Achieving Success in Meeting the Common Core State Standards in Algebra Part I

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New Jersey Algebra teachers are challenged with meeting standards described in the Common Core State Standards and New Jersey Algebra I Core Content Standards. This workshop will share numerous activities that will help the students visualize and build understanding for many of the goals delineated in these two documents. The activities will show how students begin to think and use algebra skills differently when integrated into the learning are manipulatives, technology, and real world applications. Both Algebra tiles and graphing calculators will be integrated into activities throughout the workshop.

The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

The high school standards specify the mathematics that all students should study in order to be college and career ready.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **REASON ABSTRACTLY AND QUANTITATIVELY**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **MODEL WITH MATHEMATICS**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **USE APPROPRIATE TOOLS STRATEGICALLY**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **ATTEND TO PRECISION**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

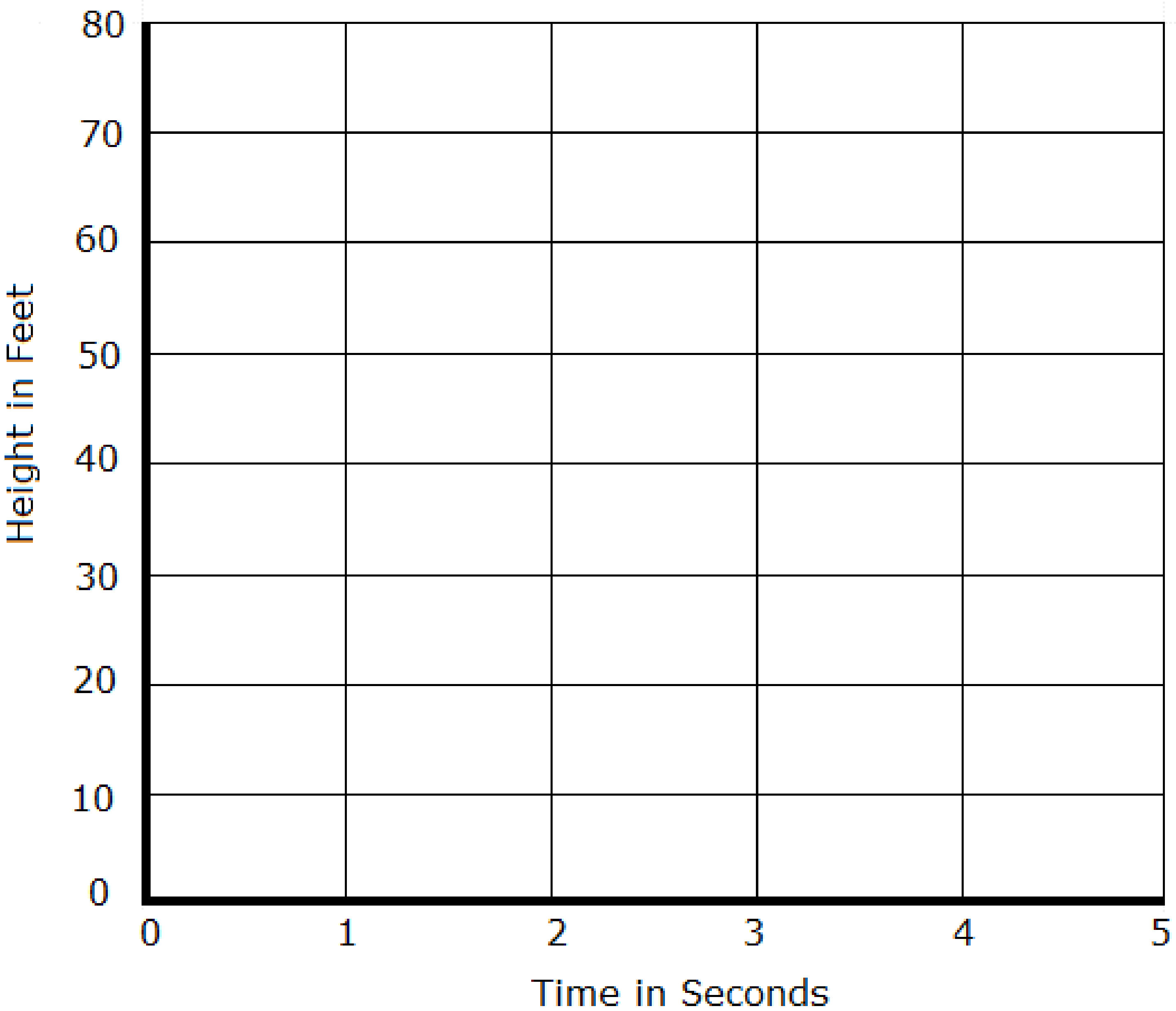
7. **LOOK FOR AND MAKE USE OF STRUCTURE**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Height of a Ball



A baseball batter pops a ball straight up. The ball reaches a height of 68 ft before falling back down. Roughly 4 s after it is hit, the ball bounces off home plate.
Make a sketch of a graph that models the ball's height in feet during its flight time in seconds. When is the ball 68 ft high? How many times will it be 20 ft high?

Points to Observe About the Graph

- When the bat hits the ball, it is a few feet above the ground. So the y-intercept is just above the origin.
- The ball's height is 0 when it hits the ground just over 4 s later. So the parabola crosses the x-axis near the coordinates (4, 0).
- The ball is at its maximum height of 68 ft after about 2 s, or halfway through its flight time.
- So the vertex of the parabola is near (2, 68).
- The ball reaches a height of 20 ft twice—once on its way up and again on its way down.

Model Rocket Science

In the metric system, the acceleration due to gravity is 9.8 m/s^2. The quadratic function $h(t)=(1/2)(-9.8)t^2+50t+25$ describes the rocket's projectile motion.

- What are the variables used in this function? What are their units of measure?
- What is the real-world meaning of $h(0)=25$?
- How is the acceleration due to gravity, or g, represented in the equation?
- How does the equation show that this force is downward?
- Graph the function $h(t)$. What viewing window shows all the important parts of the parabola?
- How high does the rocket fly before falling back to Earth? When does it reach this point?
- How much time passes while the rocket is in flight, after the engine shuts down?
- What domain and range values make sense in this situation?
- Write the equation you must solve to find when $h(t)=60$.
- When is the rocket 60 m above the ground? Use a calculator table to approximate your answers to the nearest tenth of a second.
- Describe how you determine when the rocket is at a height of 60 feet graphically.
- Summarize two new ideas you learned about quadratic equations from this activity.
- Solve this equation symbolically: $4(x-1)^2+9=37$ Check your answer graphically and using a table on the graphing calculator.

All Tied Up

Step 1: Measure the length of the thinner rope without any knots. Then tie a knot and measure the length of the rope again. Continue tying knots until no more can be tied. Knots should be of the same kind, size, and tightness. Record the data for number of knots and length of rope in a table.

All Tied Up

Number of Knots	Length of Thinner Rope	Length of Thicker Rope

Step 2: Define variables and write an equation in intercept form to model the data you collected in Step 1. What are the slope and y-intercept, and how do they relate to the rope?

Step 3: Repeat Steps 1 and 2 for the thicker rope.

Step 4: Suppose you have a 9-meter-long thin rope and a 10-meter-long thick rope. Write a system of equations that gives the length of each rope depending on the number of knots tied.

Step 5: Solve this system of equations using the substitution method.

Step 6: Select an appropriate window setting and graph this system of equations. Estimate coordinates for the point of intersection to check your solution. Compare this solution with the one from Step 5.

Step 7: Explain the real-world meaning of the solution to the system of equations.

Step 8: What happens to the graph of the system if the two ropes have the same thickness? The same length?

All Tied Up

Number of Knots	Length of Thinner Rope	Length of Thicker Rope

Make a Conjecture

A conjecture is a statement that might be true but has not been proven. Your group's goal is to come up with a conjecture relating two things and to collect and analyze the numeric evidence to support your conjecture or cast doubt on it.

In this activity you'll review the measures and graphs you have learned. Along the way, you will be faced with questions that statisticians face every day.

Step 1: Your group should select two books on different subjects or with different reading levels. Flip through the books, but do not examine them in depth. State a conjecture comparing these two books. Your conjecture should deal with a quantity that you can count or measure—for example, "The history book has more words per sentence than the math book."

Step 2: Decide how much data you'll need to convince yourself and your group that the conjecture is true or doubtful. Design a way to choose data to count or measure.
For example, you might use your calculator to randomly select a page or a sentence.

Step 3: Collect data from both books. Be consistent in your data collection, especially if more than one person is doing the collecting. Assign tasks to each member of your group.

Step 4: Find the measures of center, range, five-number summary, and IQR for each of the two data sets.

Step 5: Create a dot plot or stem-and-leaf plot for each set of data.

Step 6: Make box plots for both data sets above the same horizontal axis.

Step 7: Make a histogram for each data set.
Be sure that you have used descriptive units for all of your measures and clearly labeled your axes and plots before going on to the next step.

Step 8: Choose one or two of the measures and one pair of graphs that you feel give the best evidence for or against your conjecture. Prepare a brief report or a poster.
Include

- Your conjecture.
- Tables showing all the data you collected.
- The measures and graphs that seem to support or disprove your conjecture.
- Your conclusion about your conjecture.

Step 9: In Step 2, you thought about your design for data collection and you might have used random numbers. In Step 3, you practiced consistency in collecting data. In Steps 4, 5, and 6, you were asked to find many measures and graphs, even though you used only a few of these in your final argument. Write a paragraph explaining how a failure at any one of these steps might have changed your conclusion.

Life Expectancy

This table shows the relationship between the number of years a person might be expected to live and the year he or she was born. Life expectancy is a prediction that is very useful in professions like medicine and insurance.

U.S. Life Expectancy at Birth

Birth year	Female	Male	Combined
1940	65.2	60.8	62.9
1950	71.1	65.6	68.2
1960	73.1	66.6	69.7
1970	74.7	67.1	70.8
1975	76.6	68.8	72.6
1980	77.5	70.0	73.7
1985	78.2	71.2	74.7
1990	78.8	71.8	75.4
1995	78.9	72.5	75.8
2000	79.5	74.1	76.9

(National Center for Health Statistics, in *The World Almanac and Book of Facts 2004*, p. 76) [Data sets: **LEYR, LEFEM, LEMAL, LECOM**]

Step 1 Choose one column of life expectancy data—female, male, or combined. Let x represent birth year, and let y represent life expectancy in years. Graph the data points.

Step 2 Choose two points on your graph so that a line through them closely reflects the pattern of all the points on the graph. Use the two points to write the equation of this line in point-slope form.

Step 3 Graph the line with your data points. Does it fit the data?

Step 4 Use your equation to predict the life expectancy of a person who will be born in 2022.
Step 5: Compare your prediction from Step 4 to the prediction that another group made analyzing the same data. Are your predictions the same? Are they close? Explain why it's possible to make different predictions from the same data.

Step 6: Compare the slope of your line of fit to the slopes that other groups found working with different data sets. What does the slope for each data set tell you?

Step 7: As a class, select one line of fit that you think is the best model for each column of data—female, male, and combined. Graph all three lines on the same set of axes. Is it reasonable for the line representing the combined data to lie between the other two lines? Explain why or why not.

Step 8: How does the point-slope method of finding a line compare to the intercept-form method you learned about in Lesson 4.2? What are the strengths and weaknesses of each method?

Summarize how you can fit a point-slope line to linear data.

How will you make sure that it fits the data you have graphed?

Wind Chill

In this investigation you'll use the relationship between temperature and wind chill to explore the concept of rate of change and its connections to tables, scatter plots, recursive routines, equations, and graphs. The data in the table represent the approximate wind chill temperatures in degrees Fahrenheit for a wind speed of 20 mi/h. Use this data set to complete each task.

Temperature (°F)	Wind chill (°F)
−5	−28.540
0	−21.980
1	−20.668
2	−19.356
5	−15.420
15	−2.300
35	23.940

Step 1 Define the input and output variables for this relationship.

Step 2: Plot the points and describe the viewing window you used.

Step 3: Write a recursive routine that gives the pairs of values listed in the table.

Step 4: Copy the table. Complete the third and fourth columns of the table by recording the changes between consecutive input and output values. Then find the rate of change.

Temperature (ºF)	Wind Chill (ºF)	Change in Input	Change in Output	Rate of Change
-5	-28.540			
0	-21.980			
1	-20.558			
2	-19.356			
5	-15.420			
15	-2.300			
35	23.940			

Step 5: Use your routine to write a linear equation in intercept form that relates wind chill to temperature. Note that the starting value, 28.540, is not the y-intercept. How does the rule of the routine appear in your equation?

Step 6: Graph the equation on the same set of axes as your scatter plot. Use the calculator table to check that your equation is correct. Does it make sense to draw a line through the points? Where does the y-intercept show up in your equation?

Step 7: What do you notice about the values for rate of change listed in your table? How does the rate of change show up in your equation? In your graph?

Step 8: Explain how to use the rate of change to find the actual temperature if the weather report indicates a wind chill of 9.5° with 20 mi/h winds.

Guesstimating

In this investigation you will estimate and measure distances around your room.

As a group, select a starting point for your measurements.

Choose between five and nine objects in the room that appear to be less than 5 m away.

Description	Actual Distance (m)	Estimated Distance (m)

Step 1: List the objects in the description column of a table like this one.

Step 2: Estimate the distances in meters or parts of a meter from your starting point to each object. If group members disagree, find the mean of your estimates. Record the estimates in your table.

Step 3: Measure the actual distances to each object and record them in the table.

Step 4: Draw coordinate axes and label actual distance on the x-axis and estimated distance on the y-axis. Use the same scale on both axes. Carefully plot your nine points.

Step 5: Describe what this graph would look like if each of your estimates had been exactly the same as the actual measurement. How could you indicate this pattern on your graph?

Step 6: Make a calculator scatter plot of your data. Use your paper-and-pencil graph as a guide for setting a good graphing window.

Step 7: On your calculator, graph the line $y = x$. What does this equation represent?

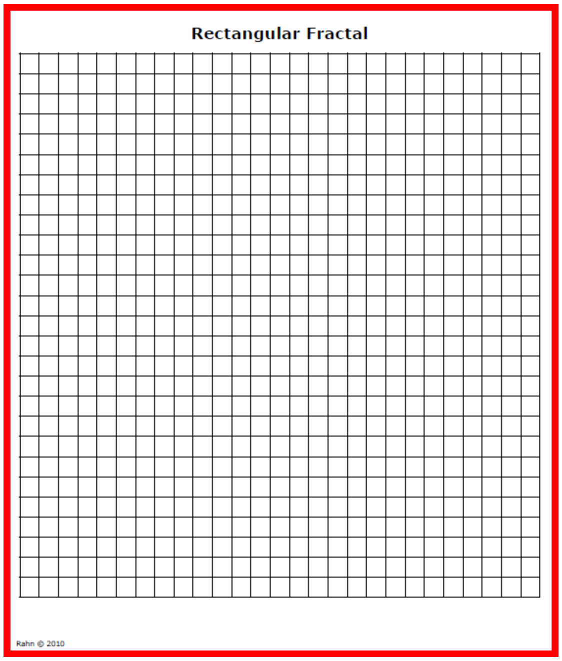
Step 8: What do you notice about the points for distances that were underestimated? What about points for distances that were overestimated?

Step 9: How would you recognize the point for a distance that was estimated exactly the same as its actual measurement? Explain why this point would fall where it does.

Description	Actual Distance (m)	Estimated Distance (m)

Loosing Area in a Fractal

In this investigation you will look for patterns in area of a rectangular fractal.



To create a fractal we will begin with a 27 x 27 rectangle. This is stage 0.

To create stage 1 draw two vertical lines and two horizontal lines to subdivide the shape into 9 equal parts. Shade any one of the parts (each person may choose a different part).

To create stage 2 draw two vertical lines and two horizontal lines in each of the remaining rectangles to subdivide the rectangle into 9 equal parts. Shade the same one part of each of these rectangles (as you did in the stage 1).

To create stage 2 draw two vertical lines and two horizontal lines in each of the remaining rectangles to subdivide the rectangle into 9 equal parts. Shade the same one part of each of these rectangles.

Use the ratio to predict the area of stage 4.

Rewrite each total unshaded area using the constant multiplier.

If x is the stage number write an expression for the unshaded area in stage x.

Create a graph for this equation.
Check the calculator table to see that it contains the same values as your table.
What does the graph tell you about the area of the rectangular fractal?

Loosing Area Fractal

Stage Number	Total Unshaded Area	Ratio of Shaded Area to Total Area

Rectangular Fractal

Description	Expression

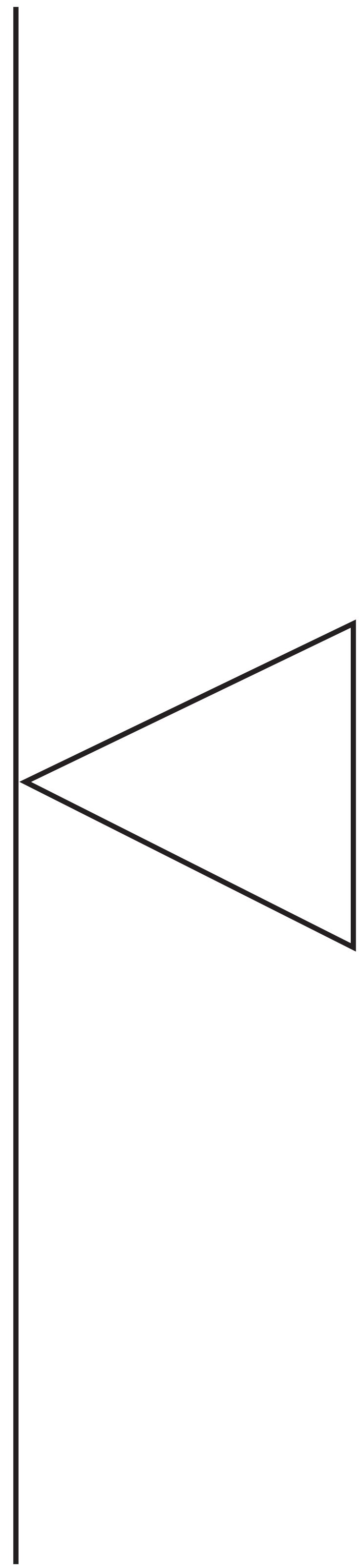
Just Undo It				
Description	**Sequence**	**Expression**	**Undo**	**Result**
Pick a Number				

Undoing an Equation Template

Equation:

Description	Undo	Result
Pick x		

Balance Scale



Distributive Property Template

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In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Algebra Introduction

Expressions

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and Inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = \frac{b_1+b_2}{2}h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra Overview

Seeing Structure in Expressions

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

   Interpret parts of an expression, such as terms, factors, and coefficients.

   Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

Arithmetic with Polynomials and Rational Functions

Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients

determined for example by Pascal's Triangle.

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original

equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.

Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are

linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.