Building a Conceptual Understanding of Algebra with Algebra Tiles

Neptune High School

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Operations with Integers - Addition

Collecting Data

DIRECTIONS: Complete Experiments 1—5 using the small yellow and red squares in the algebra tile set.

By addition we mean we are start with a certain amount and add additional tiles to the table. Remember that every pair of a red square tile and a yellow square tile the pair it is called a zero pair. You may remove the pair from the table without affecting the value of the table number. For each problem, record both the number and the color of the tile. You may use a Y for yellow and a R for red.

Experiment 1:

How many tiles, and what color will the tiles be altogether, if you combined...

1. 3 red square tiles and 5 red square tiles?_____
2. 9 red square tiles and 2 red square tiles?_____
3. 4 red square tiles and 6 red square tiles?_____
4. 8 red square tiles and 5 red square tiles?_____

Experiment 2

How many tiles, and what color will the tiles be altogether, if you combined...

5. 5 yellow square tiles and 5 yellow square tiles? _____
6. 6 yellow square tiles and 3 yellow square tiles? _____
7. 9 yellow square tiles and 6 yellow square tiles? _____
8. 8 yellow square tiles and 4 yellow square tiles? _____

Experiment 3

How many tiles, and what color will the tiles be altogether, if you combined...

9. 7 yellow square tiles and 3 red square tiles? _____
10. 7 red square tiles and 9 yellow square tiles? _____
11. 6 red square tiles and 10 yellow square tiles?_____
12. 4 yellow square tiles and 1 red square tile?_____

Experiment 4

How many tiles, and what color will the tiles be altogether, if you combined...

13. 5 yellow square tiles and 6 red square tiles?_____
14. 7 red square tiles and 2 yellow square tiles? _____
15. 9 yellow square tiles and 6 red square tiles?_____
16. 8 red square tiles and 5 yellow square tiles? _____

Experiment 5

How many tiles, and what color will the tiles be altogether, if you combined...

17. 5 yellow square tiles and 5 red square tiles? _____
18. 6 red square tiles and 6 yellow square tiles? _____
19. 9 yellow square tiles and 9 red square tiles? _____
20. 8 red square tiles and 8 yellow square tiles? _____

Analyzing the Data

DIRECTIONS: Answer each question your journal.

1. What do you notice about the colors used in the problems in Experiment 1? What do you notice about the colors found in the answers?
2. What do you notice about the colors used in the problems in Experiment 2? What do you notice about the colors found in the answers?
3. What do you notice about the colors used in the problems in Experiment 3? What do you notice about the colors found in the answers?
4. What do you notice about the colors used in the problems in Experiment 4? What do you notice about the colors found in the answers?
5. What do you notice about the colors used in the problems in Experiment 5? What do you notice about the colors found in the answers?
6. How do the problems in Experiments 1 and 2 differ from those in Experiments 3, 4, and 5?
7. Describe a pattern that exists between the problems and the answers in Experiments 1 and 2.
8. Describe a pattern that exists between the problems and the answers in Experiments 3 and 4.
9. Why do you think there are no tiles in any of the answers for the problems in Experiment 5?
10. Create a set of rules that would help someone find the total number of tiles for each problem in Experiments 1—5.

Using Symbols to Replace the Tiles

DIRECTIONS: Because writing the words "yellow" and "red" is time consuming, symbols for yellow and red can be used. So that all students in your class will use the same symbols, a yellow square tile will be represented by placing a positive (+) sign in front of a number or no sign at all. The symbol for red square tiles will be a negative (-) sign.

Example 1: Three red square tiles will be recorded as (-3).
Example 2: Four yellow square tiles will be recorded as (+4) or (4).

To show that we are beginning with a certain number of tiles and then placing additional tiles on the table. we will use the plus (+) sign between the two different, sets of tiles. Use an equals (=) sign to separate a problem from its answer.
In the space to the right of each problem in Experiments 1—5, use symbols (+, —, =) to represent each problem and its answer.

Example 3: Nine red square tiles and two red square tiles would be recorded as (-9) + (-2) = -11.

Applying What You Know

DIRECTIONS: Think about the tiles to find answers for each of the following.

1. (-6) + (2) = _____
2. (-3) + (-2) = _____
3. (-12) + (8) = _____
4. 4 + (7) =_____

5. Based on your knowledge of red and yellow square tiles, create a set of rules that might help you to find sums of integers that would be too large to complete easily with actual tiles. Write the rules you create in your journal.
6. Using only what you know about collecting tiles, determine ONLY THE SIGN of the answer for each of the following. Be able to connect the concept of the tiles to how you determined the sign of the answer.

   a. 4234 + 987=_____
   b. -981 + (599) =_____
   c. (-1562) + (- 222) =_____
   d. 96 + (-873) = _____

7. Based on your knowledge of red and yellow square tiles, determine ONLY THE SIGN for each of the following. Then use a fraction capable calculator to compute each of the problems below. Verify the sign you predicted for the answer.

Problem	Sign You Predict	Sign Shown on Calculator
a. (-0.4) + (-0.05)		
b. 3.032 + (-8.123)		
c. -12.030 + 16.003		
d. 0.121 + 2.1021		
e. $\left(\frac{1}{4}\right)+\left(\frac{5}{6}\right)$		
f. $\left(-\frac{3}{4}\right)+\left(\frac{1}{8}\right)$		
g. $\left(-1\frac{3}{5}\right)+\left(-\frac{3}{4}\right)$		
h. $\left(-\frac{5}{8}\right)+\left(\frac{13}{16}\right)$		

8. Based on your knowledge of red and yellow square tiles, predict ONLY THE SIGN of the answers for each of the problems below.

Problem	Sign You Predict
a. -9 + 17 + (-18)	
b. -10 + (-11) + (-3)	
c. 14 + 18 + 25	
d. -10 + 15 + (10) + (-17)	
e. 12 + (-16) + (-32) + 4	
f. 12 + (-4) + 18 + (-8)	

Operations with Integers - Subtraction

Collecting Data

DIRECTIONS: In this activity, you will be using manipulatives to discover rules for subtracting signed numbers. Often, in order to subtract, you must use an equivalent representation of the number. For example, the number six can be represented as...

6 6 + 3 + (-3) 7 + (-1) 6 + (-4) + 4 -4+10

Each of these expressions represent 6, but some are in simpler forms than others. Which expressions in this example incorporate the use of the zero rule of addition? Circle them.
In these exercises we will be subtracting. You will notice that the questions ask you to remove certain tiles from the table. This is subtraction.

For each problem in Experiments 1-6, record the results of the problem in the space provided. Your answer should include the number of tiles remaining after the operation is performed, along with the color of the tiles. You may use a Y for yellow and R for red.

Experiment 1

1. Remove 2 yellow square tiles from 5 yellow square tiles._____
2. Remove 1 yellow square tile from 2 yellow square tiles._____
3. Remove 4 yellow square tiles from 6 yellow square tiles._____
4. Remove 10 yellow square tiles from 15 yellow square tiles._____

Experiment 2

5. Remove 5 red square tiles from 10 red square tiles._____
6. Remove 3 red square tiles from 9 red square tiles._____
7. Remove 9 red square tiles from 16 red square tiles._____
8. Remove 7 red square tiles from 11 red square tiles._____

Experiment 3

9. Remove 5 yellow square tiles from 2 yellow square tiles._____
10. Remove 6 yellow square tiles from 2 yellow square tiles._____
11. Remove 4 yellow square tiles from 1 yellow tile._____
12. Remove 10 yellow square tiles from 9 yellow square tiles._____

Experiment 4

13. Remove 10 red square tiles from 5 red square tiles._____
14. Remove 9 red square tiles from 7 red square tiles._____
15. Remove 6 red square tiles from 3 red square tiles._____
16. Remove 7 red square tiles from 1 red tile._____

Experiment 5

17. Remove 7 red square tiles from 3 yellow square tiles._____
18. Remove 5 red square tiles from 9 yellow square tiles._____
19. Remove 6 red square tiles from 10 yellow square tiles._____
20. Remove 4 red square tiles from 1 yellow tile._____

Experiment 6

21. Remove 6 yellow square tiles from 5 red square tiles._____
22. Remove 7 yellow square tiles from 8 red square tiles._____
23. Remove 10 yellow square tiles from 5 red square tiles._____
24. Remove 8 yellow square tiles from 4 red square tiles._____

Analyzing Data

DIRECTIONS: Compare your results with those of the other members of your group. Discuss any problems for which your answers differ. Make sure all the members of your group agree on the answer to each problem and on the process of finding the answer. Then respond to the following questions in your journal.

1. Describe general strategies that were employed to solve the problems in Experiments 3—6.
2. How did the solutions in Experiments 3-6 differ from those in Experiments 1 and 2?
3. How are the problems in Experiments 1—6 similar to the addition problems you solved in Operations with Integers - Addition?
4. What rule could you create that would help you subtract signed numbers easily?

Using Symbols to Replace the Tiles

DIRECTIONS: Because writing the words "yellow" and "red" is time consuming, symbols for the colors can be used. So that all students in your class will use the same symbols, a red tile will be represented by placing a negative (—) sign in front of a number. The symbol for yellow square tiles will be a positive (+) sign or no sign at all.

Example 1: Three red square tiles will be recorded as (-3).
Example 2: Four yellow square tiles will be recorded as (+4) or (4).

To show different sets of tiles being subtracted, a minus (—) sign is placed between the two numbers representing the tiles. Use an equal (=) sign to separate a problem from its answer.

In the space to the right of each problem in Experiments 1—6, use symbols (+, —, =) to represent each problem and its answer.

Example 3: Remove 2 red square tiles from 5 red square tiles. (-5) — (-2) = -3

Applying What You Know

DIRECTIONS: Use the rule you created for subtraction in Part 2 to complete the problems below.

1. (7) - (+3)= _____
2. (+3) - (-7) =_____
3. (+4) - (+5) = _____
4. (+5) -(+4)=_____
5. (-9) -(-3) =_____
6. (10) - (11) =_____
7. (+4) - (-2) =_____
8. (-3) -(+5) =_____
9. (13) - (+10) =_____
10. (+9) - (-6) =_____

11. Use a calculator to check your answers to problems 1—10. Discuss errors with other members of your group to discover strategies that will yield correct answers. Record your answers to the following questions in your journal: If you made any errors, what kind did you make? What strategies can you use to avoid making the same kind of mistake in the future?

Operation with Signed Numbers - Multiplication and Division

Collecting Data

DIRECTIONS: Multiplication is often thought of as a shortcut for addition, or as thinking of groups of things. In this activity, you will use manipulatives to discover rules for multiplying and dividing positive and negative integers. Start each problem with a zero board.

For each problem in Experiments 1-4, record the results in the space provided. Your answer should include the number of tiles in the result, along with the color of the tiles. You may use a Y for yellow and a R for red.

Experiment 1

1. Show 3 groups of 2 red square tiles. _____
2. Show 2 groups of 4 red square tiles._____
3. Show 4 groups of 3 red square tiles._____
4. Show 2 groups of 5 red square tiles._____

Experiment 2

5. Show 2 groups of 3 yellow square tiles._____
6. Show 4 groups of 2 yellow square tiles._____
7. Show 3 groups of 5 yellow square tiles._____
8. Show 3 groups of 4 yellow square tiles._____

Experiment 3

9. Remove 3 groups of 4 red square tiles._____
10. Remove 2 groups of 3 red square tiles. ____
11. Remove 4 groups of 3 red square tiles. ____
12. Remove 2 groups of 5 red square tiles._____

Experiment 4

13. Remove 2 groups of 3 yellow square tiles.____
14. Remove 4 groups of 2 yellow square tiles.____
15. Remove 5 groups of 3 yellow square tiles.____
16. Remove 3 groups of 4 yellow square tiles.____

Analyzing Data

DIRECTIONS: Compare your results with those of the other members of your group. Discuss any differing answers. For each problem, make sure all the members of your group agree on one answer and on the method for finding the answer. Then answer the following questions in your journal.

1. Study the problems and the answers in Experiments 1 and 4.

    a. What color tiles appear in every answer?
    b. What do you notice about each of the problems in Experiment 1?
    c. What do you notice about each of the problems in Experiment 4?

2. Study the problems and answers in Experiments 2 and 3.

    a. What color tiles appear in every answer?
    b. What do you notice about each of the problems in Experiment 2?
    c. What do you notice about each of the problems in Experiment 3?

3. Based on your observations, what rule could you create to help determine the sign of the product of TWO factors?

Using Symbols to Replace the Tiles

DIRECTIONS: Because writing the words "yellow" and "red" is time consuming, symbols for the colors can be used. So that all students in your class will use the same symbols, a red square tile will be represented by placing a negative (—) sign in front of a number. The symbol for yellow square tiles will be a positive (+) sign or no sign at all.

Example 1: Three red square tiles will be recorded as (-3).
Example 2: Four yellow square tiles will be recorded as (+4) or (4).

When using symbols to indicate multiplication, place the number of groups to be displayed first, then the number of tiles that are to be in each group second. In these multiplication problems it is customary to place each of the numbers in parentheses or separate them by a "•". Use a positive (+) sign to indicate that the groups are to be added and a minus sign (-) to indicate that groups of numbers are to be removed.

In the space to the right of each problem in Experiments 1—5, use symbols (+,-,•,=) to represent the problem and its answer.

Example 3: Show 3 groups of 2 red square tiles. (3)(-2) = -6 Example 4: Remove 4 groups of 3 red square tiles. (-4)(-3) = 12

Applying What You Know

1. Based on the rule you developed in Part 2, predict ONLY THE SIGN of the answer for each of the following problems. Use your calculator to verify the results.

a. $\left(-\frac{1}{4}\right)\left(\frac{3}{4}\right)$ _____

b. (-0.O1)(-0.2) =_____

c. (3.14)(2.02)= _____

d. $\left(5\frac{1}{2}\right)\left(-\frac{3}{16}\right)$ = _____

e. (—3)2=_____

f. (5)2=_____

2. What will be sign of the product when three positive factors are multiplied together? Why?
3. What will be sign of the product when three negative factors are multiplied together? Why?
4. What will be sign of the product when two positive and one negative factor are multiplied together? Why?
5. What will be sign of the product when two negative and one positive factor are multiplied together? Why?
6. Based on the conclusions you reached in answering questions 2—5, predict ONLY THE SIGN of the answer to each of the following problems. Use your calculator to test your predictions.

a. (-3)(-2)(-1) =_____

b. (-2)(3)(4) =_____

c. (5)(-2)(-5) =_____

d. (4)(3)(5) =_____

e. (-2)3 =_____

f. (4)3 =_____

7. Since $(-3)\bullet(+2) = -6$ we can write $\frac{-6}{-3} = +2$ and $\frac{-6}{+2} = -3$. Return to the problems at the beginning of this activity. Write one division problem for each multiplication problem.

8. Study the division problems you have written. What can you write about the sign of the quotient of...

a. a positive divisor and a positive dividend?

b. a negative divisor and a negative dividend?

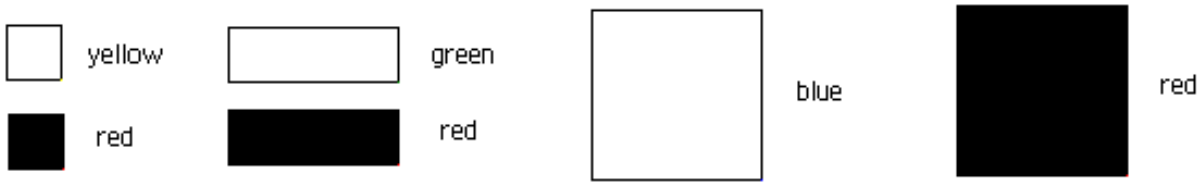
c. a positive divisor and a negative dividend?

d. a negative divisor and a positive dividend?

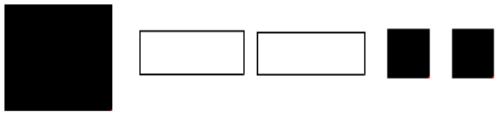
9. Write a rule that will help you determine the sign when two integers are divided.

Using Symbols to Represent Algebra Tiles

The yellow square and small red squares represent +1 and -1 respectively. The green rectangle and the red rectangle represent +x and -x respectively, and the blue square and large red square represent $+x^2$ and $-x^2$ respectively.



Write the symbolic representation for questions 1-5 and sketch the concrete representation for Problems 6 – 10.

	Symbolic Representation	Concrete Representation
1		
2		
3		
4		
5		
6	$3x - 2$	
7	$x^2 - x$	
8	$2x^2 + x + 1$	
9	$3 - x^2$	
10	$x^2 + 2x - 2$	

Simplifying Expressions

Use Algebra Tiles as needed to complete the following problems.

Simplify:

1. $2x + 3 + 5x - 4 =$ _____

2. $2x^2 + 3x - 5 + 4x^2 + x =$ _____

3. $3x^2 + 2x - 4x^2 + 2 + 5x + 1 =$_____

4. $x^2 + 2x + x^2 + 3x^2 - 4x - x^2 =$ _____

5. $2x^2 + 3 - 4x - 4x^2 =$_____

6. $2x^2 + 3x^2 + 5x - 2x =$ _____

7. $2(x^2 + 3) =$ _____

8. $3(x - 2) =$ _____

9. $4(x^2 + 3x - 2) =$ _____

10. $3(x^2 - 5) =$ _____

11. $2(3x^2 + 4) - 2x^2 =$ _____

12. $2(x - 1) + 4x + 3 =$ _____

Algebra Table