

## Achieving Success in Meeting the Common Core State Standards in Algebra Part II

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## Schedule

- ▶ 8:30–9:00 Introduction to Common Core State Standards Initiative and review of the Standards for Mathematical Practices
- ▶ 9:00 – 10:15 Standards for Mathematical Content Functions Interpreting Functions – Three Lessons
- ▶ 10:15 – 10:30 Break
- ▶ 10:30–11:45 Interpreting Functions – Three Lessons
- ▶ 11:45–12:45 Lunch
- ▶ 12:45–1:45 Graphs of Functions – Four Lessons
- ▶ 1:45–2:45 Graphs of Functions – Three Lessons/Examples
- ▶ 2:45–3:00 Closure

## Standards for Mathematical Practice

- ▶ 1. MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM
- ▶ 2. REASON ABSTRACTLY AND QUANTITATIVELY
- ▶ 3. CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS
- ▶ 4. MODEL WITH MATHEMATICS
- ▶ 5. APPROPRIATE TOOLS STRATEGICALLY
- ▶ 6. ATTEND TO PRECISION
- ▶ 7. LOOK FOR AND MAKE USE OF STRUCTURE
- ▶ 8. LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING

- ▶ The Standards for Mathematical Content are a balanced combination of procedure and understanding.
- ▶ Expectations that begin with the word “understanding are often especially good opportunities to connect the practices to the content.
  - Students who lack understanding of a topic may rely on procedures too heavily
  - A lack of understanding effectively prevents a student from engaging in the mathematical practices.

## Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

- ▶ The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years.
- ▶ Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

- ▶ The high school standards specify the mathematics that all students should study in order to be college and career ready.
- ▶ The high school standards are listed in conceptual categories:
  - • Number and Quantity
  - • Algebra
  - • Functions
  - • Modeling
  - • Geometry
  - • Statistics and Probability
- ▶ Conceptual categories portray a coherent view of high school mathematics; a student’s work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

- ▶ The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Based upon:
  - the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections.
  - the strands of mathematical proficiency specified in the National Research Council's report Adding It Up

## Function Overview

- ▶ **Interpreting Functions**
  - Understand the concept of a function and use function notation
  - Interpret functions that arise in applications in terms of the context
  - Analyze functions using different representations
- ▶ **Building Functions**
  - Build a function that models a relationship between two quantities
  - Build new functions from existing functions

- ▶ **Linear, Quadratic, and Exponential Models**
  - Construct and compare linear, quadratic, and exponential models and solve problems
  - Interpret expressions for functions in terms of the situation they model
- ▶ **Trigonometric Functions**
  - Extend the domain of trigonometric functions using the unit circle
  - Model periodic phenomena with trigonometric functions
  - Prove and apply trigonometric identities

## Interpreting Functions

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

## Example of a lesson

- ▶ In this lesson you will
  - Use a coding grid to write a coded message
  - Create and use a letter-shift code
  - Determine whether given relationships are functions
- ▶ You will learn about a special type of relationship called a function

The letter A is coded into the letter Q  
 The letter B is coded into the letter R  
 The letter U is coded into the letter K  
 This is an example of a *letter-shift code*.  
 How would you use the code to write a message?

Original input	A	B	C	D	E	F	G	H	I	J	K	L	M
Coded output	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

Original input	N	O	P	Q	R	S	T	U	V	R	X	Y	Z
Coded output	D	E	F	G	H	I	J	K	L	M	N	O	P

Use the coding grid to write a two-word or three-word message.

Exchange your coded message with a partner. Use this grid to decode each other's messages.

Original input	A	B	C	D	E	F	G	H	I	J	K	L	M
Coded output	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

Original input	N	O	P	Q	R	S	T	U	V	R	X	Y	Z
Coded output	D	E	F	G	H	I	J	K	L	M	N	O	P

Create a new code by writing a rule that shifts letters a certain specified number of places. Put the code on a grid like the one shown on the last slides. Do not let your partner see the grid.

Use your new grid to code the same message you wrote in the previous slide.

Exchange your newly coded message. Use it, along with the first message, to try to figure out each other's new code.

Original input	A	B	C	D	E	F	G	H	I	J	K	L	M
Coded output													

Original input	N	O	P	Q	R	S	T	U	V	R	X	Y	Z
Coded output													

- Compare your grid to your classmates' new grid.
- In what ways are the grids the same?
- How are they different?
- For one grid, how many coded outputs are possible for one input letter?
- How many ways are there to decode any one letter in a coded message?

Original input	A	B	C	D	E	F	G	H	I	J	K	L	M
Coded output													

Original input	N	O	P	Q	R	S	T	U	V	R	X	Y	Z
Coded output													

- Use the grid at the right to send a new two- or three-word message to your partner. Exchange and decode each other's message.
- Did your partner successfully decode your message? Why or why not?

- How is the grid above different from the grid used in step 1? Code the word FUNCTION to help you answer this question.
- Which grid makes it easier to decode message? Which coded output letters are difficult to decode into their original letters?

- Create a new coding scheme by shading squares that don't touch each other on the grid.
- Make the grid so that there is exactly one output for each input.
- How is it similar to the grid in step 1?
- How is it different?

- Letter–shift codes are relationships –
  - Any relationship between two variables is called a **relation**.
- Codes that have exactly one output letter for every input letter are examples a **function**.
  - The set of all input values is called the **domain**.
  - The set of all output values is called the **range**.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

### Example

Tell whether each table of values represents a function. Give the domain and range of each relation.

Table 1

Input	Output
1	2
2	4
3	6

Table 2

Input	1	0	1
Output	1	2	5

Table 3

Input	1	2	3	4	5	6
Output	0	0	0	0	0	0

- ▶ In parts a–d, identify which function,  $f(x)$ ,  $g(x)$ , or  $h(x)$  is used in each (input, output) pair.
 
$$f(x) = 7 + 4x \quad g(x) = |x| + 6 \quad h(x) = 18(1 + 0.5)^x$$
  - ▶ a. (5, 11)
  - ▶ b. (1, 27)
  - ▶ c. (2, 8)
  - ▶ d. (3, 19)

- ▶ In this investigation you will learn how the absolute–value function tells how much an item of data or a whole set of data deviates from the mean.
- ▶ Step 1: The data in the chart represents the pulse rate for 10 students.

Student	A	B	C	D	E	F	G	H	I	J
Pulse Rate	27	33	42	22	37	20	35	33	31	18
Difference from Mean										

Student	A	B	C	D	E	F	G	H	I	J
Pulse Rate	27	33	42	22	37	20	35	33	31	18
Difference from Mean										
Distance from the Mean										

- Enter the data in List L1 in your graphing calculator.
- Step 2: Find the **difference** between each data and the mean of the data in list L1.
- Record these numbers in the third row of the table and enter them in to list L2. What do these number represent?

- ▶ Step 3: Make a dot plot of the list L1 data and note the **distance** from each data point to the mean. Record your results in a fourth row and enter them into list L3.

- How are these entries different from those in list L2?
- How are they alike?



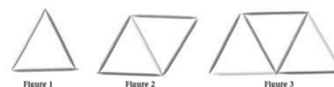
- ▶ Step 4: Next, plot points in the form (L2, L3). What numbers are in the domain and range of the graph?
- ▶ Step 5: Use the trace function on your calculator and use the arrow keys to step through the data points. Which input numbers are unchanged as output numbers?

- ▶ Step 6: Which input numbers are changed?
- ▶ Step 7: Does it make sense to connect these points with a continuous graph? Why or why not?
- ▶ Step 8: How does this graph compare to the graph of  $y = \text{abs}(x)$  on your calculator?
- ▶ Step 9: Find the mean of the deviations stored in list L2.
  - Compare it to the mean of the distances stored in list L3.
  - Which do you think is a better measure of the spread of data?

- ▶ Step 10: In your own words, write the rule for the function you graphed in Step 8.
  - What number is output as  $y$  when the input,  $x$ , is positive or equal to zero?
  - What number is output when  $x$  is negative?
  - How can you use operations to change these numbers?

3. Recognize that sequences of functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by

$$f(0) = f(1) = 1$$

$$f(n+1) = f(n) + f(n-1) \text{ for } n \geq 1.$$


- ▶ Step 1: Make Figures 1–3 of the pattern using as few toothpicks as possible. How many toothpicks does it take to reproduce each figure? How many toothpicks lie on the perimeter of each figure?
- ▶ Step 2: Copy the table with enough rows for six figures of the pattern. Make Figures 4–6 from toothpicks by adding triangles in a row and complete the table.

	Number of toothpicks	Perimeter
Figure 1		
Figure 2		

- ▶ Step 3 What is the rule for finding the number of toothpicks in each figure? What is the rule for finding the perimeter? Use your calculator to create recursive routines for these rules. Check that these routines generate the numbers in your table.
- ▶ Step 4: Now make Figure 10 from toothpicks. Count the number of toothpicks and find the perimeter. Does your calculator routine give the same answers? Find the number of toothpicks and the perimeter for Figure 25.

## Break Time

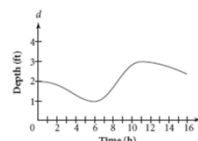


## Interpret functions that arise in applications in terms of the context

- ▶ For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

## Example

- ▶ The graph shows the depth of the water in a leaky swimming pool. Tell what quantities are varying and how they are related. Give possible real-world events in your explanation.



## The Graph Shows:

- ▶ the water level, or depth, changes over a 15-hour time period
- ▶ At the beginning ( $t=0$ ) the depth of the water in the pool is 2 feet
- ▶ The leak seems to get worse as time passes.
- ▶ When  $t = 6$  and  $d = 1$ , it seems that someone starts to refill the pool. The water level rises for the next 5 hours, during the interval  $6 \leq t \leq 11$ .
- ▶ At  $t = 11$ , the water reaches its highest level at just about 3 feet, so  $d = 3$ .
- ▶ At the 11-hour mark, the in-flowing water is apparently turned off. The pool still has a leak, so the water level starts to drop again.

- ▶ The depth of the water depends on how much time has passed
  - The depth is called the **dependent variable**.
  - The Time is the **independent variable**.
- ▶ On the graph
  - The independent variable on the horizontal axis
  - The dependent variable on the vertical axis.

- ▶ The domain is the set of all times from 0 through 16 hours. You express this interval as  $0 \leq x \leq 16$ , where  $x$  is the independent variable representing time.
- ▶ The range are the values that the dependent variable can take on or  $1 \leq y \leq 3$ , where  $y$  is depth of the water.

### Matching Up

These are graphs of increasing functions. What do the three graphs have in common? How would you describe the rate of change in each?

### Matching Up

These are graphs of decreasing functions. What do they have in common? How are they different from those in the previous step? How would you describe the rate of change of these graphs?

In the next few steps you will use this vocabulary to find and describe the graph that matches each of these real world situations. Two of the graphs will not be used.

In Situation A decide which quantities are varying. Also decide which variable is independent and which is dependent.

Match and describe the graph that best fits the situation. Write a description of the function and its graph using words such as *linear*, *nonlinear*, *increasing*, *decreasing*, *rate of change*, *maximum* or *greatest value*, and *minimum* or *least value*. Tell why you think the graph and your description match the situation

**Situation A**  
During the first few years, the number of deer on the island increased by a steady percentage. As food became less plentiful, the growth rate started slowing down. Now, the number of births and deaths is about the same.

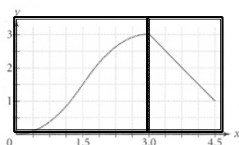
**Situation B**  
In the Northern Hemisphere the amount of daylight increases slowly from January through February, faster until mid-May, and then slowly until the maximum in June. Then it decreases slowly through July, faster from August until mid-November, and then slowly until the year's end.

**Situation C**  
If you have a fixed amount of fencing, the width of your rectangular garden determines its area. If the width is very short, the garden won't have much area. As the width increases, the area also increases. The area increases more slowly until it reaches a maximum. As the width continues to increase, the area becomes smaller more quickly until it is zero.

**Situation D**  
Your cup of tea is very hot. The difference between the tea temperature and the room temperature decreases quickly at first as the tea starts to cool to room temperature. But when the two temperatures are close together, the cooling rate slows down. It actually takes a long time for the tea to finally reach room temperature.

- ▶ What makes a function increasing?
  - when the variables change in the same way
  - the  $y$ -values *grow* when reading the graph from left to right.
- ▶ What makes a function decreasing?
  - when the variables change in different directions
  - the  $y$ -values *drop* when reading the graph from left to right.

Describe this graph, telling how the quantities in the graph relate to each other.



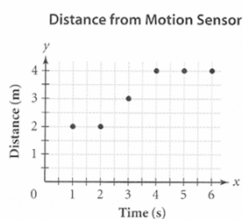
On the interval  $0 \leq x < 3.0$ , the function is nonlinear and increasing. As  $x$  increases steadily,  $y$  changes at a varying rate, so the graph is nonlinear. The  $y$ -values grow and the function is increasing.

On the interval  $3.0 \leq x \leq 4.5$ , the function appears linear and is decreasing. Because  $y$  appears to change at a constant rate on the graph, the function is linear. The  $y$ -values drop and the function is decreasing.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.

## Two-Variable Data

▶ This scatter plot shows how the distance from a motion sensor to a person varies over a period of 6 seconds. Describe where the person is in relation to the sensor at each second.



6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Internet Use


Month	Time (h)	Total fee (\$)
September	40	16.55
October	50	19.45
November	80	28.15

▶ James recently signed up with a limited-usage Internet provider. There is a flat monthly charge and an hourly rate for the number of hours he is connected during the month. The table shows the amount of time he spent using the Internet for the first three months and the total fee he was charged.

Internet Use

Month	Time (h)	Total fee (\$)
September	40	16.55
October	50	19.45
November	80	28.15


▶ Step 1: Is there a linear relationship between the time in hours that Hector uses the Internet and his total fee in dollars? If so, why do you think such a relationship exists?  
 ▶ Step 2: Use the numbers in the table to find the hourly rate in dollars per hour.  
 ▶ Explain how you calculated this rate.



The slope triangle shows the vertical and horizontal change from one point to another.

Internet Use		
Month	Time (h)	Total fee (\$)
September	40	16.55
October	50	19.45
November	80	28.15

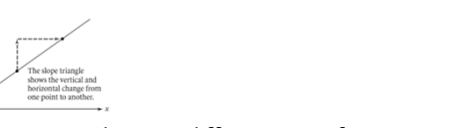
- Step 3: Draw a pair of coordinate axes on graph paper. Use the  $x$ -axis for time in hours and the  $y$ -axis for total fee in dollars. Plot and label the three points the table of data represents. Draw a line through the three points. Does this line support your answer in Step 1?
- Step 4: Choose two points on your graph. Use arrows to show how you could move from one point to the other using only one vertical move and one horizontal move. How long is each arrow? What are the units of these values?



The slope triangle shows the vertical and horizontal change from one point to another.

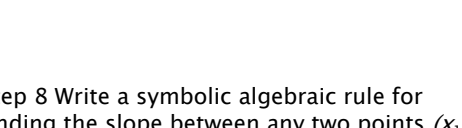
Internet Use		
Month	Time (h)	Total fee (\$)
September	40	16.55
October	50	19.45
November	80	28.15

- Step 5: How do the arrow lengths relate to the hourly rate that you found in Step 2? Use the arrow lengths to find the hourly rate of change, or slope, for this situation. What units should you apply to the number?




The slope triangle shows the vertical and horizontal change from one point to another.

- Step 6: Choose a different pair of points on your graph. Create a slope triangle between them and use it to find the slope of the line. How does this slope compare to your answers in Step 2 and Step 5?
- Step 7: Think about what you have done with your slope triangles. How could you use the coordinates of any two points to find the vertical change and the horizontal change of each arrow? Write a single numerical expression using the coordinates of two points to show how you can calculate slope.



The slope triangle shows the vertical and horizontal change from one point to another.

- Step 8: Write a symbolic algebraic rule for finding the slope between any two points  $(x_1, y_1)$  and  $(x_2, y_2)$ . The subscripts mean that these are two distinct points of the form  $(x, y)$ .



## Lunch Student Center

## Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- ▶ Maria starts her exercise routine by jogging to the gym. Her trainer says this activity burns 215 calories. Her workout at the gym is to pedal a stationary bike. This activity burns 3.8 calories per minute.

Pedaling time (min) $x$	Total calories burned $y$
0	215
1	
2	
20	
30	
45	
60	

- ▶ Step 1: Use calculator lists to write a recursive routine to find the total number of calories Maria has burned after each minute she pedals the bike. Include the 215 calories she burned on her jog to the gym.
- ▶ Step 2: Copy and complete the table using your recursive routine.
- ▶ Step 3: After 20 minutes of pedaling, how many calories has Maria burned? How long did it take her to burn 443 total calories?

Pedaling time (min) $x$	Total calories burned $y$
0	215
1	
2	
20	
30	
45	
60	

- ▶ Step 4: Write an expression to find the total calories Maria has burned after 20 minutes of pedaling. Check that your expression equals the value in the table.
- ▶ Step 5: Write and evaluate an expression to find the total calories Maria has burned after pedaling 38 minutes. What are the advantages of this expression over a recursive routine?

Pedaling time (min) $x$	Total calories burned $y$
0	215
1	
2	
20	
30	
45	
60	

- ▶ Step 6: Let  $x$  represent the pedaling time in minutes, and let  $y$  represent the total number of calories Maria burns. Write an equation relating time to total calories burned.
- ▶ Step 7: Check that your equation produces the corresponding values in the table.

Pedaling time (min) $x$	Total calories burned $y$
0	215
1	
2	
20	
30	
45	
60	

- ▶ Step 8: Plot the points from your table on your calculator. Then enter your equation into the Y menu. Graph your equation to check that it passes through the points. Give two reasons why drawing a line through the points realistically models this situation.
- ▶ Step 9: Substitute 538 for  $y$  in your equation to find the elapsed time required for Maria to burn a total of 538 calories. Explain your solution process. Check your result.

Pedaling time (min) $x$	Total calories burned $y$
0	215
1	
2	
20	
30	
45	
60	

- ▶ Step 10: How do the starting value and the rule of your recursive routine show up in your equation? How do the starting value and the rule of your recursive routine show up in your graph? When is the starting value of the recursive routine also the value where the graph crosses the  $y$ -axis?

Pedaling time (min) $x$	Total calories burned $y$
0	215
1	
2	
20	
30	
45	
60	

- Maria's workout showed the linear relationship between total calories burned and the number of minutes pedaling on the bike.

$$y = 215 + 3.8x \text{ or } y = 3.8x + 215$$

- This form  $y=a + bx$  is called the **INTERCEPT FORM**. The value of  $a$  is the **y-intercept**, which is the value of  $y$  when  $x = \text{zero}$ . The intercept gives the location where the graph crosses the  $y$ -axis. The number multiplied by  $x$  is  $b$ , which is called the **coefficient** of  $x$ .

7. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

### Making the Most of It

- Find the dimensions of at least eight different rectangular regions, each with perimeter 24 meters. You must use all of the fencing material for each garden.
- Find the area of each garden. Make a table to record the width, length, and area of the possible gardens. It's okay to have widths that are greater than their corresponding lengths.

Width (m)	Length (m)	Area (m <sup>2</sup> )

- Enter the data for the possible widths into list L1. Enter the area measures into list L2. Which garden width values would give no area? Add these points to your lists.
- Label a set of axes and plot points in the form  $(x, y)$ , with  $x$  representing width in meters and  $y$  representing area in square meters. Describe as completely as possible what the graph looks like. Does it make sense to connect the points with a smooth curve?

### Making the Most of It

- Where does your graph reach its highest point? Which rectangular garden has the largest area? What are its dimensions?

Width (m)	Length (m)	Area (m <sup>2</sup> )

- Create a graph of (width, length) data. What is the length of the garden that has a width of 2 meters? Width 4.3 meters? Write an expression for length in terms of width  $x$ .
- Using your expression for the length from the previous step, write an equation for the area of the garden. Enter this equation into Y1 and graph it. Does the graph confirm your answer for the size of the rectangle with the largest area?

- Locate the points where the graph crosses the  $x$ -axis. What is the real-world meaning for these points?
- Do you think the general shape of a garden with maximum area would change for different perimeters? Explain your answer.

- The two points on the  $x$ -axis are the  $x$ -intercepts. The  $x$ -values of those points are the solutions to the equation  $y=f(x)$  when the function value is equal to zero. These solutions are the **roots of the equation  $f(x)=0$** .

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

### Maria's Exercise Program

- Maria starts her exercise routine by jogging to the gym. Her trainer says this activity burns 215 calories. Her workout at the gym is to pedal a stationary bike. This activity burns 3.8 calories per minute.

Pedaling time (min) $x$	Total calories burned $y$
0	215
1	
2	
20	
30	
45	
60	

First you'll model this scenario with your calculator.

Step 1 Use calculator lists to write a recursive routine to find the total number of calories Maria has burned after each minute she pedals the bike. Include the 215 calories she burned on her jog to the gym.

Step 2 Copy and complete the table using your recursive routine.

Step 3 After 20 minutes of pedaling, how many calories has Maria burned? How long did it take her to burn 443 total calories?

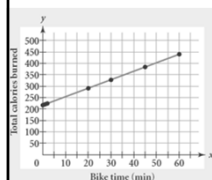
- Next you'll learn to write an equation that gives the same values as the calculator routines.
- Step 4 Write an expression to find the total calories Maria has burned after 20 minutes of pedaling. Check that your expression equals the value in the table.
- Step 5 Write and evaluate an expression to find the total calories Maria has burned after pedaling 38 minutes. What are the advantages of this expression over a recursive routine?
- Step 6 Let  $x$  represent the pedaling time in minutes, and let  $y$  represent the total number of calories Maria burns. Write an equation relating time to total calories burned.

- ▶ Step 7 Check that your equation produces the corresponding values in the table. Now you'll explore the connections between the linear equation and its graph.
- ▶ Step 8 Plot the points from your table on your calculator. Then enter your equation into the Y menu. Graph your equation to check that it passes through the points. Give two reasons why drawing a line through the points realistically models this situation.

- ▶ Step 9 Substitute 538 for  $y$  in your equation to find the elapsed time required for Maria to burn a total of 538 calories. Explain your solution process. Check your result.
- ▶ Step 10 How do the starting value and the rule of your recursive routine show up in your **equation**? How do the starting value and the rule of your recursive routine show up in your **graph**? When is the starting value of the recursive routine also the value where the graph crosses the  $y$ -axis?

## Maria's Exercise

$$y = 215 + 3.8x \text{ or } y = 3.8x + 215$$



Pedaling time (min) $x$	Total calories burned $y$
0	215
1	
2	
20	
30	
45	
60	

## Building Functions

- Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.

- ▶ A minivan is 220 mi from its destination, Cape May. It begins traveling toward Cape May at 72 mph.
  - a) Define variables and write an equation in intercept form for this relationship.
  - b) Use your equation to calculate the location of the minivan after 2.5 hours.
  - c) Use your equation to calculate when the minivan will be 130 miles from Cape May.
  - d) Graph the relationship and locate the points that are the solutions to parts b and c.
  - e) What is the real-world meaning of the rate of change in this relationship? What does the sign of the rate of change indicate?

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

### Build new functions from existing functions

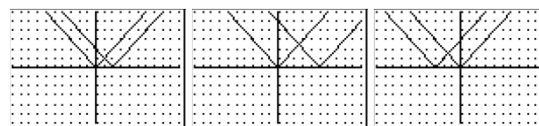
3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

### Translations of Functions

- First you'll transform the absolute-value function by making changes to  $x$ .
- Enter  $y = |x|$  into  $y1$  and graph it on your calculator.
- If you replace  $x$  with  $x-3$  in the function  $y = |x|$ , you get  $y = |x-3|$ . Enter  $y2=|x-3|$  and graph it.
- How have you transformed the graph of  $y = |x|$ ?
- Name the coordinates of the vertex of the graph of  $y = |x|$ . Name the coordinates of the vertex of the graph of  $y = |x-3|$ . How did these two points help verify the transformation you just performed?

- ▶ Find a function for  $y2$  that will translate the graph of left 4 units. What is the function? In the equation  $y = |x|$ , what did you replace  $x$  with to get your new function?

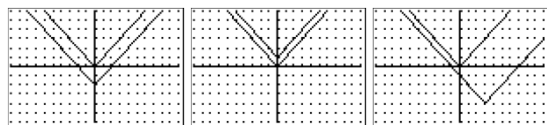
- ▶ Write a function for  $y2$  to create each graph below. Check your work by graphing both  $y1$  and  $y2$ .



- Next you will transform the absolute-value function by making changes to  $y$ .
- Clear all of the functions. Enter  $y1 = |x|$  and graph it.
- If you replace  $y$  with  $y-3$  in the function  $y = |x|$ , you get  $y-3 = |x|$ . Solve it for  $y$  and you get  $y = |x| + 3$ . Enter  $y2 = |x| + 3$ . Graph it.
- Think of the graph of  $y = |x|$  as the original figure and the graph of  $y = |x| + 3$  as its image. How have you transformed the graph of  $y = |x|$ ?
- Name the coordinates of the vertex of both graphs. How do these two points help verify the transformation you just found.

- ▶ Find a function  $y2$  that will translate the graph of  $y = |x|$  down 3 units. What is the function? In the function,  $y = |x|$ , what did you replace  $y$  with to get your new function?

- Write a function for  $y_2$  to create each graph below. Check your work by graphing both  $y_1$  and  $y_2$ .



- Summarize what you have learned about translating the absolute-value graph vertically and horizontally.

#### 4. Find inverse functions.

#### How do you find the inverse of a function?

- The equation  $y = \frac{9}{5}x + 32$

converts temperature from  $x$  in degrees C to  $y$  in degrees F.

- If you want to write an equation that converts temperature from degrees F to degrees C, you can swap the two variables in the equation and solve for  $y$ .

$$\begin{array}{ll}
 y = \frac{9}{5}x + 32 & \text{Original Equation} \\
 x = \frac{9}{5}y + 32 & \text{Interchange } x \text{ and } y \\
 x - 32 = \frac{9}{5}y & \text{Subtract 32 from both sides} \\
 5(x - 32) = 9y & \text{Multiply both sides by 5} \\
 \frac{5}{9}(x - 32) = y & \text{Divide both sides by 9} \\
 y = \frac{5}{9}(x - 32) & \text{Isolate } y \text{ on the left side}
 \end{array}$$

$$y = \frac{9}{5}x + 32 \qquad y = \frac{5}{9}(x - 32)$$

Note that after the switch,  $x$  represents degrees in F and  $y$  represents degrees in C.

The domain of the inverse is the range of the original function, and vice versa.

Is this a function?

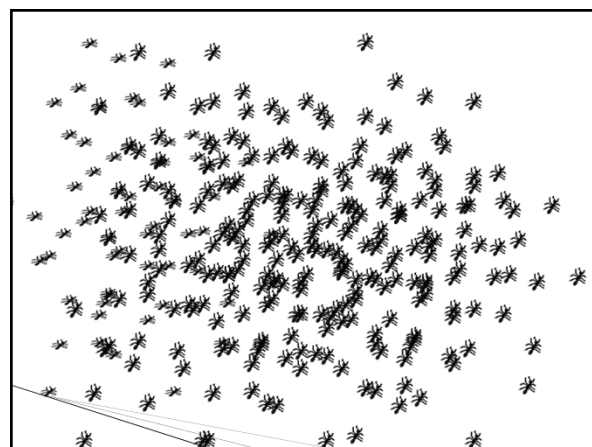
Does each input in degrees F give exactly one output in degrees C?

- ▶ How can you tell from the graph of a function whether it has an inverse that's a function?
- ▶ Does every linear function in the form  $y = a + bx$  have an inverse function?
- ▶ Does the squaring function or the absolute value function have an inverse function?
- ▶ Look for patterns in the graphs of these functions and others.
- ▶ Can you restrict the value on the domain of a function so that the inverse is a function?

- Linear, Quadratic, and Exponential Models
- ▶ Construct and compare linear, quadratic, and exponential models and solve problems
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

**Bugs, Bugs, Everywhere Bugs**

- ▶ Imagine that a bug population has invaded your classroom.
- ▶ One day you noticed 16 bugs.
- ▶ Every day new bugs hatch, increasing the population by 50% each week.
- ▶ In the first week the population increases by 8 bugs.



In a table, record the total number of bugs at the end of each week for 4 weeks.

Bugs, Bugs, Everywhere Bugs	
WEEKS ELAPSED	TOTAL NUMBER OF BUGS
START (0)	
1	
2	
3	
4	

The increase in the number of bugs each week is the population's rate of change per week. Calculate each rate of change. What are the units?

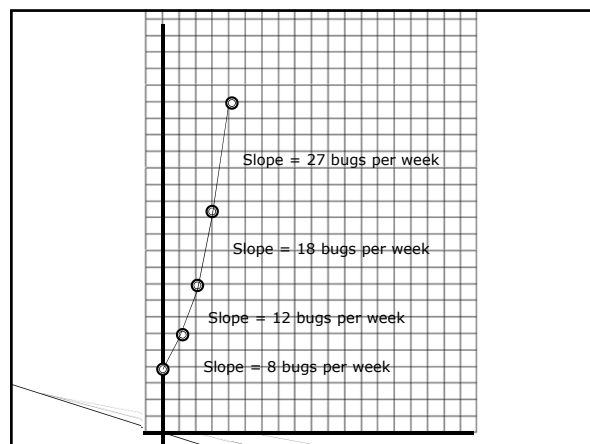
Bugs, Bugs, Everywhere Bugs			
WEEKS ELAPSED	TOTAL NUMBER OF BUGS	INCREASE IN THE NUMBER OF BUGS FROM PREVIOUS WEEK	RATIO OF THIS WEEK'S TOTAL
START (0)			
1			
2			
3			
4			

Does the rate of increase show a linear pattern? Why or why not?

Let  $x$  represent the number of weeks elapsed and let  $y$  represent the total number of bugs. Graph the data using  $(0,16)$  for the first point.

Bugs, Bugs, Everywhere Bugs		
WEEKS ELAPSED	TOTAL NUMBER OF BUGS	INCREASE IN THE NUMBER OF BUGS FROM PREVIOUS WEEK
START (0)		
1		
2		
3		
4		

Connect the points with line segments. Describe how the slope changes from point to point.



Calculate the ratio of the number of bugs each week to the number of bugs the previous week. Record it in the table.

Bugs, Bugs, Everywhere Bugs			
WEEKS ELAPSED	TOTAL NUMBER OF BUGS	INCREASE IN THE NUMBER OF BUGS FROM PREVIOUS WEEK	RATIO OF THIS WEEK'S TOTAL TO LAST WEEK'S TOTAL
START (0)			
1			
2			
3			
4			

How do the ratios compare? Explain what the ratios tell you about the bug population growth.

What is the constant multiplier for the bug population?

Bugs, Bugs, Everywhere Bugs			
WEEKS ELAPSED	TOTAL NUMBER OF BUGS	INCREASE IN THE NUMBER OF BUGS FROM PREVIOUS WEEK	RATIO OF THIS WEEK'S TOTAL TO LAST WEEK'S TOTAL
START (0)			
1			
2			
3			
4			

How can you use this number to calculate the population when 5 weeks have elapsed?

Write a recursive routine that models the population's growth for the growing number of bugs.

Describe what each part of this calculator command does.

By pressing ENTER a few times, check that your recursive routine gives the sequence of values in your table.

Use the routine to find the bug population at the end of weeks 5 to 8.

```

{0, 16}
{Ans(1)+1, Ans(2)
 *1.5}
{1 24}

{0, 16}
{Ans(1)+1, Ans(2)
 *1.5}
{1 24}
{2 36}
{3 54}

{2 36}
{3 54}
{4 81}
{5 121.5}
{6 182.25}
{7 273.375}
{8 410.0625}
    
```

What is the population after 20 weeks?  
After 30 weeks?  
What happens in the long run?

## Comparing Growths

- ▶ Maria has saved \$10,000 and wants to invest it for her daughter's college tuition. She is considering to options.
  - Plan A guarantees a payment, or return, of \$550 each year.
  - Plan B grows by 5% each year.
- ▶ With each plan, what would Maria's new balance be after 5 years? After 10 years?

## Plan A

YEAR	CURRENT BALANCE	RETURN	BALANCE
1	10,000	+ 550	= 10,550
2	10,550	+ 550	= 11,100
3	11,100	+ 550	= 11,650
4	11,650	+ 550	= 12,200

Write a recursive routine to do this on your calculator.

```

(0,10000)
(0 10000)
(Ans(1)+1,Ans(2)
+550)
(1 10550)
(Ans(1)+1,Ans(2)
+550)
(2 11100)
(Ans(1)+1,Ans(2)
+550)
(3 11650)
(Ans(1)+1,Ans(2)
+550)
(4 12200)
    
```

## Create a chart

- ▶ Record your totals on the chart for Plan A for the first 20 years.

Year	Plan A
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

## Plan B

The factor  $1+0.05$  is called a constant multiplier

YEAR	CURRENT BALANCE	RETURN	BALANCE
1	10,000	+ $10000 \times 0.05$	= $10000(1+0.05)$ = 10,500
2	10,500	+ $10500 \times 0.05$	= $10500(1+0.05)$ = 11,025
3	11,025	+ $11025 \times 0.05$	= $11025(1+0.05)$ $\approx$ 11,576
4	11,576	+ $11576 \times 0.05$	= $11576(1+0.05)$ $\approx$ 12,155

Write a recursive routine to do this on your calculator.

```

(0,10000)
(0 10000)
(Ans(1)+1,Ans(2)
*(1+.05))
(1 10500)
(Ans(1)+1,Ans(2)
*(1+.05))
(2 11025)
(Ans(1)+1,Ans(2)
*(1+.05))
(3 11576.25)
(Ans(1)+1,Ans(2)
*(1+.05))
(4 12155.0625)
(Ans(1)+1,Ans(2)
*(1+.05))
(5 12762.815625)
    
```

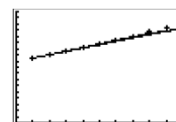
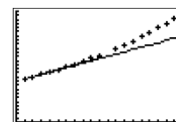
## Create a chart

- ▶ Record your totals on the chart for Plan B for the first 20 years.

Year	Plan A	Plan B
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

## Create a graph of Plan A vs. Plan B

- Enter the data from the chart in your calculator.
  - Place the year in L1
  - Balance for Plan A in L2
  - Balance for Plan B in L3
- Create a graph of
  - L1 vs. L2 - Linear Growth
  - L1 vs. L3 - Exponential Growth
- Compare the graphs. What statements can you make about the two graphs?



2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
  3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
  4. For exponential models, express as a logarithm the solution to  $abct = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- Interpret expressions for functions in terms of the situation they model
5. Interpret the parameters in a linear or exponential function in terms of a context.

Do you think activities like we looked at today will help students:

- ▶ 1. MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM
- ▶ 2. REASON ABSTRACTLY AND QUANTITATIVELY
- ▶ 3. CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS
- ▶ 4. MODEL WITH MATHEMATICS
- ▶ 5. APPROPRIATE TOOLS STRATEGICALLY
- ▶ 6. ATTEND TO PRECISION
- ▶ 7. LOOK FOR AND MAKE USE OF STRUCTURE
- ▶ 8. LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING

And Build the Understanding described in the



Key Takeaways from the High School Common Core State Standards Initiative in Mathematics

- ▶ The high school standards
  - call on students to practice applying mathematical ways of thinking to real world issues and challenges: they prepare students to think and reason mathematically
  - Set a rigorous definition of college and career readiness, by helping students develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees regularly do.
  - Emphasize mathematical modeling, the use of mathematics and statistics to analyze empirical situations, understand them better, and improve decisions.

Achieving Success in  
Meeting the Common Core  
State Standards in Algebra  
Part II

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