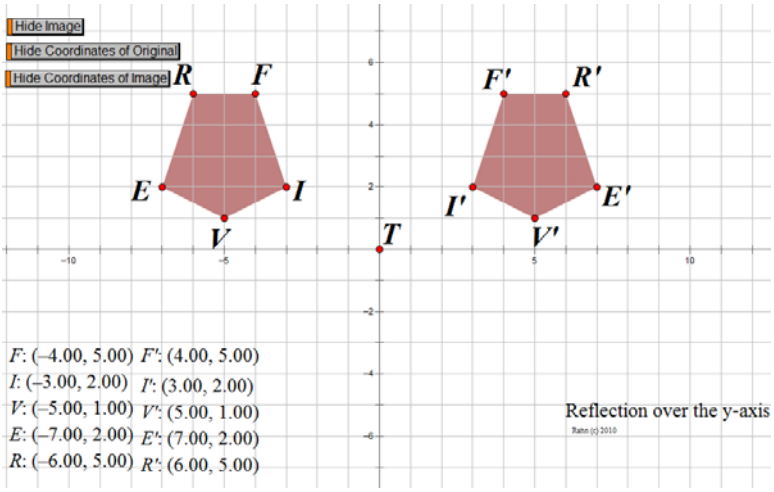
Geometry and Algebra 2

So Happy Together

Part II



Red Bank Regional High School

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Jim Rahn
www.jamesrahn.com
james.rahn@verizon.net

Exploring Transformations with the Geometer's Sketchpad
Part I: Translations

In part I of this three part activity you will explore the properties of a translation by creating a figure on a coordinate grid, translate the figure, and then making observation about what changes and what does not change.

Open Exploring Translations.gsp. Open the page entitled "Creating a Translation" at the bottom left of the document.

A rectangular grid has been placed on the document. Click on Graph and make sure Snap Points is checked off.

Use the Segment Tool to create a triangle in the first quadrant. Click on the Text Tool and then on each of the vertices to name the vertices A, B, and C.

Click on the Selection Tool and then on the three vertices. While the three vertices are highlighted, click on Measure and then Coordinates. The coordinates for the three vertices should appear on the screen. Move these to a location that does not interfere with the triangle. Click on any vertex and more the vertex. Notice that the coordinates for that vertex are updated as you move the vertex.

Click on the Segment Tool and then construct a segment that begins at the origin and extends to some point in the second quadrant. Click on the Text Tool and then on the origin to label the origin D. Click on the other endpoint to label it E. While point E is highlighted measure the coordinates of point E.

Click on the three vertices of the triangle using the Selection Tool and then click on the Construct menu and select Triangle Interior. While the interior of the triangle is highlighted, click on Measure and then click on Area. When the area is revealed, move that measure to be near the list of coordinates.

Create a Translation

Let's perform a transformation of the triangle ABC by translating the triangle. First click on D and then E and while both points are highlighted, click on Transform and select Mark Vector. Although the segment DE still appears like a segment, it now has new characteristics. It is a translation vector. You should notice that the program has illustrated a movement from D toward E. Let's try translating the triangle by using that transformation. Select the triangle, its vertices, and the interior of the triangle.

Click on Transform and select Translate. Notice the Translate window pops up to tell you that the triangle will use the marked vector from point D to point E. Since this is the translation we want to use click on Translate.

Click on the three new vertices using the Text Tool to name the points. What do you notice about their names? While the three vertices are highlighted, measure their coordinates. Place these names near the other names. Click on the interior of the translated triangle and measure its area. Move this near the other area measurement.

Keep point D located at the origin. Click on the point E and move it around the coordinate axes. Study the coordinates of A', B', and C'. Think about the following questions as you move point E to new locations?

Describe how the two triangles are the same and how they are different.

As you move point E, what is it doing to the vector DE? What two characteristics of the vector are changing?

As you study the coordinates of point E, how does that affect the coordinates of points A, B, and C?

If you move point E to the first quadrant, describe the change in triangle ABC.

Describe how the coordinates of point E control this change.

If you move point E to the third quadrant, describe the change in triangle ABC.

Describe how the coordinates of point E control this change.

If you move point E to the fourth quadrant, describe the change in triangle ABC.

Describe how the coordinates of point E control this change.

Properties of a Translation

Open the page entitled "Properties of a Translation". This time you are presented with a Quadrilateral QUAD and its image Q'U'A'D'. Move point P around the coordinate plane to confirm the properties you developed on the previous part about a translation.

Click on Show Coordinates of Original Polygon, Show Coordinates of Translated Polygon, and Show Coordinates of P to help you make observations.

Hide the coordinates by clicking on the same buttons that now say Hide.

Click on Show Segments between Corresponding Points. Describe what segments were drawn.

Move point P to different locations. Make some observations about the relationship of these segments and the translation vector.

To confirm these observations, click on the some of the various buttons at the bottom of the screen.

Explain how these new pieces of information support your observations.

What is a Translation?

Open the page entitled "What is the Translation. You will see one polygon PENTA. To view its image from some translation, click on Show Translated Pentagon. You should now see its image P'E'N'T'A'.

First move some points on the original pentagon to see if you can determine the translation that has taken place. Try to predict the direction and magnitude of the translation.

As you think you can describe the translation that changed PENTA to P'E'N'T'A', click on some the Hide/Show buttons to confirm you prediction.

Explain how you can predict the translation by just viewing the original coordinates and the coordinates of the image.

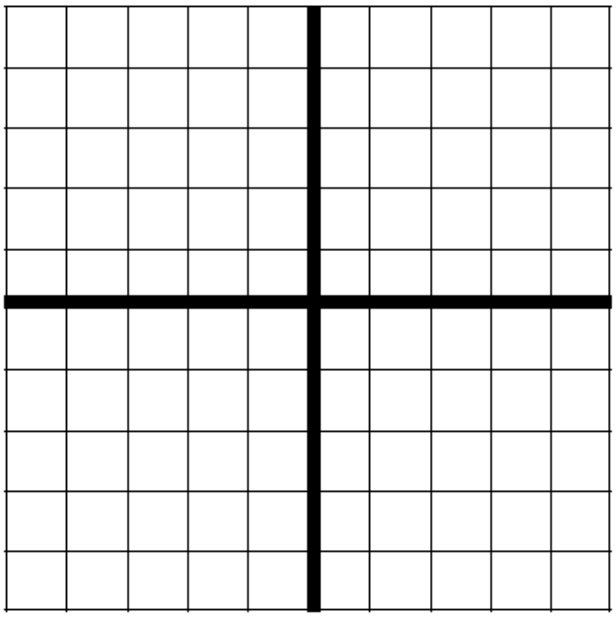
Explain how you can predict the translation by just viewing the segments between the corresponding points on the original pentagon and its image.

Explain how you can draw the translation vector from the two figures.

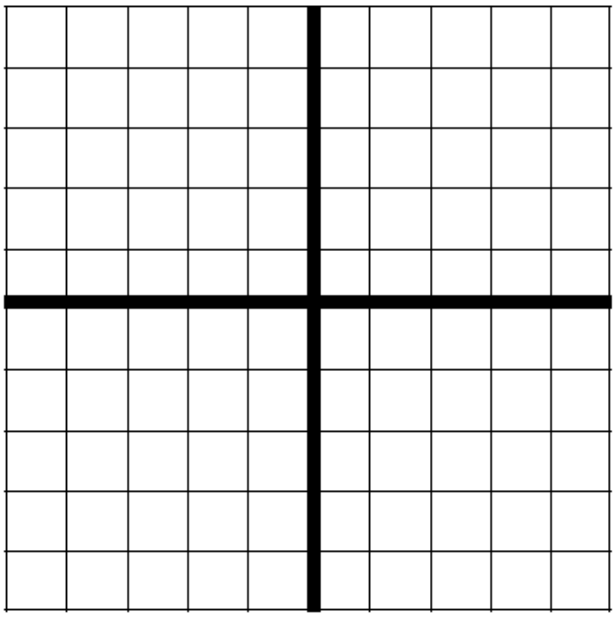
Explain how you can predict the endpoint of the translation vector by studying the two figures.

If a point (x,y) is translated using a translation vector that begins at (0,0) and ends at (a,b). Describe the coordinates of the new point after (x,y) is translated.

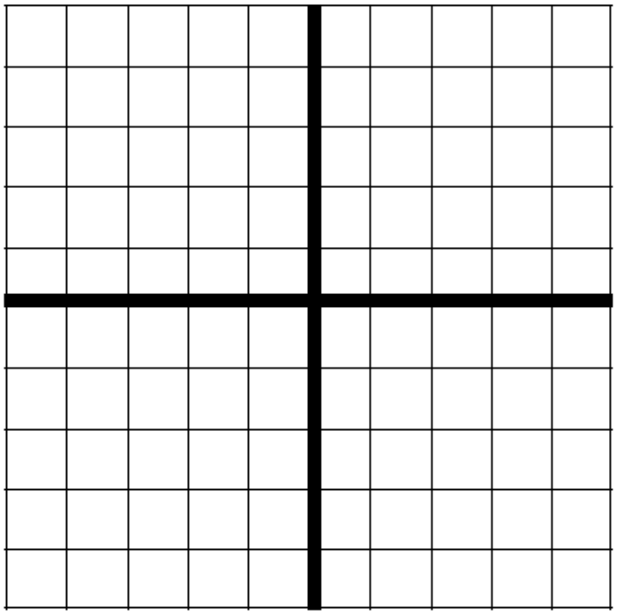
Applying What You Learned



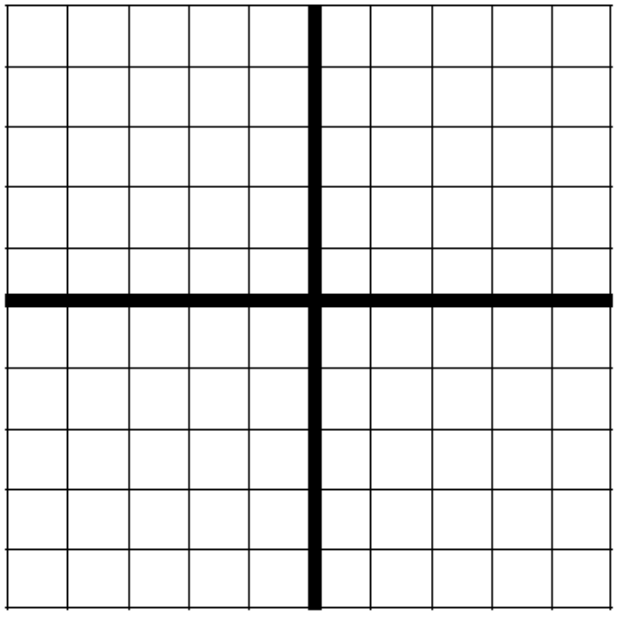
1. A triangle ABC has vertices (-4,-4), (-3,-2) and (-1,-2). If the triangle is translated using the translation vector that begins at (0,0) and ends at (0,4), find the coordinates of A', B', and C', the vertices of the image of triangle ABC under the translation.



2. A quadrilateral ABCD has been translated using a translation vector that begins at (0,0) and ends at (-3,+2). If the image of the quadrilateral has vertices A'(-4,3), B'(-3,1), C'(-1,3), and D'(-3,2), find the coordinates of the original quadrilateral ABCD.



3. A translation vector begins at (0,0) and ends at (2, -4). Create a triangle that is in quadrant II and show its translated image.



4. A rectangle has vertices A(1,3), B(2,4), C(4,1), and D(3,1). Its image has been translated to a new position: A'(-2,-1), B'(-1,0), C'(1,-2), and D'(0,-3). Find the translation vector.

Exploring Transformations with the Geometer's Sketchpad
Part II: Rotations

In part II of this three part activity you will explore the properties of a Rotation by creating a figure on a coordinate grid, rotating the figure, and then making observation about what changes and what does not change.

Open Exploring Rotations.gsp. Open the page entitled "Creating a Rotation" at the bottom left of the document.

A rectangular grid has been placed on the document. Click on Graph and make sure Snap Points is checked off.

Use the Segment Tool to create a triangle in the fourth quadrant. Click on the Text Tool and then on each of the vertices to name the vertices A, B, and C.

Click on the Selection Tool and then on the three vertices. While the three vertices are highlighted, click on Measure and then Coordinates. The coordinates for the three vertices should appear on the screen. Move these to a location that does not interfere with the triangle. Click on any vertex and more the vertex. Notice that the coordinates for that vertex are updated as you move the vertex.

Click on the three vertices of the triangle using the Selection Tool and then click on the Construct menu and select Triangle Interior. While the interior of the triangle is highlighted, click on Measure and then click on Area. When the area is revealed, move that measure to be near the list of coordinates.

Create a Rotation

Let's perform a rotation of the triangle ABC about the origin. Double click on the origin. You should see a bull's eye. Select the triangle, its vertices, and the interior of the triangle. Click on Transform and select Rotate. Notice the Rotate window pops up to tell you that the rotation will be about a fixed angle. Select 90 degrees and then click on Rotate.

Click on the three new vertices using the Text Tool to name the points. What do you notice about their names? While the three vertices are highlighted, measure their coordinates. Place these names near the other names. Click on the interior of the translated triangle and measure its area. Move this near the other area measurement.

Move the original vertices A, B, and C and study how the new coordinates A', B', and C' are related to those of A, B, and C when a figure is rotated 90 degrees counter clockwise (+90 degrees) around the origin.

If figure ABC is rotated + 90 degrees and the coordinates of A are (3,4), describe how you find the coordinates of A'.

If figure ABC is rotated + 90 degrees around the origin and the coordinates of C' are (-2,-4). Describe the coordinates of C.

If figure ABC had an area of 2 square units and was rotated about the origin by 90 degrees around the origin, what would the area of the new figure A'B'C' be?

Describe how the two triangles are the same and how they are different.

Study the order of the letters that name the figure. Is there any change in the order when the figure is rotated?

If one order of letters is clockwise, are the letters on the image figure also clockwise?

Rotation of 180°

Open the page entitled “Rotation of 180 degrees”. A quadrilateral FOUR is drawn in the first quadrant.

To create the rotation of this quadrilateral by 180 degrees, click on Show Rotated Figure. Change the shape of the quadrilateral by moving the various vertices. Make some observations about how the figure has changed.

Click on the Show Coordinates of Original Figure. Again move the original figure around and make an observation about the coordinates on the image F’O’U’R’.

Click on the Show Coordinates of Rotated Figure to study the two sets of coordinates next to each other. Make an observation about the relationship between the coordinates.

Study the two figures: the original and its image after being rotated 180 degrees around the origin. Move the points around on the original figure. What do you notice about the name of the points on the image?

If point (2,3) is rotated around the origin by 180 degrees, name the coordinates of the new point.

If the coordinates of a point rotated around the origin by 180 degrees are (4,-3), what were the coordinates of the original point.

Suppose the coordinates of point A were (-5,2) and the coordinates of the new point, creating through a rotation around the origin were (2,-5), by how many degrees were the point rotated around the origin?

Suppose the coordinates of point A were (6, -1) and the coordinates of the new point, creating through a rotation around the origin were (-6,1), by how many degrees were the point rotated around the origin?

Suppose the area of the original quadrilateral is 4.5 square units. What would be the area of the image of this quadrilateral if it was rotated around the original by 180 degrees?

If the original quadrilateral was named RAID by reading the vertices in a clockwise direction, how would the vertices of the image be named if the quadrilateral was rotated 180 degrees around the origin?

Click on the box that says “Show Segments to Matching Vertices”. You see 8 segments drawn from the center of rotation to each of the vertices. Study each pair of segments (match by colors). What angle is formed by each pair of segments? How is this related to the type of rotation that took place?

What type of rotation?

Click on the page entitled “What type of Rotation?” You will see a pentagon FIVER in the second quadrant.

Click on the button Show Image 1 to see one transformation of the original figure. Explain why you believe this transformation is a rotation.

What do you notice around the arrangement of the vertices on both figures.

Click on the button that says Show Segments to I and I’. How large of a rotation was made when figure FIVER was rotated to F’I’V’E’R’. Click on the button that says Show Angle of Rotation 1 to see if you are right.

Hide the information about Image 1. Click on Show Image 2 to see another transformation of the original figure.

Explain why you believe this transformation is a rotation.

What do you notice around the arrangement of the vertices on both figures.

Click on the button that says Show Segments to E and E”. How large of a rotation was made when figure FIVER was rotated to F’I’V’E’R’. Click on the button that says Show Angle of Rotation 2 to see if you are right.

Describe in your own words how you can determine the angle of rotation that was performed on a figure if it revolved around the origin.

Other Rotations

Click on the page entitled Other Rotations. You will see a quadrilateral TRAP in the third quadrant.

Click on Show Rotation 1. Predict the type of rotation that moved TRAP to the fourth quadrant.

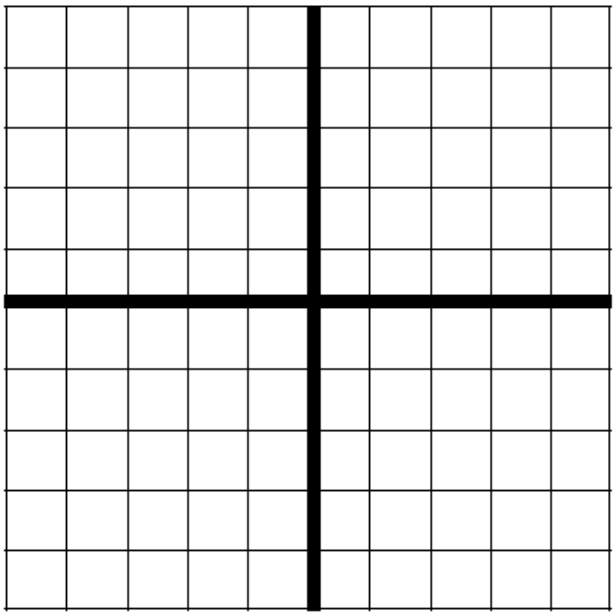
To confirm your conjecture, click on Show Segments to P and P’. Does this confirm your conjecture about the rotation?

Hide the information about rotation 1. Click on Show Rotation 2. Predict the type of rotation that moved TRAP to the fourth quadrant.

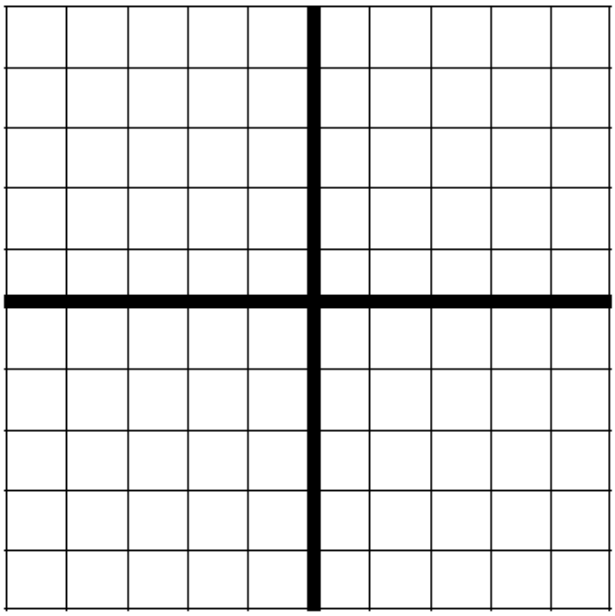
To confirm your conjecture, click on Show Segments to A and A”. Does this confirm your conjecture about the rotation?

Explain how you can determine the number of degrees a figure has been rotated around the origin if you are given both the original figure and its image.

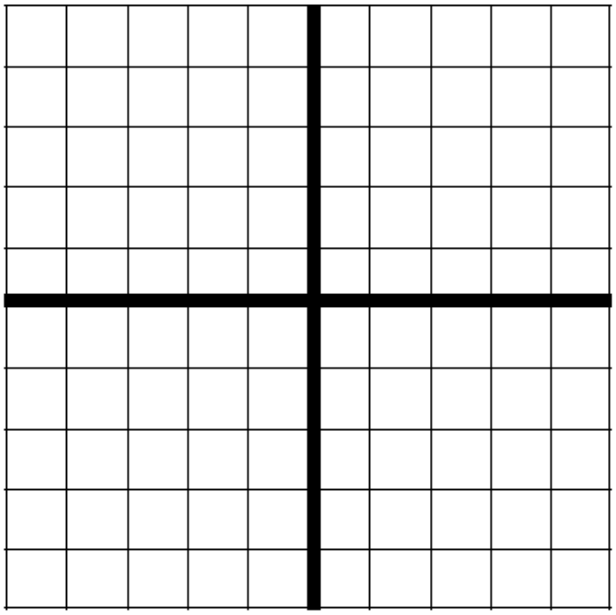
Applying What You Learned



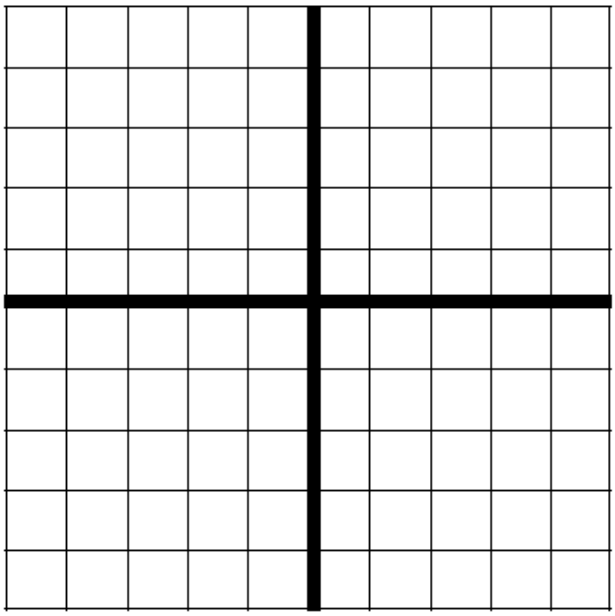
1. A triangle ABC has vertices (-4,-4), (-3,-2) and (-1,-2). If the triangle is rotated around the origin by 90 degrees, find the coordinates of A', B', and C', the vertices of the image of triangle ABC under the translation.



2. A quadrilateral ABCD has been rotated 180 degrees around the origin. If the image of the quadrilateral has vertices A'(-4,3), B'(-3,1), C'(-1,3), and D'(-3,2), find the coordinates of the original quadrilateral ABCD.



3. Create a triangle in quadrant II and show it image if it is rotated 90 degrees around the origin.



4. A rectangle is been rotated around the origin by 180 degrees and the image is located at A'(1,0), B'(2,1), C'(0,3) and D'(-1,2). What were the coordinates of the original rectangle?

Exploring Transformations with the Geometer's Sketchpad
Part III: Reflections

In part III of this three part activity you will explore the properties of a Reflections by creating a figure on a coordinate grid, reflecting the figure, and then making observation about what changes and what does not change.

Open Exploring Reflections.gsp. Open the page entitled "Creating a Reflection over the x-axis" at the bottom left of the document.

A rectangular grid has been placed on the document. Click on Graph and make sure Snap Points is checked off.

Use the Segment Tool to create a triangle in the first quadrant. Click on the Text Tool and then on each of the vertices to name the vertices A, B, and C.

Click on the Selection Tool and then on the three vertices. While the three vertices are highlighted, click on Measure and then Coordinates. The coordinates for the three vertices should appear on the screen. Move these to a location that does not interfere with the triangle. Click on any vertex and more the vertex. Notice that the coordinates for that vertex are updated as you move the vertex.

Click on the three vertices of the triangle using the Selection Tool and then click on the Construct menu and select Triangle Interior. While the interior of the triangle is highlighted, click on Measure and then click on Area. When the area is revealed, move that measure to be near the list of coordinates.

Create a Reflection

Let's perform a reflection of the triangle ABC over the x-axis. Double click on the x-axis. You should see two square bull's eye. Select the triangle, its vertices, and the interior of the triangle. Click on Transform and select Reflect. The original triangle is reflected over the x-axis.

Click on the three new vertices using the Text Tool to name the points. What do you notice about their names? While the three vertices are highlighted, measure their coordinates. Place these names near the other names. Click on the interior of the translated triangle and measure its area. Move this measurement near the other area measurement.

Move the original vertices A, B, and C and study how the new coordinates A', B', and C' are related to those of A, B, and C when a figure is reflected over the x-axis.

Questions:

If point A(3,4) is reflected over the x-axis, describe how you find the coordinates of A'.

If point B'(-2,-4) is the reflection of point B over the x-axis, describe the coordinates of B.

If figure ABC had an area of 2 square units and was reflected over the x-axis, what would the area of the new figure A'B'C' be?

Describe how the two triangles are the same and how they are different.

Study the order of the letters that name the figure. Is there any change in the order when the figure is reflected over the x-axis? If one order of letters is clockwise, are the letters on the image figure also clockwise?

Reflection over the y-axis

Double click on the y-axis. You should see double square bull's eyes on the y-axis. Select the image A'B'C' and reflect it over the y-axis. Move the points A', B', and C' to see how they change the points A", B", and C".

Describe how the figure A'B'C' is related to its reflection A"B"C".

Questions

If point A'(2,-4) is reflected over the y-axis, what are the coordinates of A"?

If point C"(1,3) is the reflection of some point C' over the y-axis, what are the coordinates of C'?

Studying Reflections over the x-axis

Open the page entitled "X-axis Reflection". A quadrilateral FOUR is drawn in the first and second quadrant. To create the reflection of this quadrilateral over the x-axis, click on the box "Show Image". Make some observations about how the figure has changed.

Click on the Show Coordinates of Original. Again move the original figure around and make an observation about the coordinates on the image F'O'U'R'. Try to predict the coordinates of the vertices on the image. Confirm you conjecture by clicking on Show Coordinates of Image. Again move the original figure around to see the relationship between the vertices.

Questions:

If point (2,3) is reflected over the x-axis, what are the coordinates of the new point?

If a point (x,y) in on the original figure and the figure is reflected over the x-axis, what will be the coordinates of the reflected point. Explain why you wrote this answer.

Studying Reflections over the y-axis

Open the page entitled "Y-axis Reflection". A pentagon FIVER is drawn in the second quadrant. To create the reflection of this quadrilateral over the x-axis, click on the box "Show Image". Make some observations about how the figure has changed.

Click on the Show Coordinates of Original. Again move the original figure around and make an observation about the coordinates on the image F'I'V'E'R'. Try to predict the coordinates of the vertices on the image. Confirm you conjecture by clicking on Show Coordinates of Image. Again move the original figure around to see the relationship between the vertices.

Questions:

If point (-2,3) is reflected over the y-axis, what are the coordinates of the new point?

If a point (x,y) in on the original figure and the figure is reflected over the x-axis, what will be the coordinates of the reflected point. Explain why you wrote this answer.

Studying Properties of Reflections over the x-axis and y-axis

Open the page entitled “Properties of Reflections”. You see a triangle ABC in one of the quadrants. There are three sets of buttons on the left and three sets on the right. You will also notice one button in the middle of the two sets.

Think about what the reflection of this triangle would look like if it was reflected over the x-axis. When you have pictured where the reflected triangle will be, click on Show Image of Reflection over the x-axis to confirm its location.

Think about the coordinates associated with each triangle. Click on Show Coordinates of Original to view the coordinates of the original triangle. Write down the coordinates of the image triangle based upon the coordinates of the original triangle. Click on Show Coordinates of x-axis Reflection to check your answers.

Hide the coordinates for both triangles. Click on Show Segments between Corresponding Points x-axis. Study the segments that connect the corresponding parts of the two triangles. Make three conjectures about these segments.

Hide the segments. Think about what the reflection of this triangle would look like if it was reflected over the y-axis. When you have pictured where the reflected triangle will be, click on Show Image of Reflection over the y-axis to confirm its location.

Think about the coordinates associated with each triangle. Click on Show Coordinates of Original to view the coordinates of the original triangle. Write down the coordinates of the image triangle based upon the coordinates of the original triangle. Click on Show Coordinates of y-axis Reflection to check your answers.

Hide the coordinates for both triangles. Click on Show Segments between Corresponding Points y-axis. Study the segments that connect the corresponding parts of the two triangles. Make three conjectures about these segments.

Studying Reflections over a Line Parallel to the x-axis

Open the page entitled “Reflections over Lines parallel to the X-axis.” Quadrilateral TRAP is drawn in the second quadrant. Click on Show Parallel Line. You will noticed that a line parallel to the x-axis was placed on the graph. Try to visualize where the image of quadrilateral TRAP will appear if it is reflected over the parallel line, instead of the x-axis. When you think you have determined the location of the image click on Show Image.

Study the coordinates of the original TRAP by clicking on Show Coordinates of Original. Then find the coordinates of the image of TRAP. Check the coordinates by clicking on Show Coordinates of Image. Make a conjecture about how you can determine the coordinates for T’R’A’P’ by studying the two sets of coordinates.

Change the position of the quadrilateral TRAP and confirm your conjecture about how you can determine the coordinates for T’R’A’P’.

Move the parallel line to the x-axis up or down and then re-check your conjecture.

Hide the coordinates of the original TRAP and change the location of T’R’A’P’. Try to determine the coordinates for TRAP by using your conjecture in reverse.

Continue to play with the original figure and the parallel line until you are comfortable with your conjecture the describes how the coordinates for TRAP are changed by reflecting the figure over the line parallel to the x-axis.

Questions:

If (1,3) is reflected over the line y = 1, what will its new position be?

If (2,6) is the point that results from reflecting (a,b) over the line y = 2, find the value of a and b.

If (-3,7) is reflected over some line parallel to the x-axis and results in the point (-3,-1), what is the equation of the line?

Complete the following: If (a,b) in a point on the original shape and that shape is reflected over the line y = k, what is the resulting point?

Studying Reflections over a Line Parallel to the x-axis

Open the page entitled "Reflections over Lines parallel to the Y-axis." Quadrilateral TRAP is drawn in the second quadrant. Click on Show Parallel Line. You will noticed that a line parallel to the y-axis was placed on the graph. Try to visualize where the image of quadrilateral TRAP will appear if it is reflected over the parallel line, instead of the y-axis. When you think you have determined the location of the image click on Show Image.

Study the coordinates of the original TRAP by clicking on Show Coordinates of Original. Then find the coordinates of the image of TRAP. Check the coordinates by clicking on Show Coordinates of Image. Make a conjecture about how you can determine the coordinates for T'R'A'P' by studying the two sets of coordinates.

Change the position of the quadrilateral TRAP and confirm your conjecture about how you can determine the coordinates for T'R'A'P'.

Move the parallel line to the y-axis left or right and then re-check your conjecture.

Hide the coordinates of the original TRAP and change the location of T'R'A'P'. Try to determine the coordinates for TRAP by using your conjecture in reverse.

Continue to play with the original figure and the parallel line until you are comfortable with your conjecture the describes how the coordinates for TRAP are changed by reflecting the figure over the line parallel to the x-axis.

Questions:

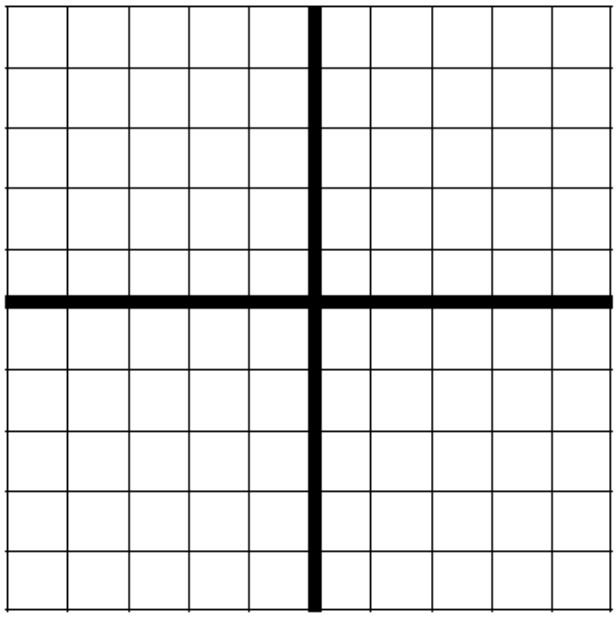
If (1,3) is reflected over the line x = 1, what will its new position be?

If (2,6) is the point that results from reflecting (a,b) over the line x = -2, find the value of a and b.

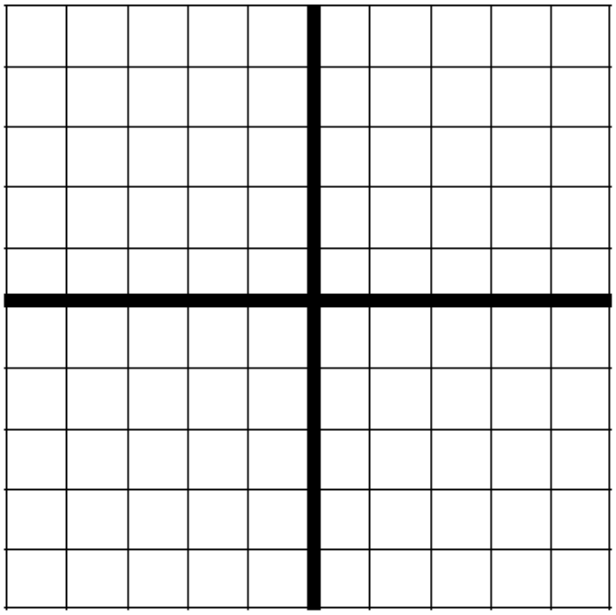
If (-4,2) is reflected over some line parallel to the y-axis and results in the point (10,2), what is the equation of the line?

Complete the following: If (a,b) in a point on the original shape and that shape is reflected over the line y = k, what is the resulting point?

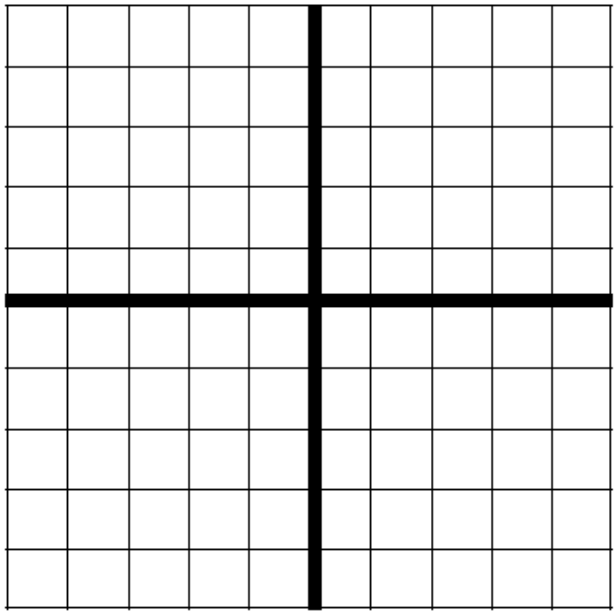
Applying What You Learned



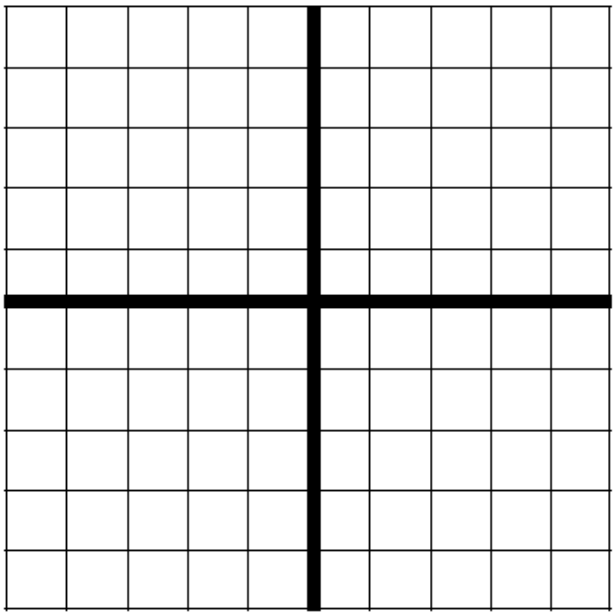
1. A triangle A(1,1), B(1,3), and C(2,1) is reflected over the y-axis. Name the coordinates of the image.



2. A quadrilateral ABCD has been reflected over the x-axis. If the image of the quadrilateral has vertices A'(-4,3), B'(-3,1), C'(-1,3), and D'(-3,2), find the coordinates of the original quadrilateral ABCD.



3. Create a triangle in quadrant II and show it image if it is reflected over the line x=1.



4. A rectangle is been reflected over the line y=-1 and the image is located at A'(1,0), B'(2,1), C'(0,3) and D'(-1,2). What were the coordinates of the original rectangle?