

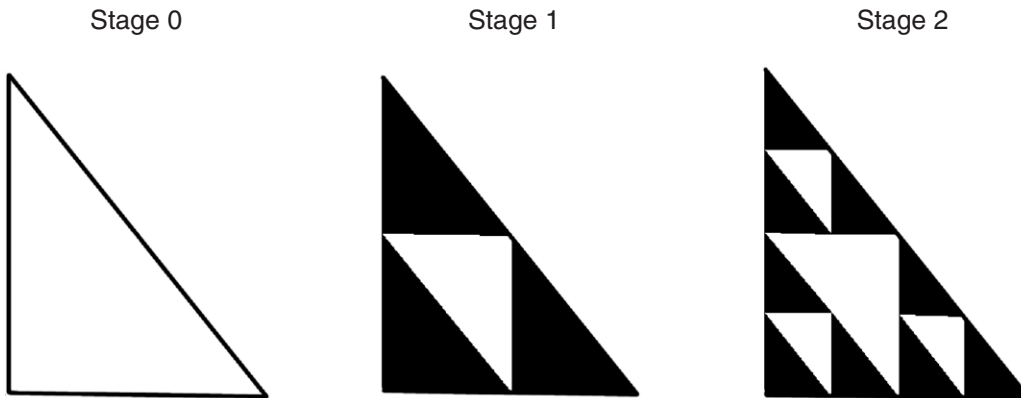


SPATIAL SENSE AND GEOMETRY

Macro IIA – Recognize, Visualize, Analyze, and Apply Geometric Properties

**Directions:** Think about what you have learned in **You Be the Judge** and **You Be the Teacher** about writing complete, accurate, and effective answers. Read and think about the task below before writing your answer and reasons in the space provided.

**The Task:** To form the pattern shown below, start with a triangle (Stage 0) and locate the midpoint of each side. Then connect the midpoints to form smaller triangles and remove the middle triangle. This completes Stage 1. To create Stage 2, locate the midpoint of each side of the remaining triangles and connect the midpoints to form smaller triangles. Again, remove the middle triangle. At each stage the triangles decrease in size, but increase in number.



- How many shaded triangles are there in Stage 3?
- If the pattern is continued, how many small shaded triangles will appear in Stage 4?
- At what stage will there be 729 shaded triangles? Support your answer.
- Will there ever be a stage with exactly 300 shaded triangles? Support your reasoning.

Below is a scoring guide or rubric that can be used to determine a score of 3, 2, 1, or 0 for a student response.

Score    Response Descriptions and Examples

3

The student indicates the correct number of shaded triangles in each stage and shows appropriate support for determining which stage has 729 shaded triangles and if any stage has exactly 300 triangles.

**Tiffany:** Each stage the triangles triple, so starting with stage 1 there are 3 shaded triangles,  $3 \times 3$  is 9 - stage 2 has 9 shaded triangles,  $9 \times 3$  is 27 so stage 3 has 27 triangles;  $27 \times 3$  is 81 - stage 4 has 81 triangles;  $81 \times 3$  is 243 - stage 5 has 243 triangles,  $243 \times 3$  is 729 - stage 6 has 729 triangles. Since stage 4 has 81 triangles and stage 5 has 243 triangles, no stage can have 300 shaded triangles because stage 6 has 729 triangles.

**Michael:** Stage 3: 27; Stage 4:  $27 \times 3 = 81$ ; Stage 6:  $81 \times 3 = 243$ ;  $243 \times 3 = 729$ ; There will never be a stage at exactly 300 because u multiply by 3 it skips 300. 300 is not in the pattern.

**Catherine:** Stage 3: 27, 81, 6th stage, No because the shaded triangles in the previous one goes up 3 times that. Therefore the number will never be 300 because 300 is not a power of 3.



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2

The student indicates the right number of shaded triangles but consistently works with the wrong stage. The student also shows adequate proof that there is never a stage with 300 shaded triangles.

**Vin:** 9 in stage 3, 27 in stage 4, 729 in stage 7; No there will never be a stage with 300 red triangles. If you follow the pattern which is all the midpoints connected to form other triangles, all the numbers are odd and since 300 is even, I don't think you will find exactly 300 shaded triangles.

**OR:** The student has the correct number of triangles for stages 3 and 4 and one of the other parts of the problem is answered correctly with appropriate support/explanation. The other part is missing, or incorrect or may be correct but has no explanation.

**Korianne:** 27 in stage 3, 81 shaded triangles in stage 4,  $729 \div 3 = 243$ , so 729 shaded triangles are in stage 243 which is stage 6. Yes, because 3 divided by 300 is 100. So, at about stage 100, there should be 300 shaded triangles.

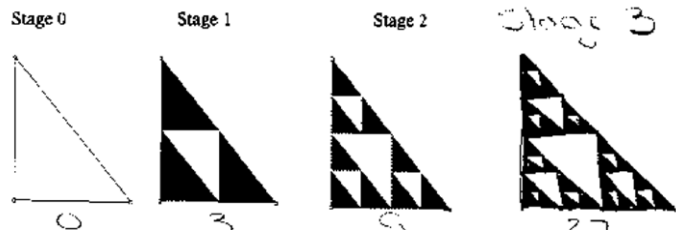
1

The student has all the correct answers but shows no work.

**Mary:** 27, 81, 6th no.

**OR:** The student shows some understanding of the pattern by completing several of the stages correctly (Stages 3 and 4, for example), but the calculations for stages with 729 and 300 triangles are incorrect or missing.

**Domina:** The number of triangles can be figured out by the equation  $f(x) = 3n$  ( $n$  = number of triangles in previous triangle,  $x$  = stage); 81 shaded triangles will be in stage 4, At stage 6 729 shaded triangles would be. No, because 300 is even.



**Lemma:** Well I saw a pattern. It is powers of 3.  $3^0, 3^1, 3^2, 3^3, 3^4, 3^5, 3^6$ , so stage 3 has 6 shaded triangles stage 4 has 9 and stage 6 has 18.

**Kris:** 27 in stage 3, 81 in stage 4.

**Joey:** 27 shaded triangles; 81, stage 2187, yes because stage increases by a numerator of 3 and 3 goes into 300 so there is a stage with 300 shaded triangles.

0

The student leaves the problem blank, answers irrelevant questions or shows work that indicates little to no understanding of the concept.

**Blanca:** There would be 13 triangles in stage 3. In stage 4 there would be 40 triangles. There would not be exactly 300 triangles because the numbers skip passed that - they go up by 3, 9, 27, 91, ext.

**Lauren:** 32 shaded triangles; 47 colored triangles in Stage 4; Stage 10 contains 729 triangles; and no its goes by adding