Activities Packet

Achieving Success in Meeting the Common Core State Standards in Geometry

Rutgers University
February 7, 2011

Jim Rahn
www.jamesrahn.com
james.rahn@verizon.net

Discovering Ways Triangles Are Congruent

Materials needed: rulers, pencils, patty paper, do now questions, closure questions.

Lesson Goal: For students to understand that there are several short cut methods that can lead to the congruency of triangles.

Do Now Activity: Ask students to identify which figures are congruent and tell why.

Make a definition with the students that congruent figures are those figures where the corresponding angles are congruent and the corresponding sides are congruent.

Next give students a copy of Parts of a Triangle. In this lesson students will discover they need at least 3 parts to get congruent triangles.

Copy angle A on a sheet of patty paper. Put the Parts of the Triangle paper away. Remember the sides of the angle you just copied are rays so you can make sides AB and AC as long as you would like. Make the angle A on the patty paper into a triangle. You cannot change the size of angle A, but you can lengthen the rays that make up the angle because that does not change the size. After students have made a triangle on the patty paper have them compare their triangles with each other by placing them on top of each other. Are the triangles congruent? (Students should conclude that the triangles are not congruent.) Have students label on the patty paper that only one angle was copied.

Copy side AB on a sheet of patty paper. Put the Parts of the Triangle aside. Make the side AB on the patty paper into a triangle. You cannot change the size of side AB, but you can attach two other sides to the given side to make a triangle. After students have made a new triangle on the patty paper have them compare their triangles with a neighbors triangle. Are the triangles congruent? (Students should conclude that the triangles are not congruent.) Have students label on the patty paper that only one side was copied.

Copy angle A on a sheet of patty paper. Have students trace side AB on to angle A. Put the Parts of the Triangle aside. Have students extend the other side of angle A to whatever length they would like. Then have the students label point C and complete the triangle. After students have made a new triangle on the patty paper have them compare it a neighbor’s triangle. Are the triangles congruent? (Students should conclude that the triangles are not congruent.) Have students label on the patty paper that two angles were copied.

Copy side AB and BC to make any triangle. After they have labeled A, B, and C have the students complete the triangle. After students have made a new triangle on the patty paper have them compare it a neighbor’s triangle. Are the triangles congruent? (Students should conclude that the triangles are not congruent.) Have students label on the patty paper that two sides were copied.

What have we learned so far? Congruent triangles cannot be made by copying one angle, one side, two angles, or two sides. There is too much unknown information to produce congruent triangles.

Since we did not get congruent triangles with one angle, one side, two angles, or two sides, let’s try three parts.

Copy side AB, BC, and CA on three different sheets of patty paper. Have students place them on top of each other so the vertices match up. Once the papers are line up have students copy the line segments onto one sheet of patty paper. After students have made a new triangle on the patty paper have them compare it with triangle ABC. Are the triangles congruent? (The triangles are congruent.) Remind students that they did not copy any of the angles, only the three sides. What can they conclude? (If the three corresponding sides of the two triangles are congruent the triangles are congruent.)

We finally have a way that makes two triangles congruent. This method is known as SSS because the three corresponding sides were congruent.

Copy angle A, side AB and angle B on a sheet of patty paper. Have students put triangle ABC aside and then complete their triangle. After students have made a new triangle on the patty paper have them compare it with their neighbor's triangle. Are the triangles congruent? (The triangles are congruent.) Remind students that they did not all the six parts. They only copied angle A, side AB, and angle B. (If two angles and the side between them on one triangle are congruent to two angles and the side between them on another triangle, the triangles are congruent.

Stop and project Congruence Shortcut on the board. Have students try to make two different triangles from two angles and the included side. What do they learn? (Only one triangle can be made from the two angles and the included side.) This method is known as ASA.

Copy angle A and sides AB and AC on a sheet of patty paper. Have students triangle ABC aside and then complete their triangle. After students have made a new triangle on the patty paper have them compare it with their neighbor's triangle. Are the triangles congruent? (The triangles are congruent.) Remind students that they did not all the six parts. They only copied angle A and the two sides that make up angle A. (If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle the triangles are congruent.

Stop and project Congruence Shortcut on the board. Have students try to make two different triangles from one angle and the two sides that make up the angle. What do they learn? (Only one triangle can be made from the two sides and the included angle.) This method is known as ASA.

What methods work so far? SSS, ASA, and SAS.

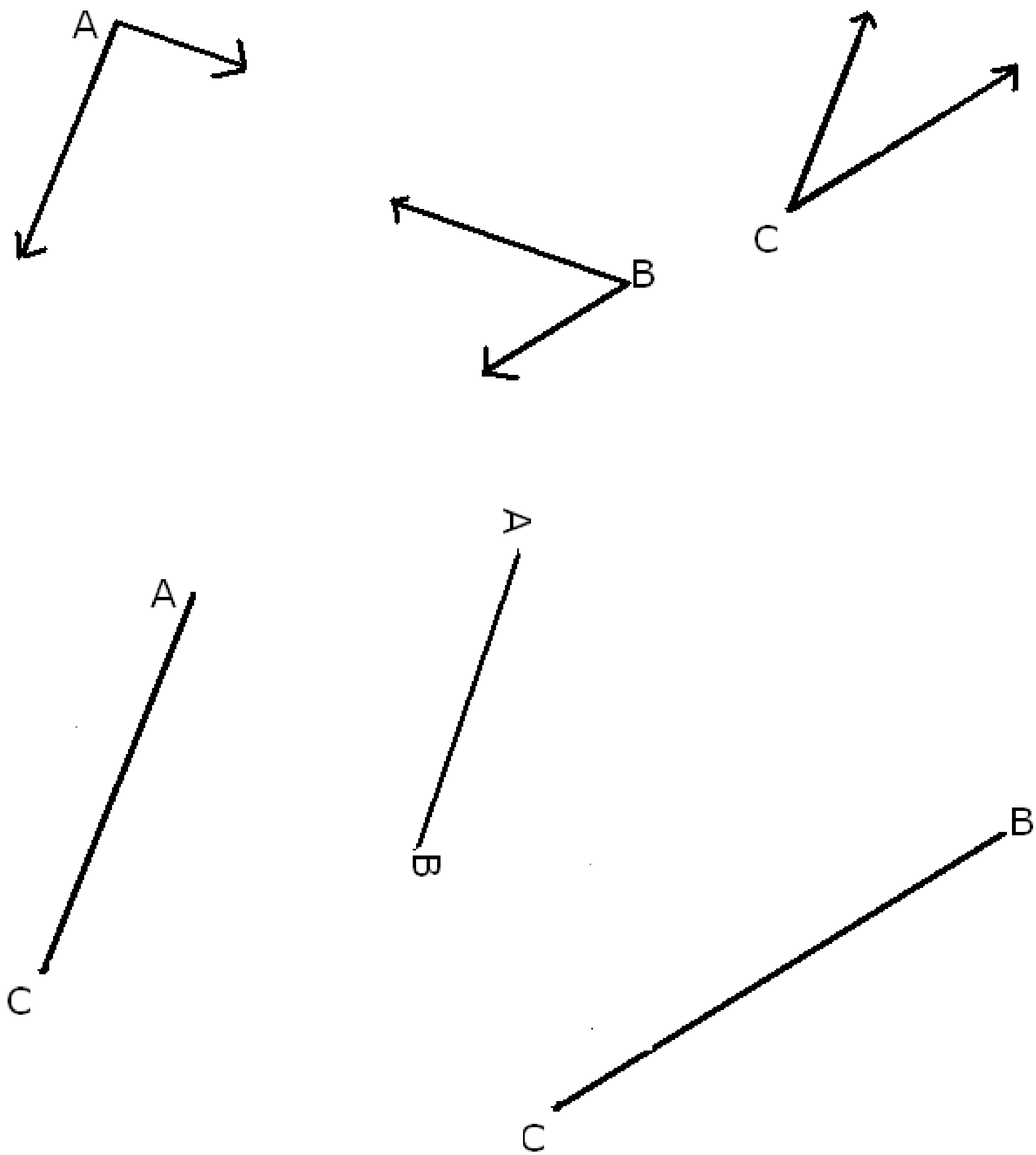
Have students copy angles A, B, and C to make a triangle. After they copy one angle, have them put triangle ABC aside, extend one of the sides of the angle, label the endpoint B and then copy angle B. This should look familiar. It is what they did for AA. Have students complete the triangle and then compare their third angle with the side of angle C. What do they notice. All three angles are congruent. Does this produce congruent triangles? (No because not side measurements were given.)

Model SSA with the computer program. Students should notice that the triangles are not congruent.

Closure:

Have students begin with triangle ABC and label parts of triangle XYZ so they are congruent by SSS, ASA, and SAS.

Parts of a Triangle



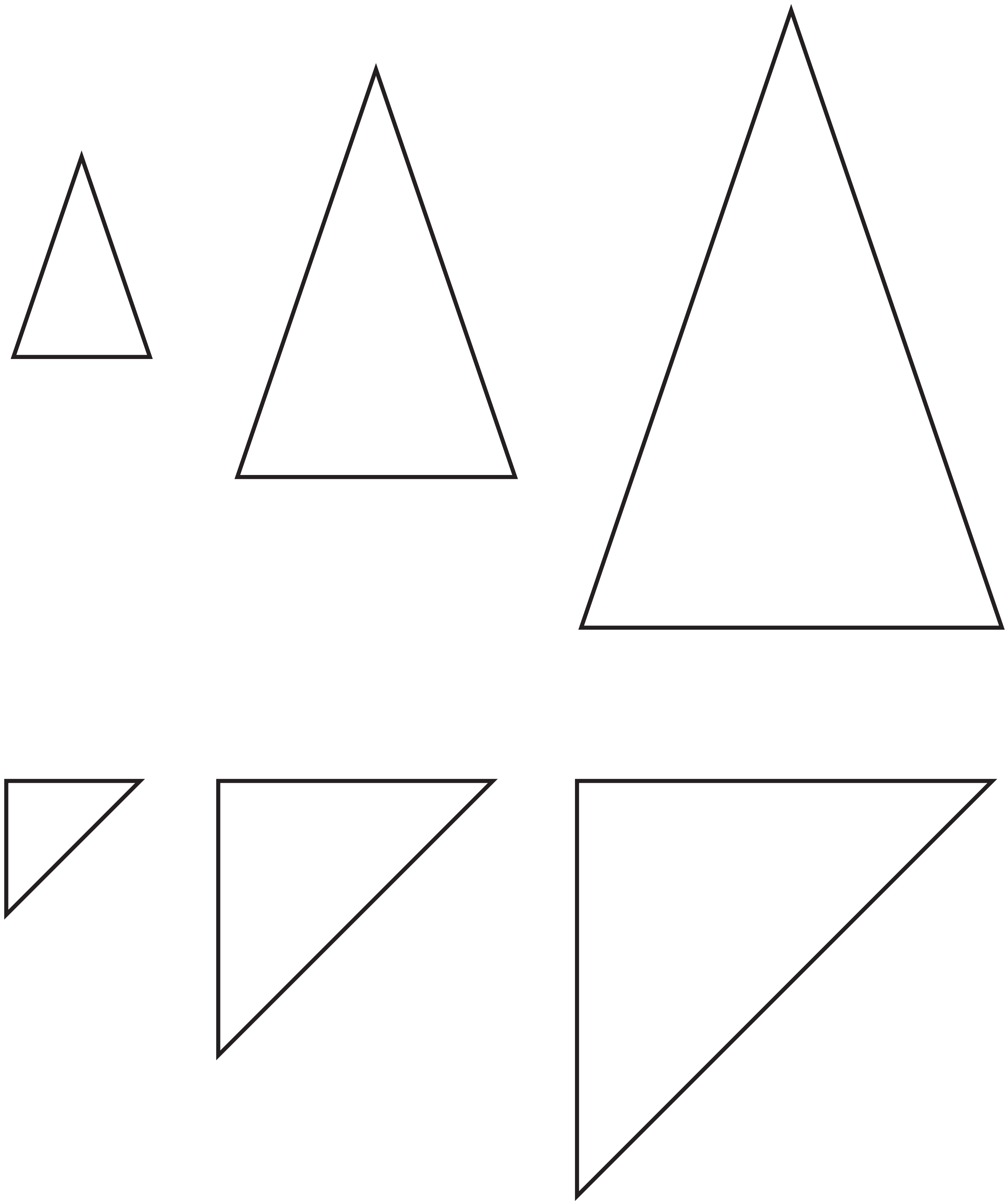
Exploring Similarity

Materials Needed: Communicators
Similar Isosceles Triangles

Procedures:

1. Place your Communicator on top of the Similar Isosceles Triangles template. Trace the smallest triangle and then slide the Communicator around to compare the angles of the smallest triangle with the angles of the other two triangles. What do you notice about the corresponding angles? (The corresponding angles are congruent.)
2. Compare the sides of the smallest triangle with those of the other two triangles. What do you notice about the corresponding sides? (They are in the same ratio.)
3. Draw a fourth isosceles triangle that is similar to the three given isosceles triangles. (Answers will vary, but should have corresponding angles congruent to those in the given picture and the corresponding sides in a given ratio.)
4. Suppose the short side of the first isosceles triangle was 1 and the longer side was 2, find the length of the sides in every other triangle.
5. Suppose the short side of the third isosceles triangle was 6 and the longer side was 8, find the length of the sides in every other triangle.
6. Suppose the area of the smallest isosceles triangle was 3 square units. Find the area of each other the other isosceles triangles.
7. Suppose the area of the third isosceles triangle was 24. What is the area of each of the other isosceles triangles.
8. Suppose the length of two corresponding sides of two isosceles triangles was in a ratio of 5/8. If the shorter leg of the smaller triangle was 10, what would the length of the corresponding side in the larger triangle?
9. Suppose the length of two corresponding sides of two isosceles triangles was in the ratio of 3/7. If the area of the smaller isosceles triangle is 9, what is the area of the larger isosceles triangle?
10. Suppose two isosceles triangle have areas that are in a ratio of 4/9. If the short side of the smaller triangle is 3, what is the length of the corresponding side in the larger triangle?

Similar Isosceles Triangles



Understanding the Properties of a Dilation

Place a grid in the communicators.

Part I:

Draw the polygon 1 whose vertices are (1,1), (1,3) and (5,3), and (5,1). Create polygon 2 whose vertices are (3,3), (3,9) and (15,9), and (15,3).

What do you observe about the two polygons?

What do you observe about the corresponding sides of the two polygons?

Find the length of their sides and find the area of each figure. What do you notice?

Polygon 2 is said to be a dilation of Polygon 1. Do the angle measurements change in the two polygons? Do the measurements of the sides change? Does the area change?

Study the coordinates for each rectangle. Can the second set of coordinates be made from the first set of coordinates by multiplying by some number? This number is called the dilation factor or scale factor.

How is this number reflected in the measurements of the polygons.

Connect the corresponding vertices with a segment. Extend these lines. What happens with all these lines? This point is called the point of dilation. Study the distances of the vertices from this point. What do you notice?

Part II:

Draw the polygon whose vertices are (1,2), (2,4), (5,4), and (4,2). Find the length of each side. Find the area of the polygon.

Draw a second polygon that is a dilation of the first polygon, but whose sides are 3 times the sides of the first polygon. How do you know what the vertices should be? Name the vertices of the new polygon.

Connect the corresponding vertices with line segments. Extend these line segments. Locate the point of dilation.

What do you notice about the relationship of the coordinates for the two polygons?

Did the angles of the polygon change?

Part III:

Draw a right triangle whose vertices are (2,2), (5,2) and (2,6). Find the length of the sides and the area of the triangle.

Draw a line segment from the origin to each of the vertices. Notice the length of these segments. Extend these segments so they are twice as long as they presently are. Form this new triangle.

Is the triangle a right triangle? What are the vertices of the triangle? How are they related to the original vertices? How do the sides of the new triangle compare to the first triangle? How does the area of the second triangle compare to the first?

What have you learned about a dilation from these three parts of the lesson.

Part IV:

Create a triangle whose one side in on the x or y-axis, whose vertices contain even numbers.

Create a dilation of the triangle using the scale factor or 1 ½.

What happened to the length of each side of the triangle, when it was dilated by a factor of 1 ½?

What happened to the area of the triangle?

What is the relationship between the corresponding sides of the triangle?

Part V:

Create a new polygon not located at the origin. Name the vertices.

Can you predict the vertices before you complete a dilation that will triple the length of the sides?

Create a dilation that triples the length of the sides.

What happened to the area? What happened to angles of the polygon?

Summary:

Ask students what a dilation does?

What does the dilation do to lines that are do not pass through the center of dilation? What does the dilation do to lines that pass through the center of dilation?

What affect does a dilation have on the lengths of the sides? What affect does a dilation have on the area of the polygon? What affect does a dilation have on the vertices of the polygon? How can you find the point of dilation?

How Large is That Tower?

Using centimeter grid paper make a tower that measures 1 cm x 1 cm x 2 cm. Call this Tower 1. Then create Tower 2 that measures 2 cm x 2 cm x 4 cm, Tower 3 that measures 3 cm x 3 cm x 6 cm, Tower 4 that measures 4 cm x 4 cm x 8 cm, and Tower 5 that measures 5 cm x 5 cm x 10 cm,

Collect the following data about each tower.

	Height of Tower	Surface Area of Tower	Volume of Tower
Tower 1			
Tower 2			
Tower 3			
Tower 4			
Tower 5			

Part I: Height or Linear Measure

1. What do you observe about the data collected in the height column?

2. Find the following ratios for the heights of the Towers. In addition, put all ratios in simplest form.

$\frac{Tower\ 2}{Tower\ 1} =$ $\frac{Tower\ 4}{Tower\ 1} =$ $\frac{Tower\ 4}{Tower\ 3} =$

$\frac{Tower\ 3}{Tower\ 1} =$ $\frac{Tower\ 3}{Tower\ 2} =$ $\frac{Tower\ 5}{Tower\ 3} =$

3. What do you observe about simplifying each ratio? Why is this?

4. By observing the above ratios, write a ratio the height of Tower 10 to Tower 7.

5. Which Tower would have a height of 48 cm.? Support your answer.
6. Knowing the height of the Tower 1, how can you find the height of Tower n?
7. If the heights of two Towers were in a ratio of $\frac{14}{12}$, which Towers would you be comparing?
8. If you were comparing the heights of a Tower 30 and Tower 12, what ratio would compare these heights?
9. Suppose a new Tower 1 was formed that was 3 un its height instead of 2 units high and both of the other measures stayed the same, describe the heights of Towers 1 to Tower 5. What do you notice about these heights?

Part II: Surface Area or Square Measure

1. What do you observe about the data collected in the surface area column?
2. Find the following ratios of the surface area of the heads. (Simplify each ratio.)

$\frac{Tower\ 2}{Tower\ 1} =$ $\frac{Tower\ 4}{Tower\ 1} =$ $\frac{Tower\ 4}{Tower\ 3} =$

$\frac{Tower\ 3}{Tower\ 1} =$ $\frac{Tower\ 3}{Tower\ 2} =$ $\frac{Tower\ 5}{Tower\ 3} =$

3. What do you observe about the data collected in the volume of the tower column?
4. By observing the above ratios write a ratio which would compare the surface area of Tower 10 to Tower 7.
5. Which Tower would have a surface area of 600 square centimeters? Support your answer.
6. Knowing the isurface area of Tower 1, how can you find the surface area of Tower n?
7. If the ratio of the surface area of a two Towers was $\frac{90}{70}$ which Towers would you be comparing?
8. If you were comparing surface area of the Tower 30 and Tower 12, what ratio would you use?
9. Suppose a new Tower 1 was formed with a visible surface area of 12 square centimeters, describe the surface area of Tower 2 to Tower 5.

Part III Volume or Cubic Measure

1. What do you observe about the data collected in the volume of the tower column?
2. Find the following ratios of the volume of one leg. (Simplify each ratio.)

$\frac{Tower\ 2}{Tower\ 1} =$ $\frac{Tower\ 4}{Tower\ 1} =$ $\frac{Tower\ 4}{Tower\ 3} =$

$\frac{Tower\ 3}{Tower\ 1} =$ $\frac{Tower\ 3}{Tower\ 2} =$ $\frac{Tower\ 5}{Tower\ 3} =$

3. What do you observe about simplifying each ratio? Why is this?
4. Write a ratio which would compare the volume of Tower 10 to Tower 7.
5. If the ratio of the volume of two Towers was $\frac{1024}{54}$, which Towers would you be comparing?
6. Which Tower would have a volume equal to 1458 cubic centimeters? Support your answer.
7. Knowing the volume of Tower 1, how can you find the volume of Tower n?
8. If you were comparing the volume of Tower 30 and Tower 12, what ratio would you use?
9. Suppose a new Tower 1 was formed with a volume equal to 4 cubic centimeters, describe the volume of Tower 2 to Tower 5.

How is the Circumference of a Circle Related to the Length of the Diameter?

- Review how students can read a centimeter ruler to find the length of the diameter of the circle. (Transparency)
- Distribute empty cans, measuring tapes, and charts to each student.
- Have students measure the diameter and circumference of their cans. As they find their two measurements have them record the data in a class chart.
- Enter the data in a graph calculator using TI-Smartview. Use L1 (Diameter) and L2 (circumference).
- Ask students to look at all the information they have recorded on the chart. Ask students what they observe. (They should notice that the circumference is about 3 times larger than the diameter.)
- Using the calculator to divide L2 by L1 in L3. Ask students to observe the values in L2. (They should notice that they are around 3. Most of them should be more than 3.) What does this mean? (The diameter fits around the outside of the circle a little more than 3 times.)
- Look at a graph of L1 vs. L2 on the calculator. What do you notice? The points form a straight line. Trace along the graph and notice that the L2 is about 3 times bigger than L1. Enter an equation in Y1 that says $y = 3x$. Graph and describe what happens. The line appears to go near all the points.
- Ask students to find the circumference for several circles by giving them either diameter measurements or radius measurements.
- Ask students to find the diameter of several circles by giving them the circumference of a circle.
- Ask students to find the radius of several circles by giving them the circumference of a circle.

Diameter (cm)	Circumference (cm)

Discovering the Area Formula for a Circle

- Distribute copies of the Activity Sheet. Distribute scissors and glue sticks. Ask students to cut out the three squares and the circle.

- Ask students to compare the radius of the circle with the side of the square.

- Have students notice that the radius of the circle equals the side the square. You can assign a random number to the length of the radius. Ask students to find the area of the three squares. ($A = lw$ or s^2) Ask students to approximate if the area of the three squares will fit in the circle. (Some students may think the three squares will not fit. Don't give it away at this point.)

- Ask students to use the glue sticks and the scissors to divide up the three squares to see if their area will fit. No overlapping pieces should be placed on the circle.

- Students should conclude that the area of the circle is a little more than three squares.

- Using the numbers that the students placed on their rectangles ask them to approximate the area of the circle. If the radius of the circle is 4 units, then each square has an area of 16 square units, and the circle has an area that is a little more than 16 x 3 or 48 square units.

- Try other circle problems. Continue to relate the picture they have just done with each problem.

- Find the area of a circle whose radius is 10 inches. The square would have an area of 100 so the circle has an area that is slightly larger than 300.

- Find the area of a circle with a diameter of 16 centimeters. The radius would be 8 centimeters and each square would have an area of 64 square centimeters. The area of the circle would be slightly larger than 3 x 64 or about 192 square centimeters.

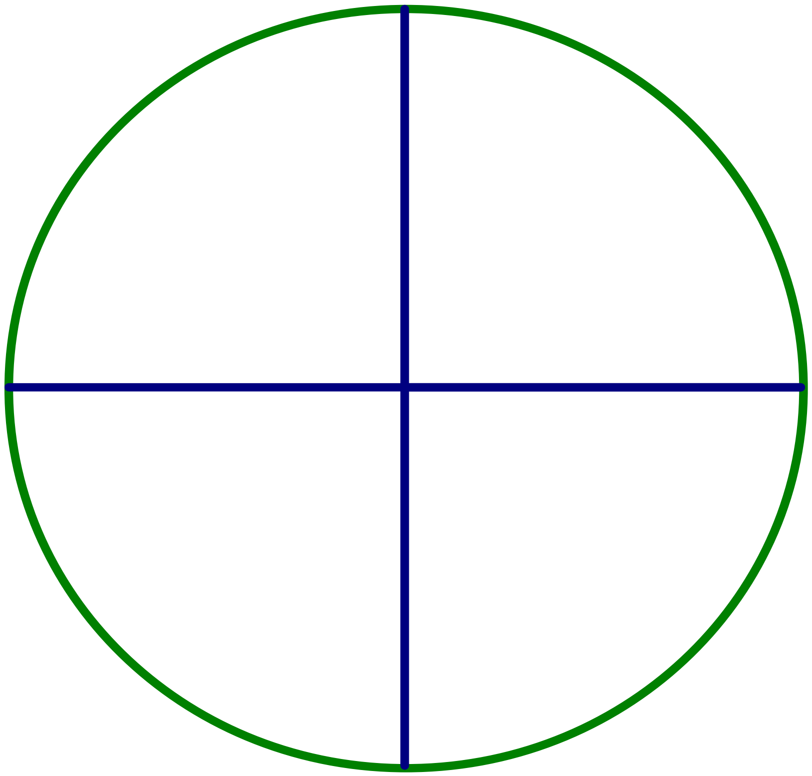
- Turn on Sketchpad and open the file circle.gsp. You will see the circle and the three squares on the file. Ask students if the three squares will fit on the circle. They should respond positively. Click and drag the point on the circle and see what happens. Continue to ask students if the area of the three squares will always fit on the circle. They should respond positively. Click on the *Show Area of Circle* button. Click on the *Show Area of Squares* button. Ask students if the sum of the three area will be more or less than the area of the circle. If students have difficulty seeing that the sum of the area of the three squares is less than the circle, click on *Show Area of Three Squares* button. Students should see that the area of the three squares is less than the area of the circle, therefore, the area of the circle is a little more than the area of the three squares.

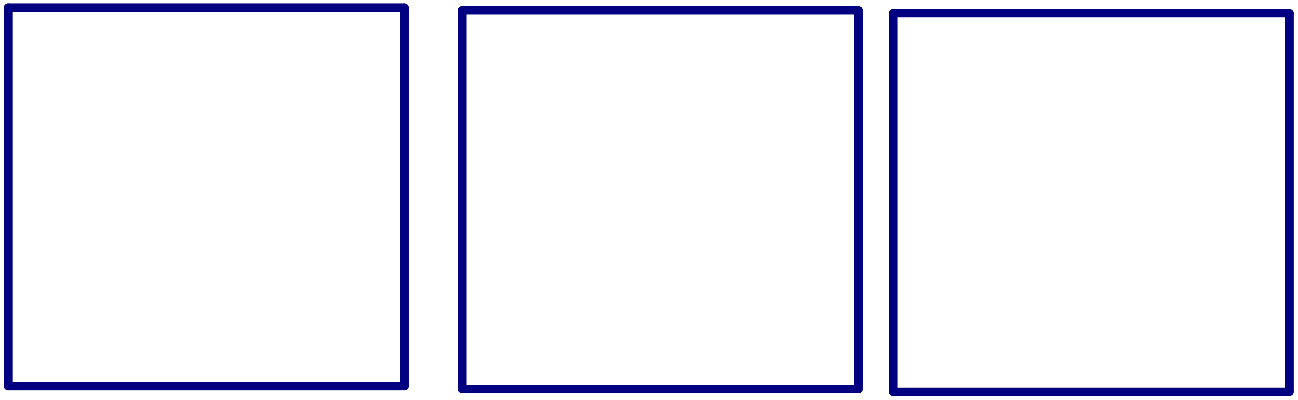
- To finally see how many times the area of the square will fit in the circle click on the *Show Ratio* button. Again pull on the point of the circle so the size of the circle changes. Ask students what they observe happening. Stop periodically so students can observe that the areas are changing, but the ratio is always 3.14. This is the exact number of times that the area of one square will fit in the circle.

- Have students practice finding the area of several other squares.

- Ask students why they are using r^2. (It is the area of one square whose side equals the radius of the square.) Why are they multiplying r^2 by 3.14. (The area of the square fits in the circle 3.14 times.)

Activity Sheet





Studying Transformations
Translations

Distribute communicators, pens, erasers, and a copy of the Transformation Grid and Chart template to all students. Have students place the communicator on top of the template.

Ask students to locate the triangle whose vertices are A(1, 1), B(2, 3), and C(4, 1). Ask students to draw the line segments between these points to create a triangle. Ask students to record the coordinates on the chart under the original figure.

Ask students to translate the figure 5 units to the left and redraw the triangle. Ask students to read the coordinates for the translated triangle and record them in the chart. Ask students to study the coordinates and write a description on how the coordinates are changing if the figure is translated 5 units to the left. (x, y)=> (___,___)

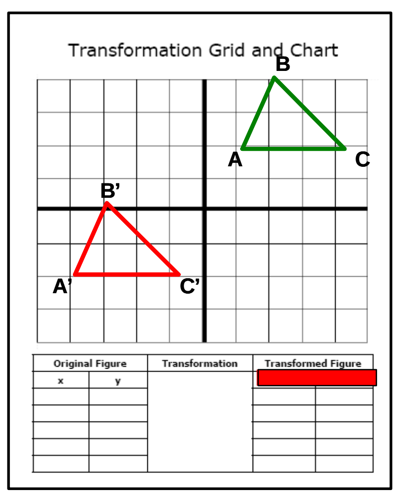
Have students erase the translated triangle, but not the original triangle. Ask students to translate the original triangle 3 units down and redraw the triangle. Ask students to read the coordinates for the translated triangle and record them in the chart. Ask students to study the coordinates and write a description on how the coordinates are changing if the figure is translated 3 units down. (x, y)=> (___,___)

Have students erase the translated triangle, but not the original triangle.
Ask students to translate the original triangle 5 units to the left and 5 units down and redraw the triangle. Ask students to read the coordinates for the translated triangle and record them in the chart. Ask students to study the coordinates and write a description on how the coordinates are changing if the figure is translated 4 units to the left and 5 units down.
(x, y)=> (___,___)

Now let's try to reverse the steps. Give the students the translation and have them move the original triangle by the rule.

Erase the translated triangle, but not the original triangle.
Ask students to translate the original triangle figure using the transformation: (x, y)=> (x-1, y-3) Ask students to explain how they translated the triangle to perform this translation.

Study the green triangle.
Describe a translation that would move the triangle to its new position described by the red triangle. (x, y)=> (___,___)



Studying Transformations
Reflections

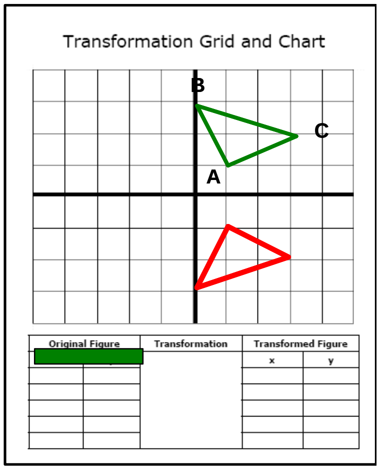
Distribute communicators, pens, erasers, and a copy of the Transformation Grid and Chart template to all students. Have students place the communicator on top of the template.

Ask students to locate the triangle whose vertices are A(1, 1), B(2, 3), and C(4, 1). Ask students to draw the line segments between these points to create a triangle. Ask students to record the coordinates on the chart under the original figure.

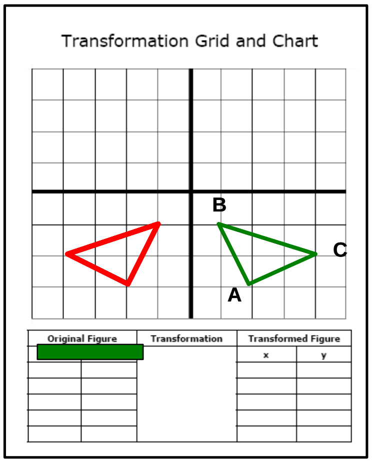
Ask the students to reflect the original triangle over the x-axis. The students can flip the communicator to help you perform this reflection. Ask students to read the coordinates for the reflected triangle and record them in the chart. Ask students to study the coordinates and write a description on how the coordinates are changing if the figure is reflected over the x-axis. (x, y)=> (___,___) Ask students to explain why this transformation makes sense.

Ask students to erase the reflected the triangle, but keep the original triangle. Ask students to reflect the original triangle over the x-axis. Students can flip the communicator to help you perform this reflection. Ask students to read the coordinates for the reflected triangle and record them in the chart. Ask students to study the coordinates and write a description on how the coordinates are changing if the figure is reflected over the x-axis.
(x, y)=> (___,___) Ask students to explain why this transformation makes sense.

Ask students to clear their communicators and then place the Communicator® on top of the Transformation Grid and Chart template. Ask students to locate the three vertices: A(2, 1), B(1, 3), and C(4, 2). Then ask them to perform the following transformation:
(x, y)=>(x, -y)
(x, y)=>(-x, y)
(x, y)=>(-x, -y)

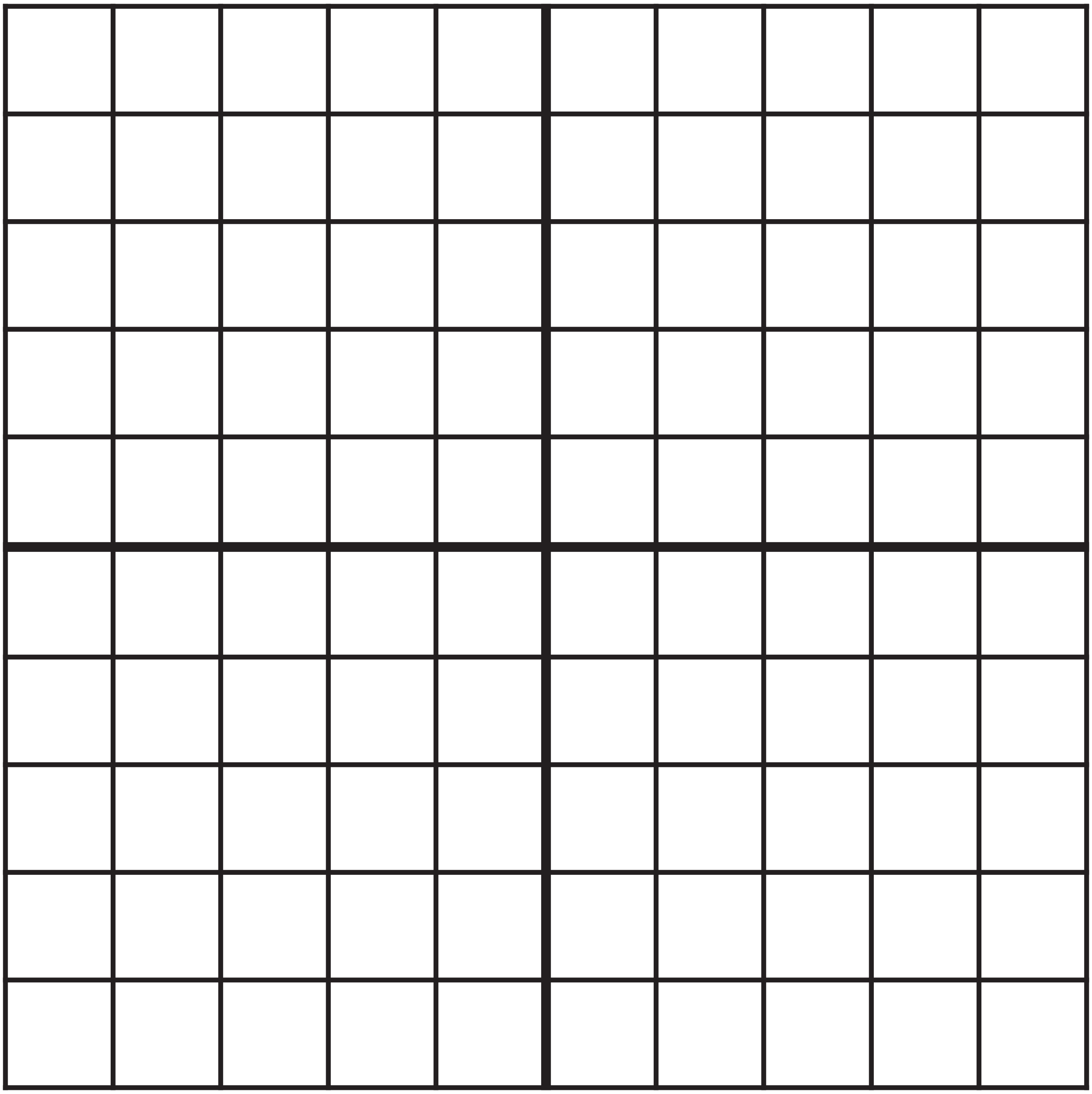


Ask students to study the transformation at the right. The green triangle is the original. The red triangle is the transformed triangle. Ask students to tell what type of transformation has taken place. Ask students to describe the transformation: (x, y)=>(___, ___)



Ask students to study the transformation at the right. The green triangle is the original. The red triangle is the transformed triangle. Ask students to tell what type of transformation has taken place. Ask students to describe the transformation: (x, y)=>(___, ___)

Transformation Grid and Chart



Original Figure		Transformation	Transformed Figure	
x	*y*		*x*	*y*