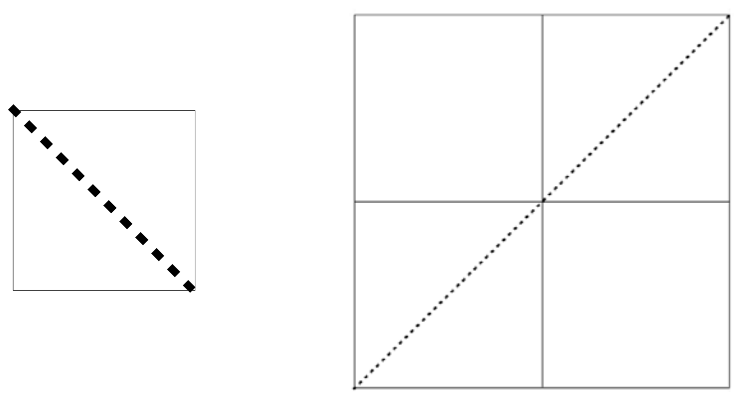
Activities that Build Understanding for Working with Radicals and Radical Equations



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Do Now

Write each of the following as a square of a number:

Examples: $4 = 2^2$

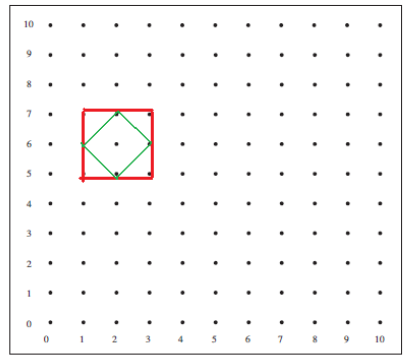
9 =	225 =
484 =	36 =
529=	625 =
196 =	576 =
81 =	289 =
25 =	144 =
361 =	324 =
49 =	441 =
121 =	1 =
400 =	16 =
169 =	64 =
256 =	100 =

Providing a Visual Model to Simplifying Radicals

Do Now: Project the page of perfect squares and have students complete the square statements for each given number.

Distribute geoboards to students while they complete the Do Now Problems. Each board should be accompanied with two geobands.

Distribute copies of the “These Squares Are Really Radical” chart for students to record various measurement as they are work with squares on the geoboard.



Ask students to make a square that contains four square units with one geoband. Ask students to describe the length of the side of the square. Tell students that because the square has an area of 4 and the side of the square is 2 we will write the following statement: $\sqrt{4} = 2$. Ask students to use another geoband to connect the midpoints of each side. Model for students why the inside figure is also a square. (All sides are equal because they are all hypotenuses of congruent right triangle. Slopes of adjacent sides are -1 and 1 therefore, the sides are perpendicular.) Ask students to find the area of the inside square. (Area is 2) If the area of the square is 2, then how long is the side? ($\sqrt{2}$).

But how long is $\sqrt{2}$? Place your thumb or index finger between to pegs on the geoboard. Can you feel both pegs on either side of your finger? Now place the same finger on the side that equals $\sqrt{2}$. What does this tell you about $\sqrt{2}$?

Take an edge of a piece of paper and make a unit ruler by marking the length of 1, 2, 3, 4, 5, 6, 7, 8 units on the edge of the paper. Line the unit ruler up along the side of the square that holds two square units. What does this tell you about $\sqrt{2}$?

Make a record of the various area and lengths from this first activity on “These Squares are Really Radical” Chart.

Now ask students to build a square that contains 16 square units. What square root statement can we make? $\sqrt{16} = 4$. Again build the square inside this square by connecting the midpoints. What is the area of the inside square? (8). What square root statement can we make about this figure? (The length of the side = $\sqrt{8}$) Look carefully at the side the inside square. Can you find another name for the length of the side? $\sqrt{8} = 2\sqrt{2}$. Approximate the length of this side using your unit ruler.

Make a record of the various area and lengths from this second activity on "These Squares are Really Radical" Chart.

Now ask students to build a square that contains 36 square units. What square root statement can we make? $\sqrt{36} = 6$. Again build the square inside this square by connecting the midpoints. What is the area of the inside square? (18). What square root statement can we make about this figure? (The length of the side = $\sqrt{18}$) Look carefully at the side the inside square. Can you find another name for the length of the side? $\sqrt{18} = 3\sqrt{2}$. Approximate the length of this side using your unit ruler.

Make a record of the various area and lengths from this third activity on "These Squares are Really Radical" Chart.

Now ask students to build a square that contains 64 square units. What square root statement can we make? $\sqrt{64} = 8$. Again build the square inside this square by connecting the midpoints. What is the area of the inside square? (32). What square root statement can we make about this figure? (The length of the side = $\sqrt{32}$) Look carefully at the side the inside square. Can you find another name for the length of the side? $\sqrt{32} = 4\sqrt{2}$. Approximate the length of this side using your unit ruler.

Make a record of the various area and lengths from this fourth activity on "These Squares are Really Radical" Chart.

Can you use a geoband to show a line that equals $5\sqrt{2}$? Can you place your line segment in such a location on the geoboard so you can see the square associated with this segment? What is the area of the square? (50) Is there a square that surrounds this square? (Yes, 10 x 10) Approximate the length of this side using your unit ruler. Make a record of the various area and lengths from this fifth activity on "These Squares are Really Radical" Chart.

Look at your completed chart. What patterns do you see?
What other predictions can you make from the chart?

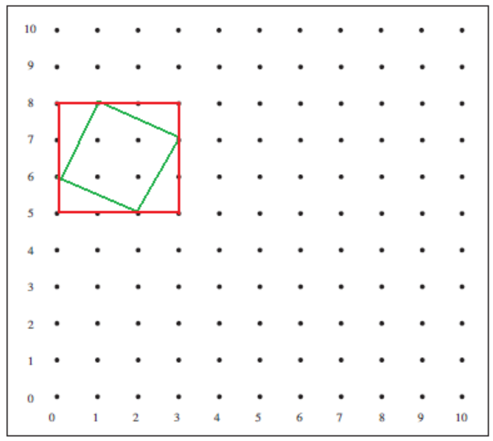
Predict what the line in the chart will look like when the side of the outside square is 20 units.

Predict what the line in the chart will look like when the area of the outside square is 484 square units.

Predict what the line in the chart will look like when the area of the inside square is 98 square units.

Predict what the line in the chart will look like when the length of the inside square is $12\sqrt{2}$ units.

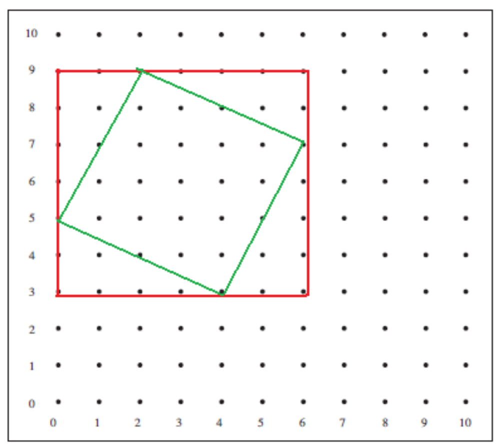
Predict what the line in the chart will look like when the length of the inside square is $\sqrt{100}$ units.



Can we see with same relationship in other squares?

Build a square that has an area of 9 square units. Write a square root statement about this square. $\sqrt{9} = 3$. Connect the points that are one unit from the vertex. Again confirm that this figure is a square. What is the area of the inside square? (5) Each triangle that surrounds the square has an area of 1 square unit, therefore, 9-4=5.

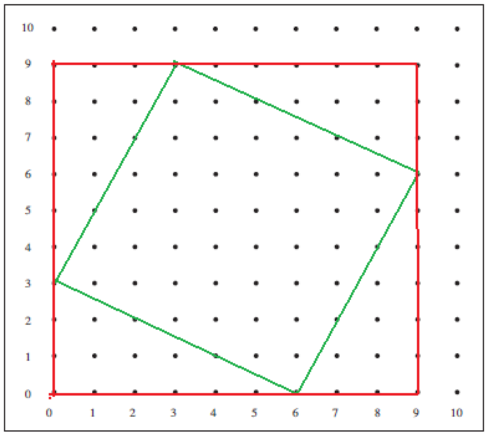
How long is the side of the square? ($\sqrt{5}$). Approximate the length of this side using your unit ruler.



Make a record of the various area and lengths from this first activity on “These Squares are Really Radical” Chart.

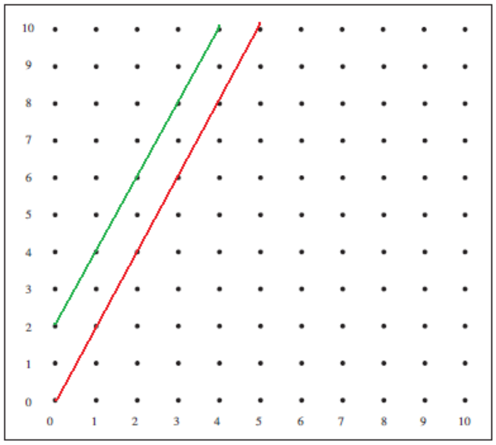
Build a square that contain 36 square units. square that has an area of 9 square units. Write a square root statement about this square. $\sqrt{36} = 6$. Connect the points that are two units from the vertex. Again confirm that this figure is a square. What is the area of the inside square? (20) Each triangle that surrounds the square has an area of 1 square unit, therefore, 36-16=20. How long is the side of the square? ($\sqrt{20}$). Look carefully at the sides. Is there another way we could write the length of the side?

($2\sqrt{5}$) Approximate the length of this side using your unit ruler.



Make a record of the various area and lengths from this second activity on "These Squares are Really Radical" Chart.

Build a square that contain 81 square units. Write a square root statement about this square. $\sqrt{81} = 9$. Connect the points that are three units from the vertex. Again confirm that this figure is a square. What is the area of the inside square? (45) Each triangle that surrounds the square has an area of 1 square unit, therefore, 81-36=45. How long is the side of the square? ($\sqrt{45}$). Look carefully at the sides. Is there another way we could write the length of the side? ($3\sqrt{5}$) Approximate the length of this side using your unit ruler.



Make a record of the various area and lengths from this third activity on "These Squares are Really Radical" Chart.

Try to draw a line segment that equals $4\sqrt{5}$ *and* $5\sqrt{5}$. Describe the size of the square associated with each. (The first square will hold 80 squares and the second will contain 125 squares.) Explain how you reasoned this. (The square whose side is $4\sqrt{5}$ would have an area of $\left(4\sqrt{5}\right)^2$ or $4 \bullet 4 \bullet 5 = 16 \bullet 5 = 80$. The square whose side is $5\sqrt{5}$ would have an area of $\left(5\sqrt{5}\right)^2$ or $5 \bullet 5 \bullet 5 = 25 \bullet 5 = 125$.) Approximate the length of this side using your unit ruler.

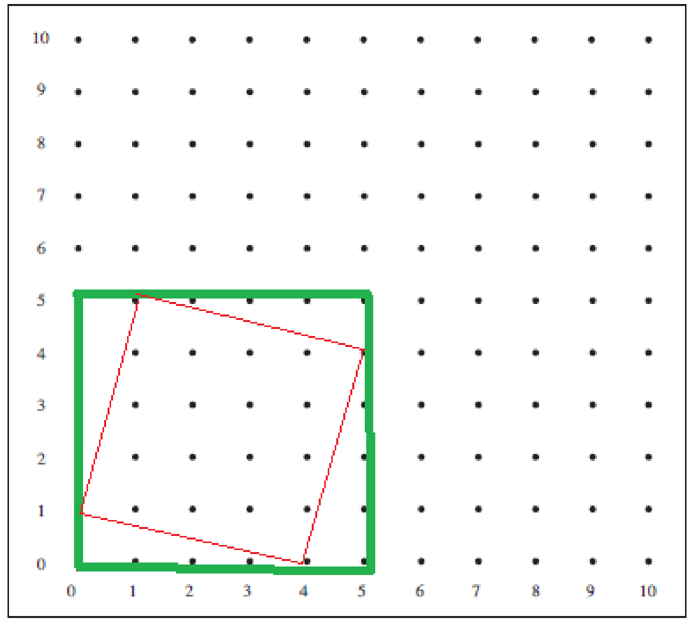
Make a record of the various area and lengths from this fourth activity on "These Squares are Really Radical" Chart.

Assignment

The geoboard at the right illustrates two squares.

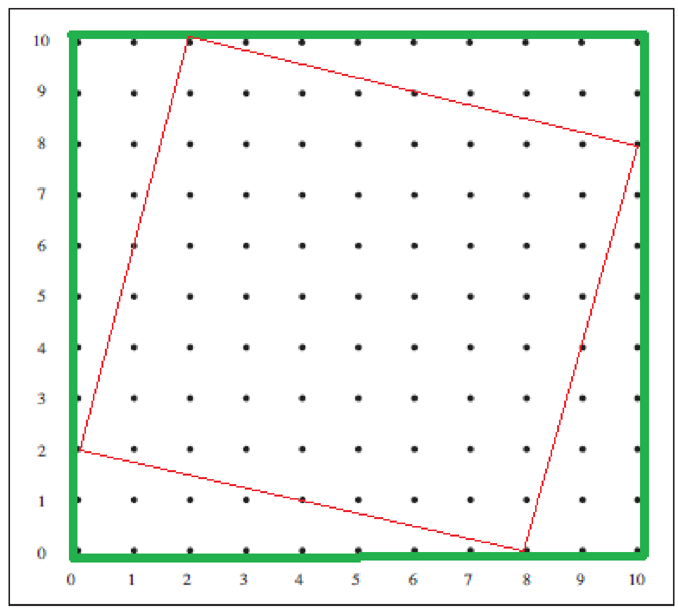
Find the area of the larger square.

Show how you find the area of the inside square.



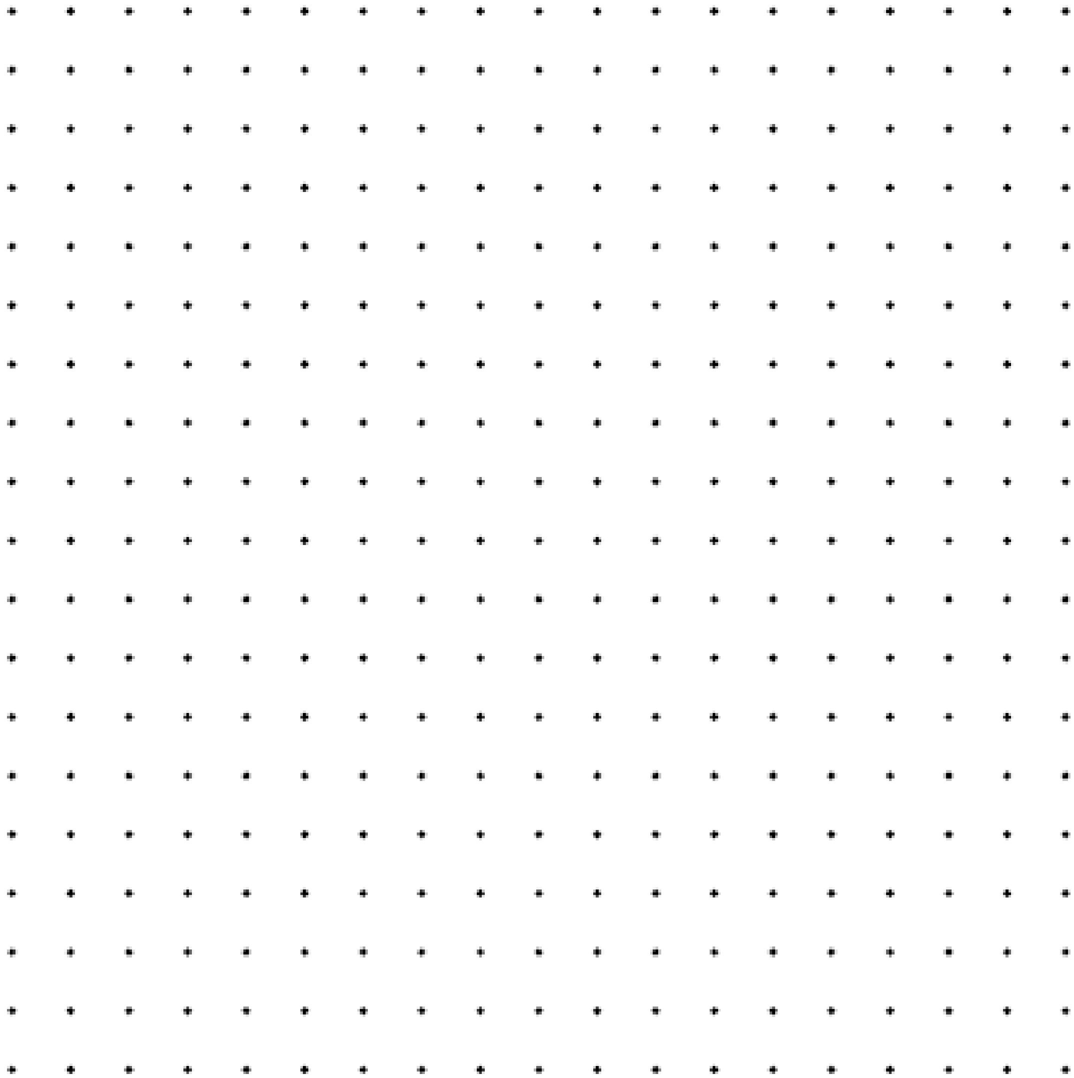
Find the length of the side of each square.

The second geoboard shows two additional squares. Using the first geoboard, describe how you can find the length of the inside square.



Find the area of the inside square from the length of the side of the inside square.

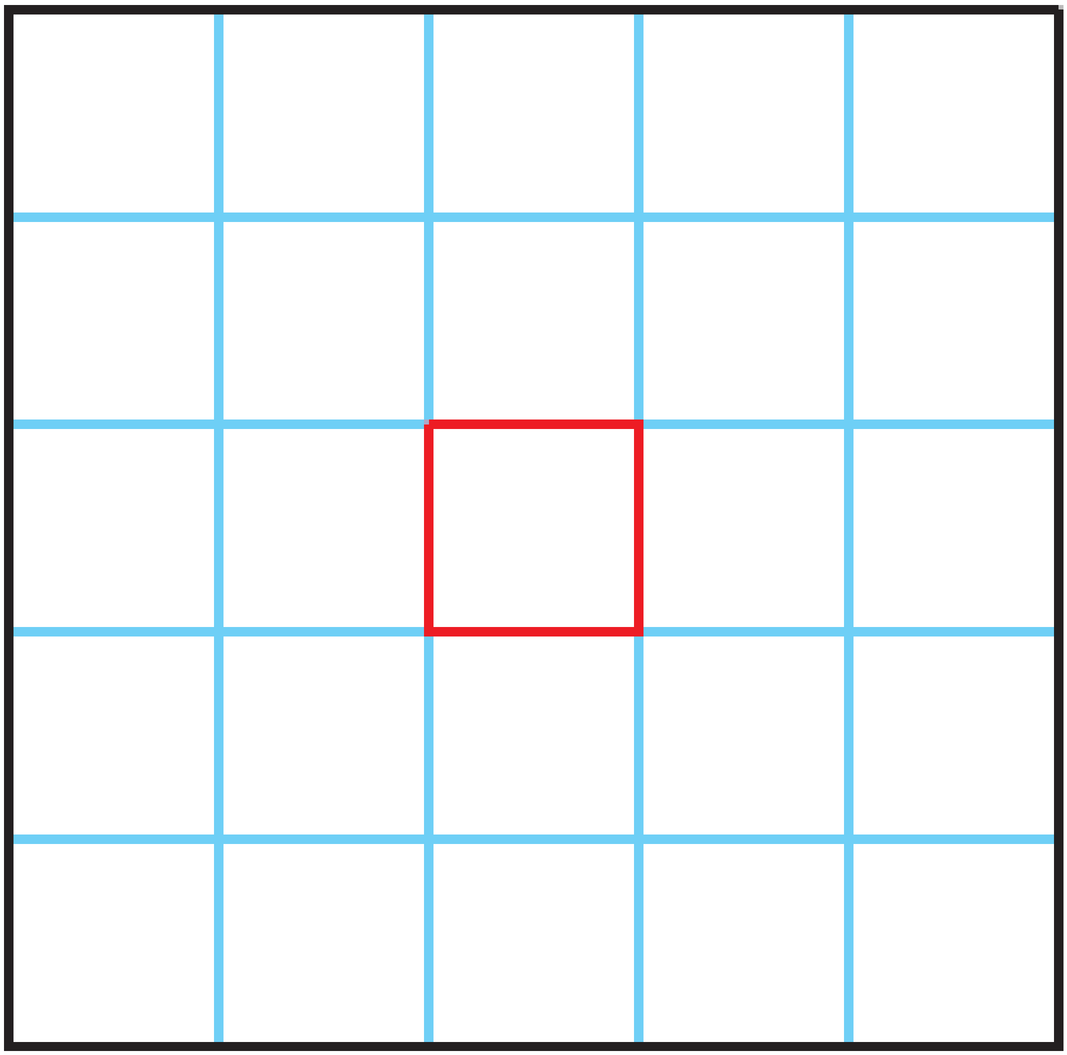
It is not possible to illustrate a square whose side is $3\sqrt{17}$ on this geoboard. Describe the size of the outside square and how to produce the inside square whose side is $3\sqrt{17}$. (If you want to illustrate the new geoboard, You can use the reverse side of this paper.) Find the area of the outside and inside squares.



These Squares Are Really Radical

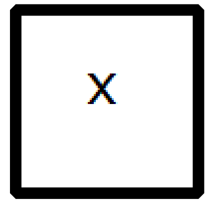
LENGTH OF SIDE ON THE OUTSIDE SQUARE	AREA OF OUTSIDE SQUARE	AREA OF INSIDE SQUARE	EXACT LENGTH OF SIDE ON THE INSIDE SQUARE	APPROXIMATE LENGTH OF SIDE ON THE INSIDE SQUARE

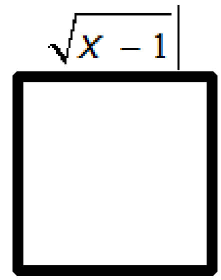
Drawing Squares



Can the Area Model Help Me Solve Radical Equations?

Recall that if a square has an area of 4, then its side equals $\sqrt{4}$ or 2. Similarly, if a square has an area of 3, then its side has a length of $\sqrt{3}$. Label the side of this square if the area of the square is x.



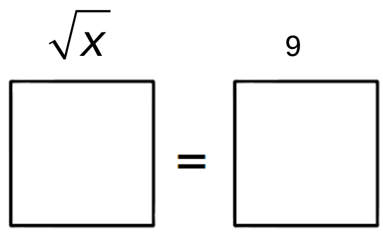


Label the area of this square if the length of the side is $\sqrt{x-1}$

The Radical Equation Template contains two sets of equations that show either two squares are equal or two sets of squares add up to two other squares.

What does that mean when the expressions are equal?

Use the Radical Equation Template to represent the following equation: $\sqrt{x} = 9$.



What is the area of each square? The area of the two squares are x and 81. What does that tell you about x? Since the areas are equal, x=81. We can check our answer by replacing x with 81 in the original equation: $\sqrt{81} = 9$

Use the Radical Equation Template to visualize what each equation is describing. Then use the model to help you solve for the value of x. Check your answer when you are done.

1. $\sqrt{x-1} = 7$
2. $\sqrt{x} - 10 = 0$
3. $\sqrt{6x} - 13 = 23$
4. $\sqrt{4x} = 20$
5. $\sqrt{x} + \frac{1}{3} = \frac{13}{3}$
6. $\sqrt{x} + 2 = \sqrt{2x} - 9$
7. $\sqrt{4x+1} + 5 = 10$
8. $\sqrt{x+2} = x$
9. $\sqrt{x-6} = \frac{1}{5}x$
10. $x = \sqrt{4x+5}$

Radical Equations