
- Have students add a circle for a 5th students, Evan, and complete the drawing to show the number of handshakes.
- Have students add a table to their communicators that they can record the number of handshakes and the number of students up front.

Number of students	3	4	5	6	7
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Number of handshakes	3	6	10	15	21
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- Ask students to work their partners to extend the chart to 6 and 7 students. (Answers have been given in the chart.)
- Ask students To look for patterns in their table that will help them determine how many handshakes will take place if the entire class were brought up front.
- Some students may extend the chart by adding the number of additional handshakes as each new student is added to the picture.
- Other students will notice that they can use the number of students to get the number of handshakes by two consecutive number of people together and dividing by 2. (For example for 4 people you need to multiply 3 people by 4 people and divide by 2 or for 5 people you need to multiply 4 people by 5 people and divide by 2. So for 20 people you need to multiply 19 people by 20 people and divide by 2 to get 190 handshakes.
- Ask students to try to extend the pattern to n people in the class.

$$\left(\frac{(n-1)(n)}{2}\right)$$

Assignment: Distribute copies of the assignment where students will solve a similar exercise to the handshake problem by drawing a vertex edge drawing and making a table.