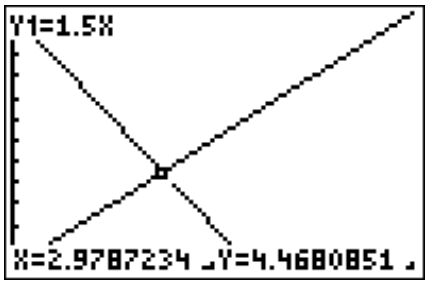
Developing an Understanding for Working with a System of Equations



X	Y1	Y2
1.5	2.25	8.25
2	3	7
2.5	3.75	5.75
3	4.5	4.5
3.5	5.25	3.25
4	6	2
4.5	6.75	.75

X=3

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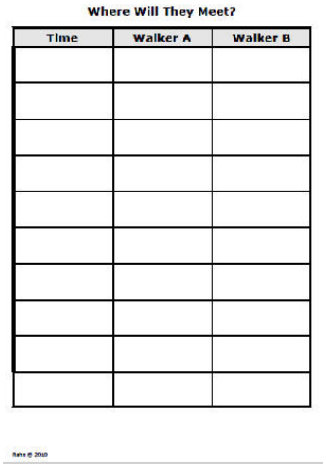
What Does It Mean to Solve a System of Equations?

In this activity you will solve a system of simultaneous equations to find the time and distance at which two walkers meet. Suppose that two people begin walking in the same direction at different average speeds. The
1st walker starts behind the slower 2nd walker.
When and where will the faster walker overtake the slower walker?


Where Will They Meet?

Time	Walker A	Walker B

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- Mark a 6 m segment at 1 m intervals.
- Have the 1st Walker start at 0.5 meters and walks toward the 6 m mark at a speed of 1 m/s. Have the 2nd walker start at the 2 m mark and walk toward the 6 m mark at 0.5 m/s.
- Have a timekeeper count 1 second counts out loud. (This does not have to be in real time.)
- Have a recorder record both the time and position of the first walker and second walker on a communicator using the "Where Will They Meet?" template.
- Enter the data in the lists of the graphing calculator.
  - L1 = time
  - L2 = position of 1st walker
  - L3 = position of 2nd walker
- Write an equation that fits each walkers data.
- Enter the equations into the graphing calculator.
- Find the approximate point where the lines intersect.
- Explain the real-world meaning of the intersection point.
- Check that the coordinates of the point of intersection satisfy both of your equations.

Questions to think about:

- Suppose that walker A walks faster than 1 m/s. How is the graph different? What happens to the point of intersection?

- Suppose that two people walk at the same speed and direction from different starting marks. What does this graph look like? What happens to the solution point?

- Suppose that two people walk at the same speed in the same direction from the same starting mark. What does this graph look like? How many points satisfy this system of equations?

Problems to Apply Idea

1. Gail leaves the trailhead at dawn to hike 12 mi toward the lake, where her friend Laura is camping. At the same time, Laura starts her hike toward the trailhead, Gail is walking uphill so she averages only 1.5 mi/h, while Laura averages 2.5 mi/hr walking downhill. When and where will they meet?

- Define variables for time and distance from the trailhead.
- Write a system of two equations to model this situation.
- Solve this system by creating a table and finding the values for the variables that make both equations true. Then locate this solution on a graph.
- Check your solution and explain its real-world meaning.

2. Use the calculator table feature to find the solution to these systems of equations. Confirm the solution by using a graph.

A. $\begin{cases} y = -9 + 2x \\ y = 3 + 0.5x \end{cases}$ B. $\begin{cases} y = 4x - 5.5 \\ y = -3x + 5 \end{cases}$ C. $\begin{cases} y = 100 + 10x \\ y = 50 + 20x \end{cases}$

3. The total tuition for students at South College and North College consists of student fees plus costs per credit. Some classes have different credit values. The table shows the total tuition for programs with different numbers of credits at each college.

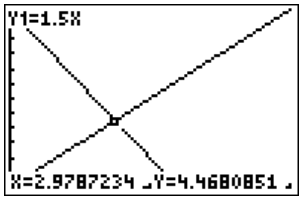
Total Tuition		
Credits	South College	North College
1	55	47
3	115	11
6	205	207
9	295	303
10	325	335
12	385	399

A. Write a system of equations that represents the relationship between credit hours and total tuition for each college.
B. Find the solution to this system of equations and check it.
C. Which method did you use to solve this system? Why?
D. What is the real-world meaning of the solution?
E. When is it cheaper to attend South College? North College?

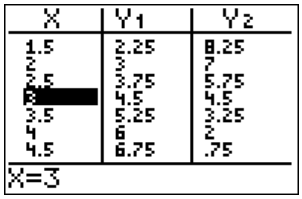
Where Will They Meet?

Time	Walker A	Walker B

Building Understanding for Solving a System of Equations by Substitution



A solution to a system of equations can be found graphically and by using tables. Often these methods can be very tedious and may sometime lead to only an approximate solution.



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X=3

To find the exact solutions, you'll need to work algebraically with the equations. Let's look at the substitution method.

On a rural highway a police officer sees a motorist run a red light at 50 mph and begins pursuit. At the instant the police officer passes through the intersection at 60 mph, the motorist is 0.2 mi down the road. When and where will the office catch up to the motorist?

- If d= the distance from the intersection and t= time traveled, write an equation in two variables to model this situation.
- When the police car catches up to the motorist what will true about the distance of the motorist from the intersection and the distance of the police car from the intersection?
- Replace the d value in one equation with the d value from the other equation. Solve the equation for time.

- What is the meaning of this time?
- How far is each vehicle from the intersection at this time?
- Check your answer by looking at a graph and a set of table values for both equations.

All Tied Up

All Tied Up

Number of Knots	Length of Thinner Rope	Length of Thicker Rope

- Locate two ropes with different thicknesses.
- Measure the thinner rope without any knots.
- Tie a knot in the rope and measure the length again.
- Record the number of knots and the length of the rope in a table using the "All Tied Up Template" with your communicator.
- Continue to tie knots and measure the length of the rope.
- Repeat for both ropes.
- By looking at the table for the length of the thinner rope, what do you notice about the change in the length. By looking at the table for the length of the thicker rope, what do you notice about the change in the length? For each rope, find a number that represents the average rate of change in the rope for each knot tied.
- Define the variables used in this chart and represent each variable with a letter.
- Suppose the length of the original thinner rope was 9 meters long and the thicker rope was 10 meters long. Using the data you have found about the change in length of each rope as a knot is added, write an equation for the length of each rope using y-intercept form.
- Notice each equation is of the form $y=b+mx$. Since both equations are equal to y, let's use substitution as we did in the previous example. Solve the resulting equation.
- What does this answer for x mean? Find the corresponding value for y in both equations.
- Enter both equations in the graphing calculator. Create an appropriate window to view both graphs. What is the approximate solution by graphing?
- Create a set of table values for both equations. What is the approximate solution for by looking at tables?
- Explain the meaning of the solution for this system of equations.

Extension:

- Suppose the two ropes were still two different thicknesses, but were the same length to begin, what would happen to the graph of the two lines?
- Suppose the two ropes were the same thickness and their original length. What would happen to the graph of the two lines?

Can we always use the Substitution Method?

So far you have seen equations written in intercept form.
These equations make it easy to use the substitution method since they are already both solved for y.
y=900 -6x
y=1000-10.3x
Sometimes it is necessary to place equations in intercept form before using substitution.

A pharmacist has 5% saline (salt) solution and 20% saline solution. How much of each solution should be combined to create a bottle of 90 ml of 10% solution.
Let x represent the amount of 5% solution you have and y represent the amount of 20% solution you have. First write an equation for the total amount of liquid in each container. Then write a second equation that describes the amount of salt in each solution.

Solve the one equation for x or y and then substitute it into the other equation.

What is the meaning of this solution.

One More Problem:

This system of equations models the profits of two home-based Internet companies.

$$\begin{cases} P = -12000 + 2.5N \\ P = -5000 + 1.6N \end{cases}$$

The variable P represents profit in dollars, and N represents hits to the company's website.

a. Use the substitution method to find an exact solution.
b. Is an approximate or exact solution more meaningful in this model?

All Tied Up

Number of Knots	Length of Thinner Rope	Length of Thicker Rope

Developing an Understanding for the Elimination Method for Solving a System of Equations

We have solved a system of equations using graphs, tables, and substitution. The substitution method was introduced to provide you an analytical (or algebraic) way to work with a system of equations to find a solution to the system. In this activity you will explore using another analytic (or algebraic) was to solve a system of equations.

First let's think about some things we have done with equations.

When you add equal quantities to each side of an equation, the resulting equation is equivalent and has the same solution as the original equation.

$$\begin{array}{rr} & y - 4 = 12 \\ + & 4 = 4 \\ \hline & y = 16 \end{array}$$

Recall that y=16 is a solution to the first equation and the last equation.

$$\begin{array}{rr} & 3y - 2x = 1 \\ + & 2x = 2x \\ \hline & 3y = 1 + 2x \end{array}$$

Recall that (1,1) (4,3), and (10,7) are solutions to first equation and the last equation.

so

$$\begin{array}{rr} & a = b \\ + & c = d \\ \hline & a + c = b + d \end{array}$$

Frank is thinking of two numbers, but he won't say what they are. Instead he plays a game. He tells you that the sum of the two numbers is 163 and that their difference is 33. Find the two numbers.

If the two numbers are x and y, you can write two equations based on Frank's statements.

- Write an equation based on the first description:
- Write an equation based on the second description:

From your equations, which is the larger number? How can you tell?

Use the property described above to add the equations together. You should see that this results in an equation with just one variable. Solve for the value of that variable.

Use this value to find the value of the other number. What is the solution to Frank's problem?

How long is a paper clip? How wide is a penny?

- To write one equation, lay one paper clip along the long side of the paper and then add enough pennies to complete the 11 inch length. Write a statement that describe the 11" length of the paper using paper clips and pennies. Replace the word paper clips with C and pennies with P. You now how your first equation.
- To write the other equation for the system, lay two paper clips along the shorter side of the paper and add pennies to complete the 8.5 inch length. Write a statement that describe the 8.5" length of the paper using paper clips and pennies. Replace the word paper clips with C and pennies with P. You now how your second equation.
- Study your two equations. Notice that each variable has different coefficients on each equation.
- What could you do to both sides of one of the equations so that the variable C is eliminated when the two equations are added?
- Perform this operation to both sides of the one equation and then add the equations together.
- Once you have eliminated the C, solve the resulting equation for P.
- Substitute your P value back into either equation to solve for C.
- Find another way to solve this system by elimination.
- Explain the real-world meaning of the solution.
- Describe other experiments in measuring that you can solve using a system of equations.

From Science:

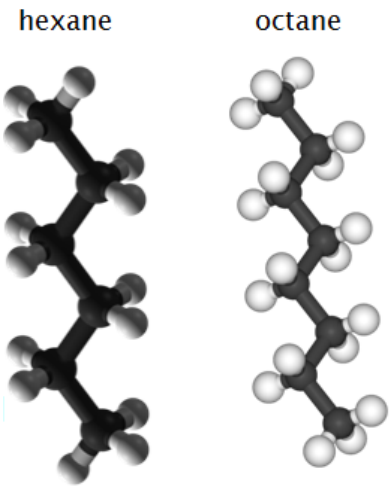
A molecule of hexane, C_6H_{14}, has six carbon atoms and fourteen hydrogen atoms.

Its molecular weight in graphs per mole, the sum of the atomic weights of carbon and hydrogen is 86.178.

The molecular weight of octane, C_8H_{18}, is 114.232 grams per mole.

Octane has eight carbon atoms and eighteen hydrogen atoms per molecule.

Find the atomic weights of carbon and hydrogen.



Try These:

1. Suppose you had the system of equations $\begin{cases} 3x - 2y = 11 \\ -x + 6y = 7 \end{cases}$

   Show how you could solve this system by eliminating the x variable.

   Show how you could solve this system by eliminating the y variable.

   Show how you could use substitution to solve this system of equations.

Developing an Understanding for Solving a System of Equations Using Matrices

In this activity you will solve systems of equations using matrices. The matrix method is similar to the elimination method, but uses matrices. You may find this quicker because the notation is shorter. The following system has been set up as a matrix.

$$\begin{cases} 5x + 3y = -1 \\ 2x - 6y = 50 \end{cases} \quad \begin{bmatrix} 5 & 3 & -1 \\ 2 & -6 & 50 \end{bmatrix}$$

When writing a matrix the equations must first be placed in general form. Notice that the matrix is made up of three columns and two rows. Each row represents one of the equations. One column shows the coefficients of x, another column the coefficients of y, and the last column the constants.

Find the solution to this system of equations by using the elimination method.

Complete the following two statements to describe the solution of the system of equations:

X = _____ Y=_____

The solution matrix for this system of equations will be: $\begin{cases} x + 0y = 4 \\ 0x + y = -7 \end{cases} \quad \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -7 \end{bmatrix}$

How do we work with a Matrix to get the solution matrix? We perform row operations.

Recall that in the elimination method you combined equations and multiplied them by numbers. In much the same way we can modify the rows of the matrix by performing row operations on each number in those rows.

- *Multiply (or divide) all numbers in a row by a non-zero number*
- *Add all numbers in a row to corresponding numbers in another row*
- *Add a multiple of the numbers in one row to the corresponding numbers in another row*
- *Exchange two rows.*

Our goal will be to change the original matrix into the solution matrix:

$$\begin{bmatrix} 5 & 3 & -1 \\ 2 & -6 & 50 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -7 \end{bmatrix}$$

Let's start:

The original matrix is formed by copying the coefficients and constants from the two equations:

$$\begin{bmatrix} 5 & 3 & -1 \\ 2 & -6 & 50 \end{bmatrix}$$

Notice that the middle column has one negative and one positive integer.
By what number can you multiply the second row (equation) by that will make the middle column opposite numbers? Once you decide, multiply the first row (first equation) by that number and rewrite the matrix.

$$\begin{bmatrix} 5 & 3 & -1 \\ & & \end{bmatrix}$$

Study the new matrix. Notice that if we add the first row to the second row, the middle column will contain a zero in the first row. So replace the second row with the addition of the first and second rows.

$$\begin{bmatrix} 5 & 3 & -1 \\ & & \end{bmatrix}$$

Notice that we can simplify the second row by dividing by a positive number. Rewrite the matrix by dividing the second row by this number.

$$\begin{bmatrix} 5 & 3 & -1 \\ & & \end{bmatrix}$$

You need just one more zero in the first column. By multiplying the second row by a negative number and adding it to the first row you can gain one more zero. So replace the first row with this combination.

$$\begin{bmatrix} & & \\ 1 & 0 & 4 \end{bmatrix}$$

Now we just need to divide the first row by a number to get a 1 in column 2. Complete that operation.

$$\begin{bmatrix} & & \\ 1 & 0 & 4 \end{bmatrix}$$

You can now switch the rows.

$$\begin{bmatrix} & \\ & \end{bmatrix}$$

The solution to the system of equations is: x = _____ and y = _____.

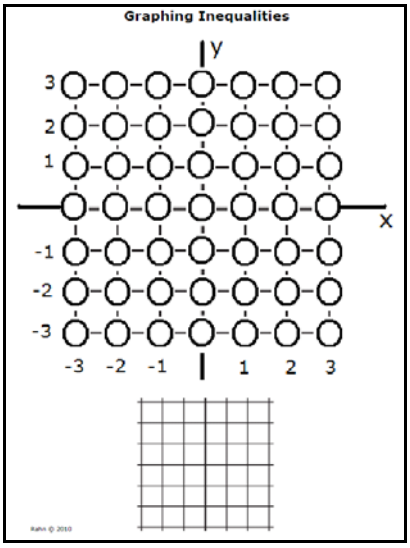
Solve this system of equations by using matrices.

$$\begin{cases} x - 2y = 3 \\ 3x + y = 23 \end{cases}$$

$$\begin{cases} 2x + y = 11 \\ 6x - 5y = 9 \end{cases}$$

On Friday, 3247 people attended the county fair. The entrance fee for an adult was $5 and for a child 12 and under the fee was $3. The fair collected $14,273. How many of the total attendees were adults and how many were children. Solve this problem using a system of equations and the matrix method.

Building Understanding for Graphing Inequalities



Place the Graphing Inequalities Template in your communicator.

Each group member should choose a different statement from the list below.

A. $y \square 1 + \frac{1}{3}x$

B. $y \square -1 - \frac{1}{3}x$

C. $y \square 2 + \frac{1}{2}x$

D. $y \square -2 - \frac{1}{2}x$

- Evaluate the right side of the statement for x = -3. This corresponds to each circle in the first column on the graph. Fill in > if the y-value is greater than your value, = if the values are equal, and < if the y-value is less than your value.
- Evaluate the right side of the statement for x = -2. This corresponds to each circle in the second column on the graph. For each circle in the second column on the graph, fill in > if the y-value is greater than your value, = if the values are equal, and < if the y-value is less than your value.
- Repeat the last step for x = -1, 0, 1, 2, and 3.
- What do you notice about the circles filled with the equal sign? Describe any other patterns you see.
- Test a point with fractional or decimal coordinates that is not represented by a circle on the grid. Compare your results with the symbols on the same side of the line of equal signs as your point.
- Next to the xy axis, at the bottom of the template, write your statement with the "less than" symbol, <. Shade the region of points that makes your statement true. If the points on the line make an inequality true, draw a solid line through them.
- Repeat the last step with >, $\leq$, $\geq$, and =.
- Compare your graphs with those of others in your group. What graphs require a solid line? A dashed line?
- What graphs require shading? Shading above the line? Below the line?
- Discuss how to use one point to check the graph of an inequality.

Graphing Inequalities