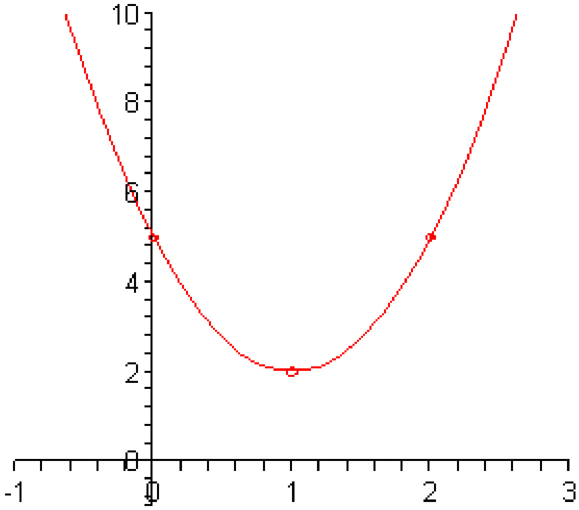
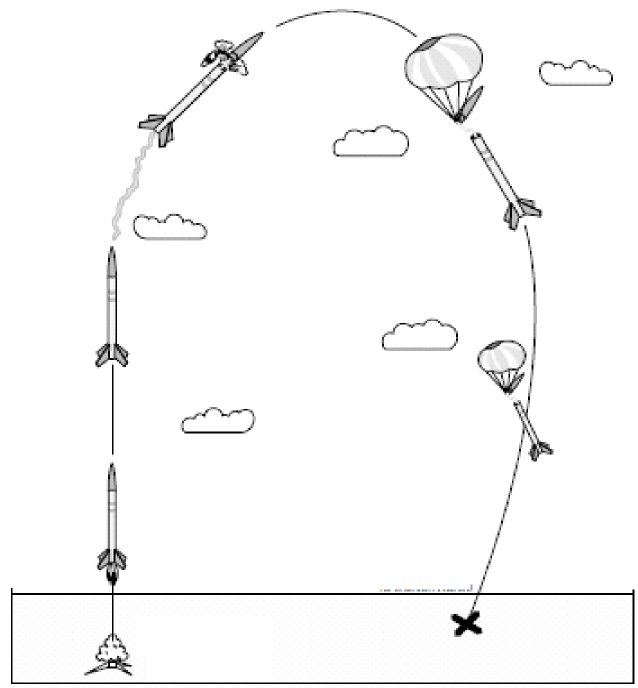
Introducing Quadratic Functions through Real Problems





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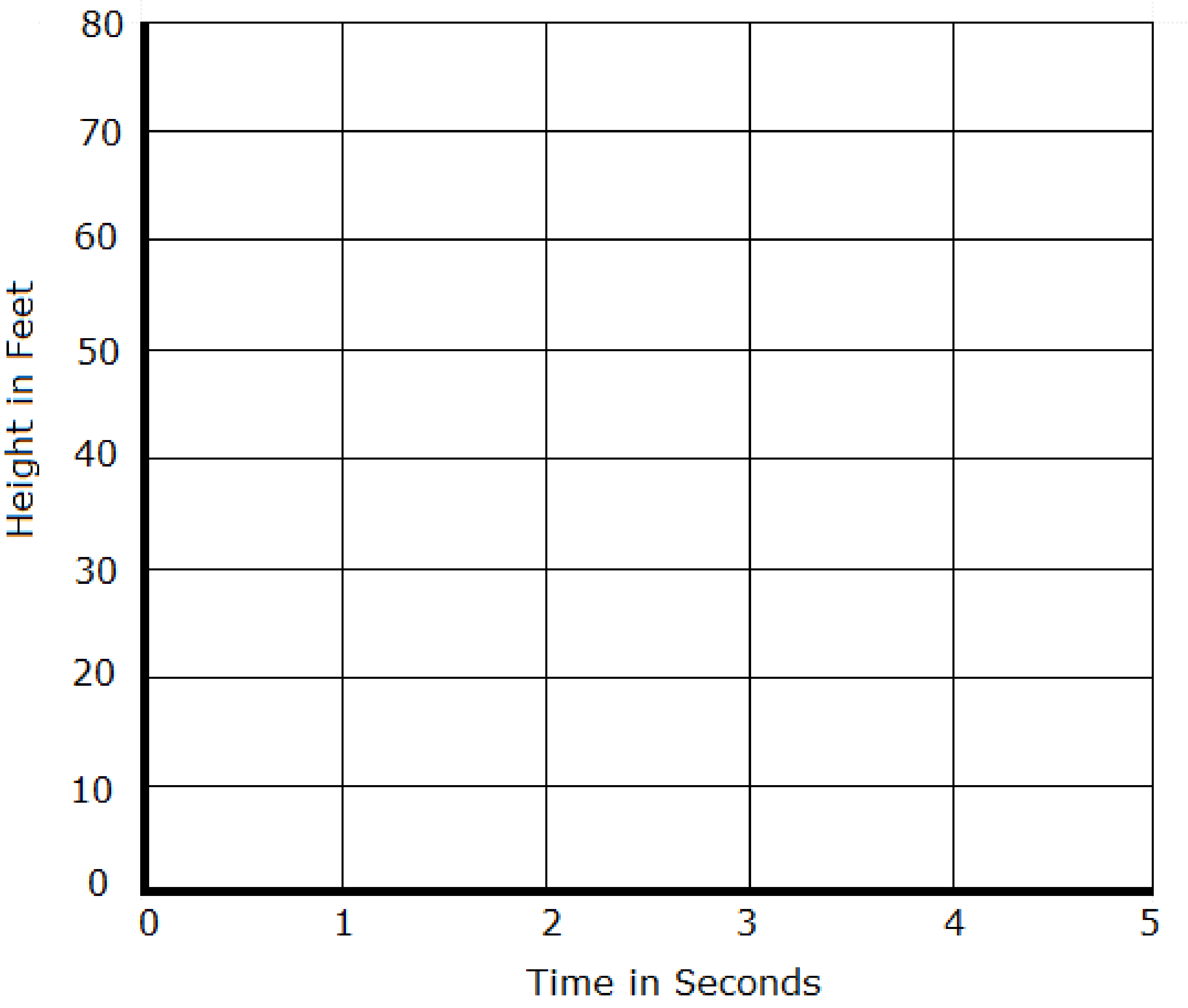
Height of a Ball



A baseball batter pops a ball straight up. The ball reaches a height of 68 ft before falling back down. Roughly 4 s after it is hit, the ball bounces off home plate.
Make a sketch of a graph that models the ball's height in feet during its flight time in seconds.

Points to Observe About the Graph

When the bat hits the ball, it is a few feet above the ground. So the y-intercept is just above the origin.

The ball's height is 0 when it hits the ground just over 4 s later. So the parabola crosses the x-axis near the coordinates (4, 0).

The ball is at its maximum height of 68 ft after about 2 s, or halfway through its flight time.

So the vertex of the parabola is near (2, 68).

The ball reaches a height of 20 ft twice—once on its way up and again on its way down.

Model Rocket Science

A model rocket blasts off and its engine shuts down when it is 25 m above the ground. Its velocity at that time is 50 m/s. Assume that it travels straight up and that the only force acting on it is the downward pull of gravity.

In the metric system, the acceleration due to gravity is 9.8 m/s^2. The quadratic function $h(t)=(1/2)(-9.8)t^2+50t+25$ describes the rocket's projectile motion.

- What are the variables used in this function? What are their units of measure?
- What is the real-world meaning of $h(0)=25$?
- How is the acceleration due to gravity, or g, represented in the equation?
- How does the equation show that this force is downward?
- Graph the function h(t). What viewing window shows all the important parts of the parabola?
- How high does the rocket fly before falling back to Earth? When does it reach this point?
- How much time passes while the rocket is in flight, after the engine shuts down?
- What domain and range values make sense in this situation?
- Write the equation you must solve to find when $h(t)=60$.

- When is the rocket 60 m above the ground? Use a calculator table to approximate your answers to the nearest tenth of a second.
- Describe how you determine when the rocket is at a height of 60 feet graphically.

- Summarize two new ideas you learned about quadratic equations from this activity.

Solve this equation symbolically: $4\left(x-1\right)^2+9=37$

Check your answer graphically and using a table on the graphing calculator.

What's the Biggest Area?

You want to build a garden whose perimeter is equal to 24 meters. If you plan to use all of the fencing material for each garden, find the dimensions of at least eight different rectangular regions that each have a perimeter 24 meters.

Find the area of each garden.

Make a table to record the width, length, and area of the possible gardens. As you find dimensions for the rectangles, the lengths can be shorter than the widths.

Think about the widths and lengths of the garden that would have an area of zero. Enter these in your lists also.

Enter the data for the possible widths into list L1. Enter the lengths in L2 and the area measures into list L3.

Label a set of axes and plot points in the form (x, y), with x representing width in meters and y representing area in square meters. Describe as completely as possible what the graph looks like. Does it make sense to connect the points with a smooth curve?

Where does your graph reach its highest point? What is the name of this point? Which rectangular garden has the largest area? What are its dimensions?

Using the graphing calculator

Create a graph of (width, length) or (L1, L2) data.

What is the length of the garden that has a width of 3 meters?

Width 5.5 meters?

Write an expression for length in terms of width x.

Using your expression for the length from the previous step, write an equation for the area of the garden. Enter this equation into Y1 and graph it. Does the graph confirm your answer for the size of the rectangle with the largest area?

Write an equation that states that the area of the rectangle is zero.

Locate the points where the graph crosses the x-axis. What is the real-world meaning for these points?

How many different widths produce an area of 30? 35?

If you were to use a different perimeter for the garden that was not 24 meters, do you think the general shape of a garden with maximum area would change? Explain your answer.

Summary:

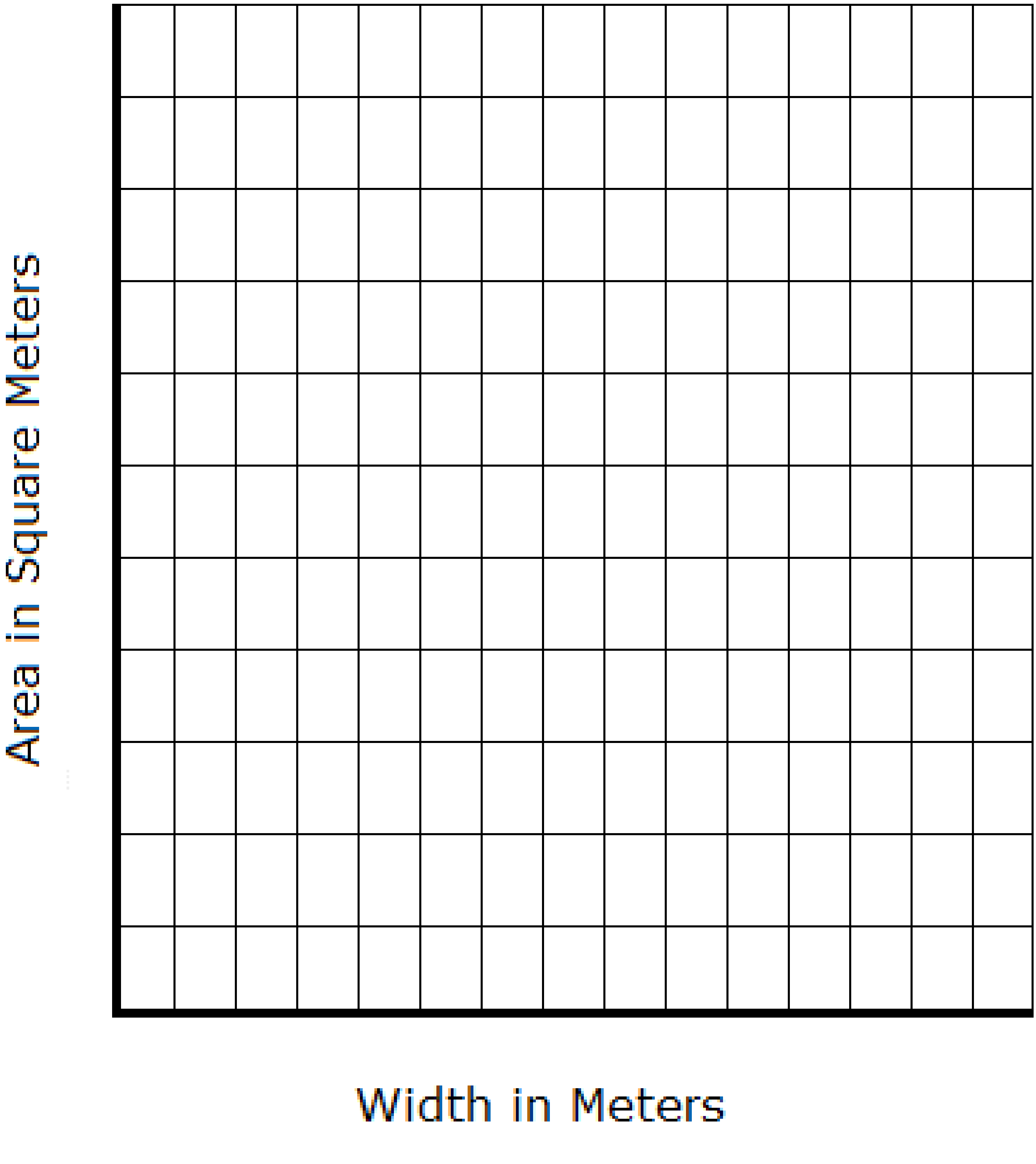
Writing an equation for the area of a rectangle whose perimeter is always 24 created what type of function?

This function had several points of interest:

- the zeros or the x-values that produced zero area
- the vertex or the x value where the maximum area occurs
- for each area between zero and the maximum of 36, there are two x-values that produce each area.

What's the Biggest Area?		
Width(m)	Length(m)	Area (m^2)

What’s the Biggest Area?



Making a Connection Between Zeros and the Vertex of a Parabola

Use a graph and your calculator's table function to approximate the roots of $0=x^2 - 2x - 8$ by placing $y1= x^2 - 2x - 8$.

Roots: x = ________ and x = _________

Create a graph of y1. The vertical line through the vertex that cuts a parabola into two mirror images is called the line of symmetry. From the roots, find the vertex and the line of symmetry.

Find the equation of the line of symmetry, and find the coordinates (h, k) of the vertex of the parabola $y=x^2 -2x - 8$. Then write the equation in the form $y=a(x - h)^2 + k$.

Enter the equation into Y2 and graph it. How does it compare to Y1? Confirm you conjecture by looking at a set of table values for Y1 and Y2. What does the table show you?

Do you need to move your vertex closer or further from the x-axis?
Do you need a stretching or shrinking factor?
Write your equation:

VERTEX FORM

Write the equation of the parabola in vertex form. The form is $y = a(x-h)^2+k$.

What is the stretching factor?

What is the vertex?

What is the equation of the parabola in vertex form?

GENERAL FORM

Expand either form to find the general form.

$y = 2(x+1)^2 - 8$

$y = 2(x+3)(x-1)$

What can you tell from each form?

Another Form for a Parabola

Another common form for a parabola or a quadratic equation is the factored form: $y = a(x - r_1)(x - r_2)$

On your calculator, graph the equations y=x+1 and y=x-5 at the same time.

What is the x-intercept of each equation you graphed in the previous step?

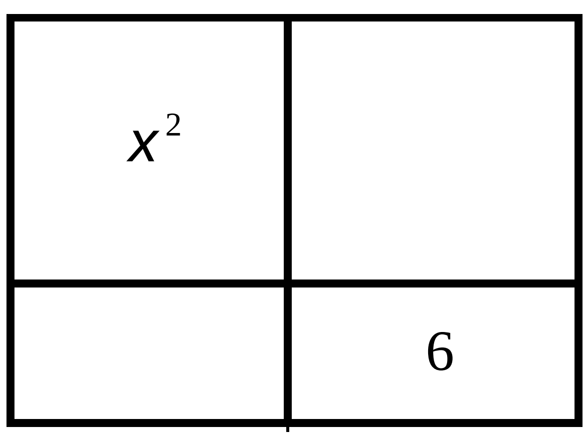
Using the same set of axes, graph y=(x+1)(x-5). Describe the graphs are related.

Describe the location of the x-intercepts on the graph of y=(x+1)(x-5).

Expand y=(x+1)(x-5) to create the general form for a parabola. Graph the equation in general form on the same set of axes. What do you notice about this parabola and its x-intercepts? Is the graph of y=(x+1)(x-5) a parabola?

Now you'll learn how to find the roots from the general form.

Complete the rectangle diagram whose sum is x^2+7x+6.



A few parts on the diagram have been labeled to get you started.

Write the multiplication expression of the rectangle diagram in factored form.

Use a graph or table to check that this form is equivalent to the original expression.

Find the roots of the equation $0=x^2+5x+6$ from its factored form.

Rewrite each of the following equation in factored form by completing a rectangle diagram. Then find the roots of each. Check your work by making a graph.

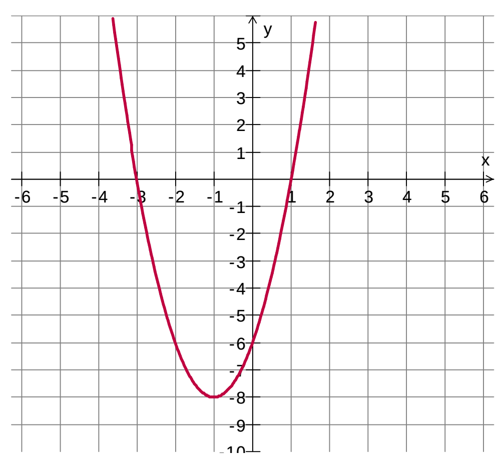
$0 = x^2 + 7x + 10$

$0 = x^2 + 2x - 15$

$0 = x^2 + 13x - 48$

$0 = x^2 - 11x + 24$

Write the equation for this parabola in vertex form, factored form, and general form.



FACTORED FORM

What are the x-intercepts?

What two factors must be used in the factored form?

Graph y=(x+3)(x-1) on your calculator.
How are the graphs similar and how are they different?

What is the vertex of your equation?