Achieving Success in Meeting the Common Core State Standards in Algebra Part I

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New Jersey algebra teachers are challenged with meeting standards described in the Common Core State Standards and New Jersey Algebra I Core Content Standards. This workshop will engage the participants in numerous activities that will help their students develop the concept of function and its connection to expressions, equations, modeling, and coordinates. The activities will explore both linear, quadratic and exponential models. The graphing calculators will be integrated into many of the activities during the workshop.

The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

The high school standards specify the mathematics that all students should study in order to be college and career ready. All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. REASON ABSTRACTLY AND QUANTITATIVELY

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **MODEL WITH MATHEMATICS**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **USE APPROPRIATE TOOLS STRATEGICALLY**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **ATTEND TO PRECISION**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **LOOK FOR AND MAKE USE OF STRUCTURE**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Interpreting Functions

• Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$

• Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

• Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

• Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and

relate these functions to the model.

c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and (t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

- Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 or f(x) = (x+1)/(x-1) for $x \neq 1$.
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

• Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions

• Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for *x*, π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

• Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

• Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(Θ) + cos2(Θ) = 1 and use it to find sin(Θ), cos(Θ), or tan(Θ) given sin(Θ), cos(Θ), or tan(Θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

ALPHABET CODING CHART

Z																										
Y																										
X																										
W																										
V																										
U																										
T																										
S																										
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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	V	X	Y	Z

INPUT													
OUTPUT													

INPUT													
OUTPUT													

Developing an Understanding for the Absolute Value Function

In this investigation you will learn how the absolute-value function tells how much an item of data or a whole set of data deviates from the mean.

Step 1: The data in the chart represents the pulse rate for 10 students.

Student	A	B	C	D	E	F	G	H	I	J
Pulse Rate	27	33	42	22	37	20	35	33	31	18
Difference from Mean										
Distance from the Mean										

Enter this data in List 1 in your graphing calculator.

Step 2: Find the difference between each data point and the mean of the data in list L1.

Record these numbers in the third row of the table and enter them into list L2.What do these numbers represent?

Step 3: Make a dot plot of the list L1 data and note the distance from each data point to the mean. Record your results in a fourth row and enter them into list L3.

How are these entries different from those in list L2?

How are they alike?

Step 4: Next, plot points in the form (L2, L3). What numbers are in the domain and range of the graph?

Step 5: Use the trace function on your calculator and use the arrow keys to step through the data points. Which input numbers are unchanged as output numbers?

Step 6: Which input numbers are changed, and how?

Step 7: Does it make sense to connect these points with a continuous graph? Why or why not?

Step 8: How does this graph compare to the graph of Y1 = abs(x) on your calculator?

Step 9: Find the mean of the deviations stored in list L2. Compare it to the mean of the distances stored in list L3.Which do you think is a better measure of the spread of the data?

Step 10: In your own words, write the rule for the function you graphed in Step 8.What number is output as y when the input, x, is positive or equal to zero? What number is output when x is negative? How can you use operations to change these numbers?

Recursive Toothpick Patterns			
	Number of Toothpicks	**Perimeter**	**Area**
Figure 1			
Figure 2			
Figure 3			
Figure 4			
Figure 5			

James' Workout	
Pedaling time (min) x	Total Calories burned y

Understanding Linear Equations and Intercept Form

Maria starts her exercise routine by jogging to the gym. Her trainer says this activity burns 215 calories. Her workout at the gym is to pedal a stationary bike. This activity burns 3.8 calories per minute.

Pedaling time (min) X	Total calories burned y
0	215
1	
2	
20	
30	
45	
60	

Step 1: Use calculator lists to write a recursive routine to find the total number of calories Maria has burned after each minute she pedals the bike. Include the 215 calories she burned on her jog to the gym.

Step 2: Copy and complete the table using your recursive routine.

Step 3: After 20 minutes of pedaling, how many calories has Maria burned? How long did it take her to burn 443 total calories?

Step 4: Write an expression to find the total calories Maria has burned after 20 minutes of pedaling. Check that your expression equals the value in the table.

Step 5:Write and evaluate an expression to find the total calories Maria has burned after pedaling 38 minutes. What are the advantages of this expression over a recursive routine?

Step 6: Let x represent the pedaling time in minutes, and let y represent the total number of calories Maria burns. Write an equation relating time to total calories burned.
Step 7: Check that your equation produces the corresponding values in the table.

Step 8: Plot the points from your table on your calculator. Then enter your equation into the Y menu. Graph your equation to check that it passes through the points. Give two reasons why drawing a line through the points realistically models this situation.

Step 9: Substitute 538 for y in your equation to find the elapsed time required for Maria to burn a total of 538 calories. Explain your solution process. Check your result.

Step 10: How do the starting value and the rule of your recursive routine show up in your equation? How do the starting value and the rule of your recursive routine show up in your graph? When is the starting value of the recursive routine also the value where the graph crosses the y-axis?

Making the Most of It

Find the dimensions of at least eight different rectangular regions, each with perimeter 24 meters. You must use all of the fencing material for each garden.

Find the area of each garden. Make a table to record the width, length, and area of the possible gardens. It's okay to have widths that are greater than their corresponding lengths.

Width (m)	Length (m)	Area (m^2)

Enter the data for the possible widths into list L1. Enter the area measures into list L2. Which garden width values would give no area? Add these points to your lists.

Label a set of axes and plot points in the form (x, y), with x representing width in meters and y representing area in square meters. Describe as completely as possible what the graph looks like. Does it make sense to connect the points with a smooth curve?

Where does your graph reach its highest point? Which rectangular garden has the largest area? What are its dimensions?

Create a graph of (width, length) data. What is the length of the garden that has a width of 2 meters? Width 4.3 meters? Write an expression for length in terms of width x.

Using your expression for the length from the previous step, write an equation for the area of the garden. Enter this equation into Y1 and graph it. Does the graph confirm your answer for the size of the rectangle with the largest area?

Locate the points where the graph crosses the x-axis. What is the real-world meaning for these points?

Do you think the general shape of a garden with maximum area would change for different perimeters? Explain your answer.

The two points on the x-axis are the x-intercepts. The x-values of those points are the solutions to the equation $y=f(x)$ when the function value is equal to zero. These solutions are the roots of the equation $f(x)=0$.

Making the Most of It

Width (m)	Length (m)	Area (m^2)

Bugs, Bugs, Everywhere Bugs

Imagine that a bug population has invaded your classroom.
One day you noticed 16 bugs. Every day new bugs hatch, increasing the population by 50% each week.

In the first week the population increases by 8 bugs.

In a table, record the total number of bugs at the end of each week for 4 weeks.

Bugs, Bugs, Everywhere Bugs			
WEEKS ELAPSED	TOTAL NUMBER OF BUGS	INCREASE IN THE NUMBER OF BUGS FROM PREVIOUS WEEK	RATIO OF THIS WEEK'S TOTAL TO LAST WEEK'S TOTAL
START (0)			
1			
2			
3			
4			

The increase in the number of bugs each week is the population's rate of change per week. Calculate each rate of change. What are the units?

Does the rate of increase show a linear pattern? Why or why not?

Let x represent the number of weeks elapsed and let y represent the total number of bugs. Graph the data using (0,16) for the first point.

Connect the points with line segments. Describe how the slope changes from point to point.

Calculate the ratio of the number of bugs each week to the number of bugs the previous week. Record it in the table.

How do the ratios compare? Explain what the ratios tell you about the bug population growth.
What is the constant multiplier for the bug population?

How can you use this number to calculate the population when 5 weeks have elapsed?

Write a recursive routine that models the populations growth for the growing number of bugs.

Describe what each part of this calculator command does.

By pressing ENTER a few times, check that your recursive routine gives the sequence of values in your table.

Use the routine to find the bug population at the end of weeks 5 to 8.

What is the population after 20 weeks? After 30 weeks? What happens in the long run?

Bugs, Bugs, Everywhere Bugs			
Weeks Elapsed	Total Number of Bugs	Increase in the number of bugs from previous week (rate of change per week)	Ratio of this week's total to last week's total
Start (0)			
1			
2			
3			
4			

Week 1 = ___ x Week 0

Week 2 = ___ x Week 1 = ___ x ___ x Week 0

Week 3 = ___ x Week 2 = ____ x ___ x Week 1 = ___ x ___ x ___ x Week 0

Week 4 = ___ x Week 3 = ____ x Week 0

Week 5 = ___ x Week 4 = ____ x Week 0

Week 10 = ___ x Week ___ = ___ x Week 0

Week 20 = ___ x Week ___

Week 30 = ___ x Week ___

Week n = ___ x Week ___