Achieving Success in Meeting the Common Core State Standards in Algebra Part I

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Schedule

- 8:30 – 8:45 Introduction to Common Core State Standards Initiative
- 8:45–10:15 Standards 1–4 for Mathematical Practices and Representative Lessons
- 10:15–10:30 Break
- 10:30–11:45 Standards 5–8 for Mathematical Practices and Representative Lessons
- 11:45 – 12:45 Lunch
- 12:45– 1:45 Building Understanding for Working with Equations
- 1:45 –2:45 Building Understanding for Working with Polynomials
- 2:45 – 3:00 Closure

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

- The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years.
- Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

- The Standards for Mathematical Content are a balanced combination of procedure and understanding.
- Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content.
  - Students who lack understanding of a topic may rely on procedures too heavily.
  - A lack of understanding effectively prevents a student from engaging in the mathematical practices.

- The high school standards specify the mathematics that all students should study in order to be college and career ready.
- The high school standards are listed in conceptual categories:
  - • Number and Quantity
  - • Algebra
  - • Functions
  - • Modeling
  - • Geometry
  - • Statistics and Probability
- Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

- The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Based upon:
  - The NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections.
  - The strands of mathematical proficiency specified in the National Research Council's report Adding It Up.

1. MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM

- Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- They analyze givens, constraints, relationships, and goals.
- They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.
- They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

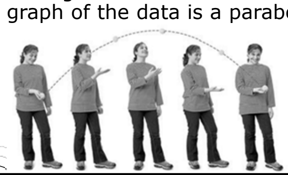
- They monitor and evaluate their progress and change course if necessary.
- Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.
- Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

- Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.
- Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”
- They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Representative Lesson

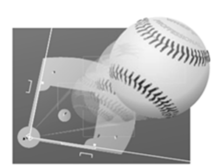
Introducing Quadratic Equations

- When you throw a ball straight up into the air, its height depends on three major factors
  - its starting position
  - the velocity at which it leaves your hand
  - The force of gravity.
- Earth’s gravity causes objects to accelerate downward, gathering speed every second.
- This acceleration due to gravity, called g, is 32 ft/s^2. It means that the object’s downward speed increases 32 ft/s for each second in flight.
- If you plot the height of the ball at each instant of time, the graph of the data is a parabola.

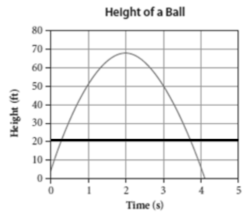


Sketch a graph

- A baseball batter pops a ball straight up. The ball reaches a height of 68 ft before falling back down. Roughly 4 s after it is hit, the ball bounces off home plate.
- Sketch a graph that models the ball’s height in feet during its flight time in seconds.
- When is the ball 68 ft high?
- How many times will it be 20 ft high?

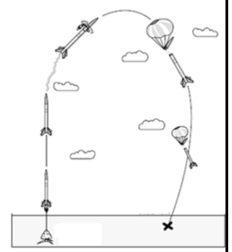


- This is a sketch of the ball's height from the time it is hit to when it lands on the ground.
- When the bat hits the ball, it is a few feet above the ground. So the *y-intercept is just above the* origin.
- The ball's height is 0 when it hits the ground just over 4 s later.
- So the parabola crosses the *x-axis near the coordinates* (4, 0).
- The ball is at its maximum height of 68 ft after about 2 s, or halfway through its flight time.
- So the vertex of the parabola is near (2, 68).
- The ball reaches a height of 20 ft twice—once on its way up and again on its way down.



The parabola above is a transformation of the equation *$y=x^2$. The function $f(x)=x^2$ and transformations of it are called* ***quadratic functions,*** *because the* highest power of *x is x-squared.*

Model Rocket Science



- A model rocket blasts off and its engine shuts down when it is **30 m** above the ground. Its velocity at that time is **40 m/s**. Assume that it travels straight up and that the only force acting on it is the downward pull of gravity.
- In the metric system, the acceleration due to gravity is 9.8 m/s^2.

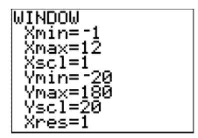
The quadratic function $h(t)=(1/2)(-9.8)t^2+\mathbf{40}t+\mathbf{30}$ *describes the* rocket's **projectile motion.**

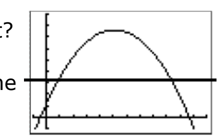
$h(t)=(1/2)(-9.8)t^2+\mathbf{40}t+\mathbf{30}$

- Define the function variables and their units of measure for this situation.
- What is the real-world meaning of h(0)=25?
- How is the acceleration due to gravity, or g, represented in the equation?
- How does the equation show that this force is downward?

$$h(t)=\frac{1}{2}(-9.8)t^2+50t+25$$

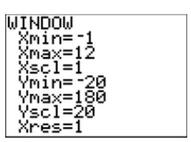
- Graph the function h(t). What viewing window shows all the important parts of the parabola?
- How high does the rocket fly before falling back to Earth? When does it reach this point?
- How much time passes while the rocket is in flight, after the engine shuts down?
- What domain and range values make sense in this situation?

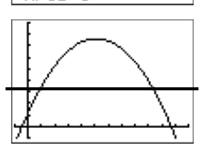




$$h(t)=\frac{1}{2}(-9.8)t^2+50t+25$$

- Write the equation you must solve to find when h(t)=60.
- When is the rocket 60 m above the ground? Use a calculator table to approximate your answers to the nearest tenth of a second.
- Describe how you determine when the rocket is at a height of 60 feet graphically.





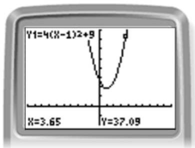
Example A

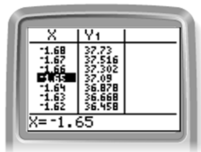
- Solve $4(x-1)^2+9=37$ symbolically. Check your answers with a graph and a table.

$$4(x-1)^2+9=37$$
$$4(x-1)^2=28$$
$$(x-1)^2=7$$
$$\sqrt{(x-1)^2}=\sqrt{7}$$
$$|x-1|=\sqrt{7}$$
$$x-1=\pm\sqrt{7}$$
$$x=+1\pm\sqrt{7}$$
$$x=1+\sqrt{7}\ or\ 1-\sqrt{7}$$
$$x\approx 3.65\ or\ -1.65$$

$$4(x-1)^2 + 9 = 37$$

The calculator screens of the graph and the table support each solution.





[-10,10,1, -10,40,5]

2. Reason abstractly and quantitatively.

- Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships:
  - the ability to *decontextualize*—
    - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—
  - and the ability to *contextualize*,
    - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.
- .

- Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Representative Lesson

All Tied Up

Collecting Data and Writing Equations

- Step 1 Measure the length of the thinner rope without any knots.
  - Then tie a knot and measure the length of the rope again. Continue tying knots until no more can be tied. Knots should be of the same kind, size, and tightness. Record the data for number of knots and length of rope in a table.

All Tied Up

Number of Knots	Length of Thinner Rope	Length of Thicker Rope

- Step 2 Define variables and write an equation in intercept form to model the data you collected in Step 1.
  - What are the slope and *y*–intercept, and how do they relate to the rope?
- Step 3 Repeat Steps 1 and 2 for the thicker rope.

- Step 4 Suppose you have a 9-meter-long thin rope and a 10-meter-long thick rope.
  - Write a system of equations that gives the length of each rope depending on the number of knots tied.

- Step 5 Solve this system of equations using the substitution method.
- Step 6 Select an appropriate window setting and graph this system of equations. Estimate coordinates for the point of intersection to check your solution. Compare this solution with the one from Step 5.
- Step 7 Explain the real-world meaning of the solution to the system of equations.
- Step 8 What happens to the graph of the system if the two ropes have the same thickness? The same length?

Sample Data I

Number of Knots	Length (cm)
0	100
1	89.7
2	78.7
3	68.6
4	57.4
5	47.8
6	38.1

Number of Knots	Length (cm)
0	90
1	83.1
2	76
3	68.8
4	61.9
5	75
6	67.8

Sample Data II

Number of Knots	Length (cm)
0	100
1	94
2	88
3	81.3
4	75.7
5	69.9
6	63.5

Number of Knots	Length (cm)
0	90
1	86
2	81.9
3	77.3
4	73
5	68.9
6	64.8

- J.P. is thinking of two numbers, but he won't say what they are.
- He tells you that the sum of the two numbers is 163 and that their difference is 33.
- Find the two numbers.
- Write a system of equations for the sum and difference of these numbers.

If f is the first number and s is the second number then

$$f + s = 163$$
$$f - s = 33$$

- Use the elimination method to solve for the two numbers.

$$\begin{array}{l} f + s = 163 \\ \underline{f - s = 33} \\ 2f \quad = 196 \\ f = 98 \end{array}$$

This means that the first number J.P. is thinking about is 98.

Can you tell me anything else about the first number?

$$f + s = 163$$
$$f - s = 33$$

So if we know the first number (f) is 98 can you use f=98 in either equation?

$$f + s = 163 \quad f - s = 33$$
$$98 + s = 163 \quad 98 - s = 33$$
$$s = 65 \qquad s = 65$$

3. Construct viable arguments and critique the reasoning of others.

- Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- They make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples.

- They justify their conclusions, communicate them to others, and respond to the arguments of others.
- They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

- Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument— explain what it is.

- Elementary students can construct arguments using concrete referents such as objects,a drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.
- Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Representative Lesson

Exploring a Conjecture

A conjecture

- A **conjecture** is a statement that might be true but has not been proven. Your group's goal is to come up with a conjecture relating two things and to collect and analyze the numeric evidence to support your conjecture or cast doubt on it.
- In this activity you'll review the measures and graphs you have learned. Along the way, you will be faced with questions that statisticians face every day.

Step 1

- Your group should select two books on different subjects or with different reading levels. Flip through the books, but do not examine them in depth. State a conjecture comparing these two books. Your conjecture should deal with a quantity that you can count or measure—for example, "The history book has more words per sentence than the math book."

Step 2

- Decide how much data you'll need to convince yourself and your group that the conjecture is true or doubtful. Design a way to choose data to count or measure.
- For example, you might use your calculator to randomly select a page or a sentence.

Step 3

- Collect data from both books. Be consistent in your data collection, especially if more than one person is doing the collecting. Assign tasks to each member of your group.

Step 4

- Find the measures of center, range, five-number summary, and IQR for each of the two data sets.

Step 5

- Create a dot plot or stem-and-leaf plot for each set of data.

Step 6

- Make box plots for both data sets above the same horizontal axis.

Step 7

- Make a histogram for each data set.
- Be sure that you have used descriptive units for all of your measures and clearly labeled your axes and plots before going on to the next step.

Step 8

- Choose one or two of the measures and one pair of graphs that you feel give the best evidence for or against your conjecture. Prepare a brief report or a poster.
- Include
  - Your conjecture.
  - Tables showing all the data you collected.
  - The measures and graphs that seem to support or disprove your conjecture.
  - Your conclusion about your conjecture.

Step 9

- In Step 2, you thought about your design for data collection and you might have used random numbers. In Step 3, you practiced consistency in collecting data. In Steps 4, 5, and 6, you were asked to find many measures and graphs, even though you used only a few of these in your final argument. Write a paragraph explaining how a failure at any one of these steps might have changed your conclusion.

4. Model with mathematics.

- Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a
- school event or analyze a problem in the community.

- By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

- They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions.
- They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Representative Lesson

- Writing Point-Slope Equations to Fit Data

- This table shows the relationship between the number of years a person might be expected to live and the year he or she was born.
- Life expectancy is a prediction that is very useful in professions like medicine and insurance.

Birth Year	Female	Male	Combined
1940	65.2	60.8	62.9
1950	71.1	65.6	68.2
1960	73.1	66.6	69.7
1970	74.7	67.1	70.8
1975	76.6	68.8	72.6
1980	77.5	70.0	73.7
1985	78.2	71.2	74.7
1990	78.8	71.8	75.4
1995	78.9	72.5	75.8
2000	79.5	74.1	76.9

- Each group will work with either Female, Male or Combined Data

Step 1: Choose one column of life expectancy data—female, male, or combined. Let *x* represent birth year, and let *y* represent life expectancy in years. Graph the data points.

Step 2: Choose two points on your graph so that a line through them closely reflects the pattern of all the points on the graph. Use the two points to write the equation of this line in point-slope form.

Birth Year	Female	Male	Combined
1940	65.2	60.8	62.9
1950	71.1	65.6	68.2
1960	73.1	66.6	69.7
1970	74.7	67.1	70.8
1975	76.6	68.8	72.6
1980	77.5	70.0	73.7
1985	78.2	71.2	74.7
1990	78.8	71.8	75.4
1995	78.9	72.5	75.8
2000	79.5	74.1	76.9

- Each group will work with either Female, Male or Combined Data

Step 3: Graph the line with your data points. Does it fit the data?

Step 4: Use your equation to predict the life expectancy of a person who will be born in 2022.

Birth Year	Female	Male	Combined
1940	65.2	60.8	62.9
1950	71.1	65.6	68.2
1960	73.1	66.6	69.7
1970	74.7	67.1	70.8
1975	76.6	68.8	72.6
1980	77.5	70.0	73.7
1985	78.2	71.2	74.7
1990	78.8	71.8	75.4
1995	78.9	72.5	75.8
2000	79.5	74.1	76.9

Step 5: Compare your prediction from Step 4 to the prediction that another group made analyzing the same data. Are your predictions the same? Are they close? Explain why it's possible to make different predictions from the same data.

Step 6: Compare the slope of your line of fit to the slopes that other groups found working with different data sets. What does the slope for each data set tell you?

Birth Year	Female	Male	Combined
1940	65.2	60.8	62.9
1950	71.1	65.6	68.2
1960	73.1	66.6	69.7
1970	74.7	67.1	70.8
1975	76.6	68.8	72.6
1980	77.5	70.0	73.7
1985	78.2	71.2	74.7
1990	78.8	71.8	75.4
1995	78.9	72.5	75.8
2000	79.5	74.1	76.9

Step 7: As a class, select one line of fit that you think is the best model for each column of data—female, male, and combined. Graph all three lines on the same set of axes. Is it reasonable for the line representing the combined data to lie between the other two lines? Explain why or why not.

Step 8: How does the point-slope method of finding a line compare to the intercept-form method you learned about in Lesson 4.2? What are the strengths and weaknesses of each method?

Birth Year	Female	Male	Combined
1940	65.2	60.8	62.9
1950	71.1	65.6	68.2
1960	73.1	66.6	69.7
1970	74.7	67.1	70.8
1975	76.6	68.8	72.6
1980	77.5	70.0	73.7
1985	78.2	71.2	74.7
1990	78.8	71.8	75.4
1995	78.9	72.5	75.8
2000	79.5	74.1	76.9

- Summarize how you can fit a point-slope line to linear data.
- How will you make sure that it fits the data you have graphed?

5. Use appropriate tools strategically.

- Mathematically proficient students consider the available tools when solving a mathematical problem.
- These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.

- Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
  - For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge.

- When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Representative Lesson

Wind Chill

- In this investigation you will use the relationship between temperature and wind chill to explore the concept of rate of change and its connection to tables, scatter plots, recursive routines, equations and graphs.
- The data in the table represents the approximate wind chill temperatures in degrees F for a wind speed of 20 mi/h.

Temperature (ºF)	Wind Chill (ºF)
-5	-28.540
0	-21.980
1	-20.558
2	-19.356
5	-15.420
15	-2.300
35	23.940

- Step 1: Define the input and output variables for this relationship.
- Step 2: Plot the points and describe the viewing window you used.
- Step 3: Write a recursive routine that gives the pairs of values listed in the table.

Temperature (ºF)	Wind Chill (ºF)
-5	-28.540
0	-21.980
1	-20.558
2	-19.356
5	-15.420
15	-2.300
35	23.940

- Step 4: Copy the table. Complete the third and fourth columns of the table by recording the changes between consecutive input and output values. Then find the rate of change.

Temperature (ºF)	Wind Chill (ºF)	Change in Input	Change in Output	Rate of Change
-5	-28.540			
0	-21.980			
1	-20.558			
2	-19.356			
5	-15.420			
15	-2.300			
35	23.940			

- Step 5: Use your routine to write a linear equation in intercept form that relates wind chill to temperature. Note that the starting value, 28.540, is not the *y*-intercept. How does the rule of the routine appear in your equation?
- Step 6: Graph the equation on the same set of axes as your scatter plot. Use the calculator table to check that your equation is correct. Does it make sense to draw a line through the points? Where does the *y*-intercept show up in your equation?

Temperature (ºF)	Wind Chill (ºF)
-5	-28.540
0	-21.980
1	-20.558
2	-19.356
5	-15.420
15	-2.300
35	23.940

- Step 7: What do you notice about the values for rate of change listed in your table? How does the rate of change show up in your equation? In your graph?
- Step 8: Explain how to use the rate of change to find the actual temperature if the weather report indicates a wind chill of 9.5° with 20 mi/h winds.

Temperature (ºF)	Wind Chill (ºF)
-5	-28.540
0	-21.980
1	-20.558
2	-19.356
5	-15.420
15	-2.300
35	23.940

6. Attend to precision.

- Mathematically proficient students try to communicate precisely to others.
  - They try to use clear definitions in discussion with others and in their own reasoning.
  - They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
  - They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem.
  - They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- In the elementary grades, students give carefully formulated explanations to each other.
- By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Representative Lesson

- Guesstimating

- In this investigation you will estimate and measure distances around your room.
- As a group, select a starting point for your measurements.
- Choose between five and nine objects in the room that appear to be less than 5 m away.

Guesstimating

- Step 1: List the objects in the description column of a table like this one.
- Step 2: Estimate the distances in meters or parts of a meter from your starting point to each object. If group members disagree, find the mean of your estimates. Record the estimates in your table.
- Step 3: Measure the actual distances to each object and record them in the table.

- Step 4: Draw coordinate axes and label actual distance on the x-axis and estimated distance on the y-axis. Use the same scale on both axes. Carefully plot your nine points.
- Step 5: Describe what this graph would look like if each of your estimates had been exactly the same as the actual measurement. How could you indicate this pattern on your graph?

- Step 6: Make a calculator scatter plot of your data. Use your paper-and-pencil graph as a guide for setting a good graphing window.
- Step 7: On your calculator, graph the line $y = x$. What does this *equation* represent?
- Step 8: What do you notice about the points for distances that were underestimated? What about points for distances that were overestimated?
- Step 9: How would you recognize the point for a distance that was estimated exactly the same as its actual measurement? Explain why this point would fall where it does.

7. Look for and make use of structure.

- Mathematically proficient students look closely to discern a pattern or structure.
- Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.

- Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.
- They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.

- They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Representative Lesson

Moving Ahead

- Suppose the population of a town is 12,800 and the town's population grows at a rate of 2.5% each year.
- An expression for the population 3 years from now is $12{,}800(1 + 0.025)^3$.
- To represent one more year, you can write the expression $12{,}800(1 + 0.025)^4$.
- You can also think about the growth from 3 years to 4 years recursively. Because the rate of growth is constant, multiply the expression for 3 years by one more constant multiplier to get $12{,}800(1 + 0.025)^3 \bullet (1 + 0.025)^1$.
- Confirm this on the graphing calculator.

Moving Ahead

- Step 1
  - Rewrite each product below in expanded form, and then rewrite it in exponential form with a single base. Use your calculator to check your answer.

$$3^4 \cdot 3^2 = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3) = 3^7$$

$$x^3 \cdot x^5 = (x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x \cdot x) = x^8$$

$$(1 + 0.05)^2 \cdot (1 + 0.05)^4 = (1 + 0.05)^6$$

$$10^3 \cdot 10^6 = (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) = 10^9$$

Moving Ahead

- Step 2
  - Compare the exponents in each final expression you got in Step 1 to the exponents in original product. Describe a way to find the exponents in the final expression without using expanded form.

$3^4 \cdot 3^2 = (3\cdot 3\cdot 3\cdot 3)\cdot(3\cdot 3) = 3^7$

$x^3 \cdot x^5 = (x\cdot x\cdot x)\cdot(x\cdot x\cdot x\cdot x\cdot x) = x^8$

$(1+0.05)^2 \cdot (1+0.05)^4 = (1+0.05)^6$

$10^3 \cdot 10^6 = (10\cdot 10\cdot 10)\cdot(10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10) = 10^9$

Moving Ahead

- Step 3
  - Generalize your observation:

$$b^m \cdot b^n = b^{m+n}$$

- Step 4:
  - The number of ants in a colony after 5 weeks is $16(1+0.5)^5$. What does the expression $16(1+0.5)^5 \bullet (1+0.5)^3$ mean in this situation? Rewrite the expression with a single exponent.
  - The depreciation value of a truck after 7 years is $11{,}500(1-0.2)^7$. What does the expression $11{,}500(1-0.2)^7 \bullet (1-0.2)^2$ mean in this situation?
  - The expression $A(1+r)^n$ can model n time periods, what does $A(1+r)^{n+m}$ model?
- Step 5:
  - How does looking ahead in time with an exponential model relate to multiplying expressions with exponents?

For any nonzero value of b and any integer value of m and n,

$$b^m \cdot b^n = b^{m+n}$$

- Lara buys a $500 sofa at a furniture store. She buys the sofa with a new credit card that charges 1.5% interest per month, with an offer for "no payments for a year."
  - What balance will Lara's credit card bill show after 6 months? Write an exponential expression and evaluate it. $500(1 + 0.015)^6$; $546.72
  - How much total interest will be added after 6 months? $46.72
  - What balance will Lara's credit card bill show after 12 months? Write an exponential expression and evaluate it. $500(1 + 0.015)^{12}$; $597.81
  - How much more interest will be added between 6 and 12 months? $51.09
  - Explain why more interest builds up between 6 and 12 months than between 0 and 6 months.

8. Look for and express regularity in repeated reasoning.

- Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.
- Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal.

- By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$.
- Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series.

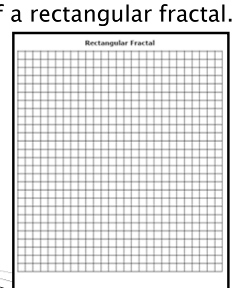
- As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Representative Lesson

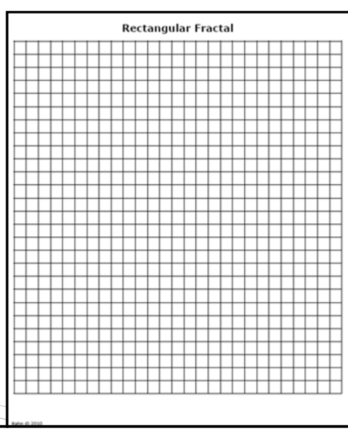
Loosing Area in a Fractal

Loosing Area in a Rectangular Fractal

- In this investigation you will look for patterns in area of a rectangular fractal.

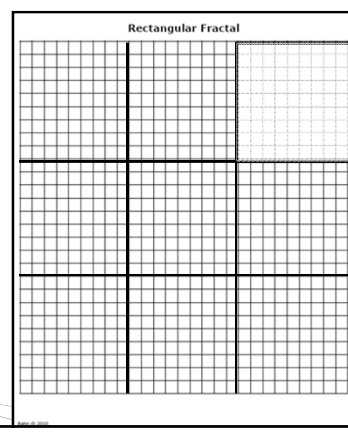


Growth of a Rectangular Fractal

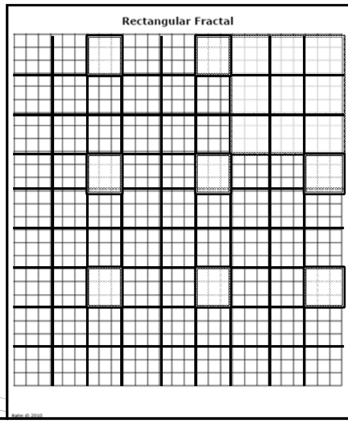


- To create a fractal we will begin with a 27 x 27 rectangle
- This is stage 0.

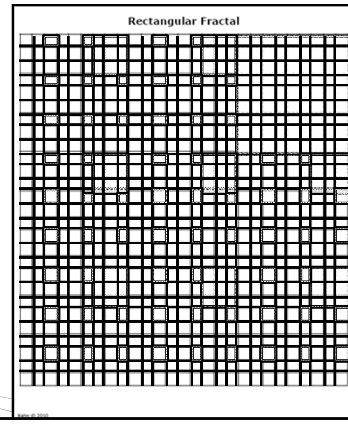
- To create stage 1 draw two vertical lines and two horizontal lines to subdivide the shape into 9 equal parts. Shade any one of the parts.



- To create stage 2 draw two vertical lines and two horizontal lines in each of the remaining rectangles to subdivide the rectangle into 9 equal parts. Shade the same one part of each of these rectangles.



- To create stage 3 draw two vertical lines and two horizontal lines in each of the remaining rectangles to subdivide the rectangle into 9 equal parts. Shade the same one part of each of these rectangles.



Stage Number	Total Unshaded Area	
0	729	
1	648	
2	576	
3	512	

Let's collect some data

Stage Number	Total Unshaded Area	Ratio of this Stage's area to the previous Stage's area
0	729	
1	648	
2	576	
3	512	

Stage Number	Total Unshaded Area	Ratio of this Stage's area to the previous Stage's area
0	729	
1	648	$\frac{648}{729}=\frac{8}{9}$
2	576	$\frac{576}{648}=\frac{8}{9}$
3	512	$\frac{512}{576}=\frac{8}{9}$
4	$512\cdot\frac{8}{9}=455\frac{1}{9}$	$\frac{455\frac{1}{9}}{512}=\frac{8}{9}$

Use the ratio to predict the area of stage 4

Stage Number	Total Unshaded Area	Ratio of this Stage's area to the previous Stage's area
0	729	
1	$729\cdot\frac{8}{9}$	$\frac{648}{729}=\frac{8}{9}$
2	$729\cdot\left(\frac{8}{9}\right)^2$	$\frac{576}{648}=\frac{8}{9}$
3	$729\cdot\left(\frac{8}{9}\right)^3$	$\frac{512}{576}=\frac{8}{9}$
4	$729\cdot\left(\frac{8}{9}\right)^4$	$\frac{455\frac{1}{9}}{512}=\frac{8}{9}$

Rewrite each total unshaded area using the constant multiplier.

Stage Number	Total Unshaded Area
0	729
1	$729 \cdot \frac{8}{9}$
2	$729 \cdot \left(\frac{8}{9}\right)^2$
3	$729 \cdot \left(\frac{8}{9}\right)^3$
4	$729 \cdot \left(\frac{8}{9}\right)^4$

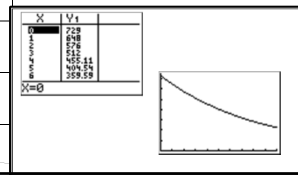
If x is the stage number write an expression for the unshaded area in stage x.

$$y = 729\left(\frac{8}{9}\right)^x$$

Stage Number	Total Unshaded Area
0	729
1	$729 \cdot \frac{8}{9}$
2	$729 \cdot \left(\frac{8}{9}\right)^2$
3	$729 \cdot \left(\frac{8}{9}\right)^3$
4	$729 \cdot \left(\frac{8}{9}\right)^4$

$$y = 729\left(\frac{8}{9}\right)^x$$

Create a graph for this equation.
Check the calculator table to see that it contains the same values as your table.
What does the graph tell you about the area of the rectangular fractal.



Total length

Stage Number

$$y = 27\left(\frac{4}{3}\right)^x$$

Constant multiplier

Starting length

This type of equation is called an exponential equation.

Lunch

Student Center

Algebra Introduction

- Expressions
- Equations and Inequalities
- Connections to Functions and Modeling

Expressions

- An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function.
- Reading an expression with comprehension involves analysis of its underlying structure.
- Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation.
- A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and Inequalities

- An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal.
- The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane.
- An equation can often be solved by successively deducing from it one or more simpler equations.
- The same solution techniques used to solve equations can be used to rearrange formulas.

Connections to Functions and Modeling

- Expressions can define functions, and equivalent expressions define the same function.
- Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation.
- Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Seeing Structure in Expressions

- Interpret the structure of expressions
  - Interpret expressions that represent a quantity in terms of its context.
  - Use the structure of an expression to identify ways to rewrite it.
- Write expressions in equivalent forms to solve problems
  - Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
  - Derive the formula for the sum of a finite geometric series.

Arithmetic with Polynomials and Rational Functions

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations

- Create equations that describe numbers or relationships
  - Create equations and inequalities in one variable and use them to solve problems.
  - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
  - Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning
  - Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.
  - Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
  - Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
  - Solve quadratic equations in one variable.

- Solve systems of equations
  - Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
  - Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
  - Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

- Represent and solve equations and inequalities graphically
  - Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
  - Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately.
  - Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Building Understanding for Order of Operations and Writing Expressions

We are all familiar with the typical Order of Operation Lesson

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

What usually takes place in this lesson?

Here's a little different way to approach this lesson that building conceptual understanding.

What is three plus five times two?

Don't immediately refer to PEMDAS.

Ask for several students to share how they approached the problem.

What is three plus five times two?

Try entering this problem on the home screen of the graphing calculator or any calculator where a string of numbers and operations can be printed on the screen. What is the result?

How many ways can you enter it on the home screen?

Is there an order for the operations when the problem is written horizontally?
3+5*2

Order of Operations

1. Evaluate expressions within parentheses or other grouping symbols.
2. Evaluate all powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

$$3 + 5 \cdot 2 = 3 + 10 = 13$$

How would you have to write
$3 + 5 \cdot 2$
so the answer is 16?

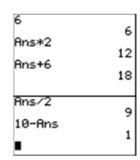
Why introduce the lesson this way?

- How many students have heard the PEMDAS lesson prior to Algebra I?
- How is this approach different from the normal PEMDAS lesson?
- Who is doing most of the thinking as the lesson is introduced this way?
- How does this approach engage the student in thinking about $3 + 5 \cdot 2$ without differently?

Learning to build mathematical expressions

- Let's try performing a string of operations to see what we get.
- On paper:
  - Start with 6.
  - Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.
  - Start with 20.
  - Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.
  - Start with −4
  - Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.

On the graphing calculator
- Start with 6.
  - Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.
- Start with 20.
  - Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.
- Start with −4
  - Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.

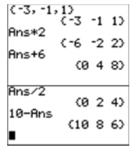


Do you understand what the calculator is doing?

Let's study several numbers.

On the home screen type three numbers in numerical order inside a brace. {-3, -1, 1}

Using the same technique on the graphing calculator Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.

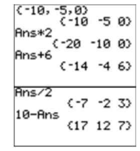


Are you more interested in what is happening?

Try three more numbers in increasing order.

On the home screen type three numbers in numerical order inside a brace. {-10, -5, 0}

Using the same technique on the graphing calculator Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.



Can you explain the change?

These problems appear pretty simple because we are giving all the directions in short steps and you are performing them in the order in which they are described.

Let's see if we can learn to write expressions through a similar activity. Start with a chart and complete each line based on the directions given.

Description	Expression

Description	Expression
	x
	$2x$
	$2x+6$
	$\frac{2x+6}{2}$
	$10-\frac{2x+6}{2}$

After we have written the expression we can test it:

On the graphing calculator homescreen type:

- 6 => x:10-(2x+6)/2

=> means STO

General Format:
Your Number => x:10-(2x+6)/2

Confirm 20, -4, and some of the other numbers we used.

What do you understand about your expression?

Set up the expression for this problem:

- Using the Description/Expression Template
- Pick any number
- Divide the number by 4
- Add 7
- Multiply the result by 2
- Subtract 8
- Compare your starting and ending numbers? Do you see what is happening?

Set up the expression for this problem:

- Using the Description/Expression Template
- Start with either 2, -5, or 8
- Divide the number by 4
- Add 7
- Multiply the result by 2
- Subtract 8
- Compare your starting and ending numbers? Can you figure out what is happening?

Set up the expression for this problem:

- Using the Description/Expression Template
- Start with x
- Divide the number by 4
- Add 7
- Multiply the result by 2
- Subtract 8
- What will you expression equal if x is 2, -5, or 8?

Description	Expression
$\left(\frac{x}{4}+7\right)2-8$	$2\left(\frac{x}{4}+7\right)-8$
$2 \to 7$	
$-5 \to 3.5$	
$8 \to 10$	

Do these number pairs make sense?

Number Tricks

- Each person pick any number from 1 to 25.
- Add 9 to it.
- Multiply the result by 3.
- Subtract 6 from the current answer.
- Divide this answer by 3.
- Now subtract your original number.
- Compare your results.
- Will the answer be the same regardless on the number you begin with?
- Why is this?
- Write out the algebraic expression for this number trick.

Description	Expression
	x
	$x+9$
	$3(x+9)$
	$3(x+9)-6$
	$\frac{3(x+9)-6}{3}$
	$\frac{3(x+9)-6}{3}-x$

This is a pretty complex expression. Can we put these in an equation and solve for x?

Analyzing a Number Trick

$$2\left(\frac{x+3}{2}+5\right)-x+8$$

- Write in words the number trick that is described above.
- Test the number trick to be sure you get the same result no matter what number you choose.
- Can you explain why this number trick work?

$$\frac{4-2(x+4)}{2}+x \qquad \frac{4+-2(x+4)}{2}+x$$

- Given the expression on the left, you might want to think of subtraction as adding the opposite and re-write the expression
- Write, in words, the number trick that is described above.
- Test the number trick to be sure you get the same result no matter what number you choose.
- Which operations that undo previous operations make this number trick work?

You create a trick

- Create your own trick that has at least 5 stages.
- Test it on your calculator with at least four different numbers to make sure all the answers are the same.
- When you think your trick works, test it on your other group members.

- Daxun, Lacy, Claudia, and Al are working on a number trick. Here are the number sequences their number trick generates:

Description	Daxun's sequence	Lacy's sequence	Claudia's sequence	Al's sequence
Pick the starting number.	14	−5	−8.6	x
	19	0		
	76	0		
	64	−12		
	16	−3		
	2	2		

a. Describe the stages of this number trick in the first column.
b. Complete Claudia's sequence.
c. Write a sequence of expressions for Al in the last column.

What does it mean to solve an equation?

Is it any more than just undoing the procedure of building an equation?

- Choose a secret number.
- Now choose four more non-zero numbers and in any random order
  - add one of them,
  - multiply by another,
  - subtract another, and
  - divide by the final number
- Record in words what you did and your final result on the communicator with a blank *Building and Undoing an Expression or Equation* template. (Do not record your secret number.)
- Switch communicators and have another students find your secret number.

Building and Undoing an Expression or Equation

Description/Sequence	Expression	Undo	Result

2,5,4,8

Add 2,
Multiply by 5,
Subtract 4,
And divide by 8

Reveal the results

What was my starting number?

Building and Undoing an Expression or Equation

$\frac{5(x+2)-4}{8} = 13\frac{2}{8}$

Description/Sequence	Expression	Undo	Result
?	x		
Add 2 Ans+2	X+2		
Multiply by 5 Ansx5	$5(x+2)$		
Subtract 4 Ans-4	$5(x+2)-4$		
Divide by 8 Ans ÷ 8	$\frac{5(x+2)-4}{8}$		$13\frac{2}{8}$

To some number, add 3, multiply by 2, add 18, and finally divide by 6.

a. Convert the description into an expression, and write an equation that states that this expression is equal to 15.
b. Find the starting number if the final result is 15.
c. Test your solution to part b using your equation from part a.

Building and Undoing an Expression or Equation

Description/Sequence	Expression	Undo	Result

Solve Equations is Just Undoing Operations

Use the Build and Undo Expression or Equations Chart to complete the following number trick. Complete the first three columns only.

- Pick a number
- Divide the number by 4
- Add 7
- Multiply the result by 2
- Subtract 8

Building and Undoing an Expression or Equation

Description/Sequence	Expression	Undo	Result

Solve Equations is Just Undoing Operations

Description / Sequence	Expression		

- Suppose the answer to this equation is 28. Find the value of x?

Solve Equations is Just Undoing Operations

Description / Sequence	Expression		
Pick a number ?	X		44
Divide by 4 /4	$\frac{x}{4}$		11
Add 7 +7	$\frac{x}{4}+7$		18
Multiply by 2 x2	$2\left(\frac{x}{4}+7\right)$		36
Subtract 8 -8	$2\left(\frac{x}{4}+7\right)-8$		28

Instead of building an equation, let's start with an equation and solve it

Here is an equation. What is it saying? First build the equation, then we'll solve it. $\frac{3+2(x-4)}{5}+6=11$

Building and Undoing an Expression or Equation

$$\frac{3+2(x-4)}{5}+6=11$$

Description/Sequence	Expression	Undo	Result
Pick a number	x		

Try solving these equations

- Place the Building and Undoing an Expression Template in your Communicator®.
- Record an equation in the cell at the top.
- Complete the description column using the order of operations.
- Complete the undo column.
- Finally, work up from the bottom of the table to solve the equation.
- Write a few sentences explaining why this method works to solve an equation

$$\frac{3+2(x-4)}{5}+6=11$$

Simplifying the Technique of Solving an Equation

Undoing an Equation Template

Equation: $7+\frac{(x-3)}{4}=42$

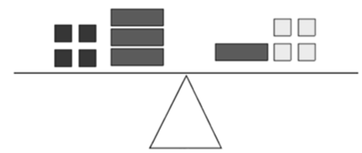
Description	Undo	Result
Pick x	x	143
-3	+3	140
÷4	x4	35
+7	-7	42

- An equation is a statement that says the value of one expression is equal to the value of another expression.
- Solving equations is the process you used to determine the value of the unknown that makes the equation true. This is called the solution.

If you were told the expression on the left describes several operations that were performed to a given number and that the result equals to 7, describe all the operations that were performed on x and what order they were performed to arrive at the answer 7?

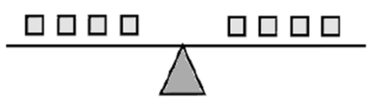
$$\frac{3(x+9)-6}{3} - x = 7$$

Balancing Equations



Actions with Balanced Scales

- Tell what would happen to the balanced scale below if each of the actions listed are taken.
- Remember, the scale is reset after each action.

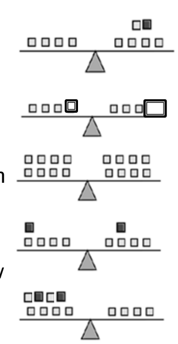


1. Three red squares are added to the right side. **unbalanced**
2. One yellow and one red square are added to the right side. balanced
3. One yellow squares is removed from the left side and one yellow square is removed from the right side. balanced
4. Two red squares are added to the right side of the scale and two yellow squares are added to the left side. **unbalanced**
5. Multiply the number of items on each side by two. balanced
6. Two red squares and two yellow squares are added to the left side of the scale. balanced
7. A red square is added to the left and a yellow square is removed from the right. balanced

8. The number of items on each side is cut in half. balanced
9. Two yellow squares are removed from the left and two yellow squares are added to the right side of the scale. **unbalanced**
10. Two red squares are removed from the left and two red squares are removed from the right side of the scale. balanced
11. One zero pair is added to the left side and one zero pair is added to the right side of the scale. balanced
12. Two yellow squares are added to the right side and two red squares are added to the left side of the scale. **unbalanced**
13. One red square is added to each side of the scale. balanced
14. Double the number of squares on the left and divide the number of squares on the right by two. **unbalanced**

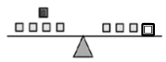
Actions that Balance a Scale

1. One yellow and one red square are added to the right side.
2. One yellow squares is removed from the left side and one yellow square is removed from the right side.
3. Multiply the number of items on each side by two.
4. One red square is added to each side of the scale.
5. Two red squares and two yellow squares are added to the left side of the scale

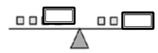


Actions that Balance a Scale

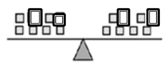
5. A red square is added to the left and a yellow square is removed from the right.



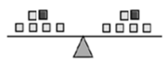
6. The number of items on each side is cut in half.



7. Two red squares are removed from the left and two red squares are removed from the right side of the scale.

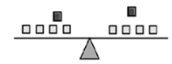


8. One zero pair is added to the left side and one zero pair is added to the right side of the scale.

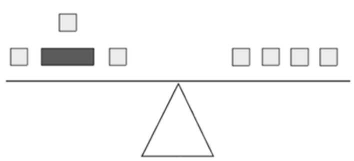


Actions that Balance a Scale

9. One red square is added to each side of the scale.



Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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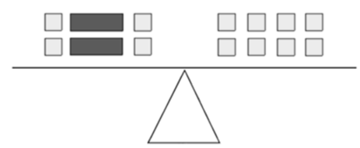
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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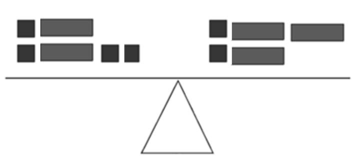
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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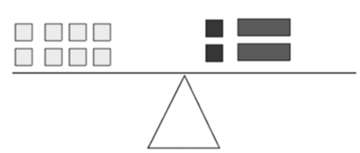
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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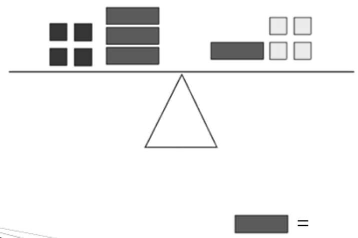
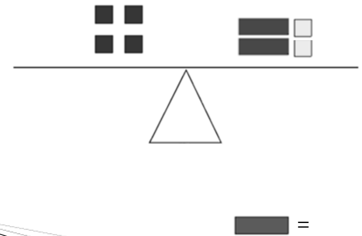
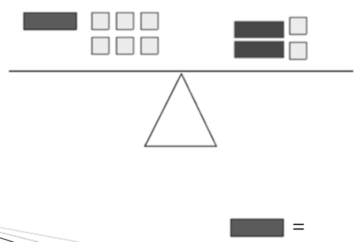
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



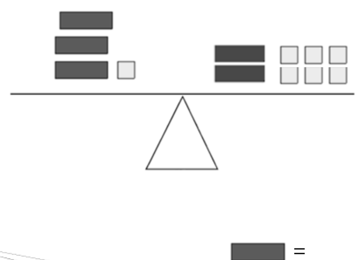
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



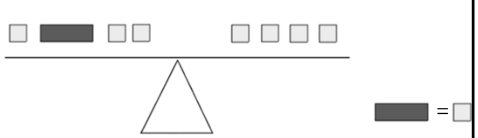
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



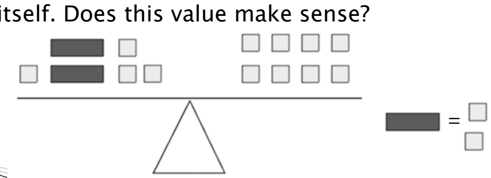
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



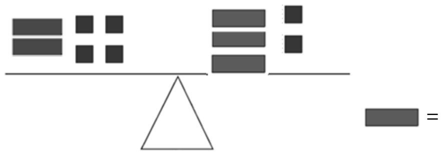
- Use the algebra models to represent
  $x + 3 = 4$
  on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



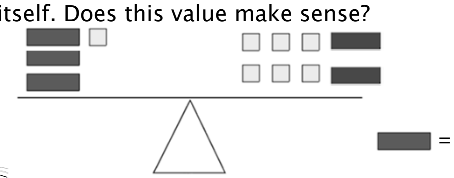
- Use the algebra models to represent
  $2x+4 = 8$
  on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



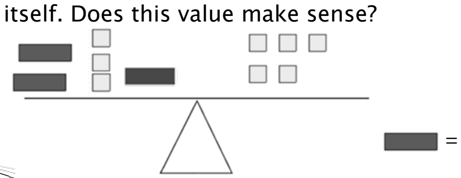
- Use the algebra models to represent
  $${}^{-}2x + {}^{-}4 = {}^{-}2 + 3x$$
  on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



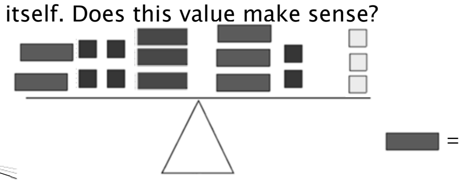
- Use the algebra models to represent
  $$3x + 1 = {}^{-}2x + 6$$
  on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



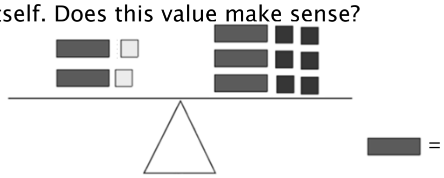
- Use the algebra models to represent
  $$2x + 3 + {}^{-}1x = 5$$
  on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



- Use the algebra models to represent
  $$2x + {}^{-}4 + 3x = {}^{-}2 + 3x + 3$$
  on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



- Use the algebra models to represent
  $$2(x + 1) = 3(x + {}^{-}2)$$
  on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?

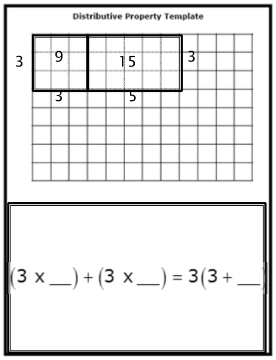


Building Understanding for Working with Polynomials

Draw a rectangle that measures 3 by 8 using the grid at the top of the page.

What is the area of the rectangle?

Separate the rectangle into two parts by drawing a vertical line so that one rectangle has an area of 9.



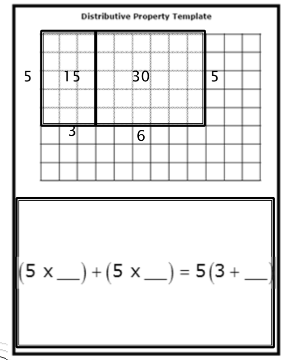
Write the dimensions of both sections and find the area of each.

Complete the following statement:

Draw a rectangle that measures 5 by 9 using the grid at the top of the page.

What is the area of the rectangle?

Separate the rectangle into two parts by drawing a vertical line so that one part is 5 by 3



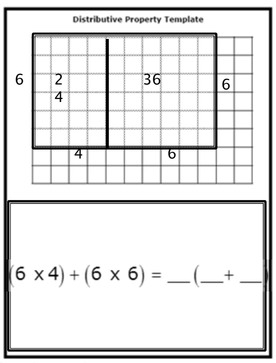
Write the dimensions of both sections and find the area of each.

Complete the following statement:

Draw a rectangle that measures 6 by 10 using the grid at the top of the page.

What is the area of the rectangle?

Separate the rectangle into two parts to illustrate 6 x 4 and 6 x 6.

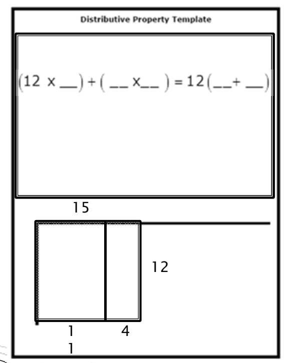


Complete the following statement:

Use the partial rectangle at the bottom of the page to draw a rectangle that is 12 x 15.

What is the area of the rectangle?

Separate the rectangle into two parts to illustrate 12 x 4 and 12 x 11

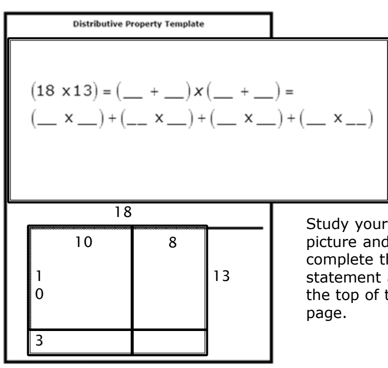


Complete the following statement:

Use the partial rectangle at the bottom of the page to draw a rectangle that is 13 x 18.

Separate the 18 side into two parts 10 and 8 using a vertical line.

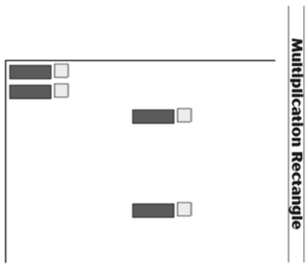
Separate the 13 side into two parts 10 and 3 by using a horizontal line.



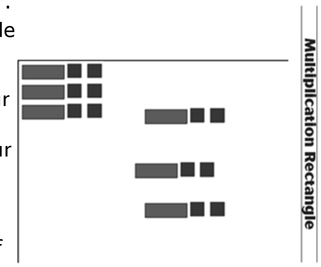
Study your picture and complete the statement at the top of this page.

Multiplying with Variables

- To complete 2(x+1) show two groups of (x + 1). Form a rectangle with the pieces on the table.
- How long is your rectangle? How wide is your rectangle? Notice how the length and width of the rectangle are part of 2(x+1).
- What is the algebraic name for the inside of your rectangle? Draw a picture of your rectangle:

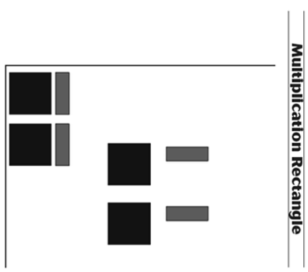


- To complete $3(x + {}^{-}2)$ show three groups of $(x + {}^{-}2)$.
- Form a rectangle with the pieces on the table.
- How long is your rectangle?
- How wide is your rectangle?
- What is the algebraic name for the inside of your rectangle?



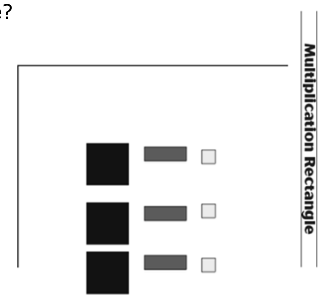
To complete $2(x^2 + x)$ what should you show on the table?

What is this equal to?



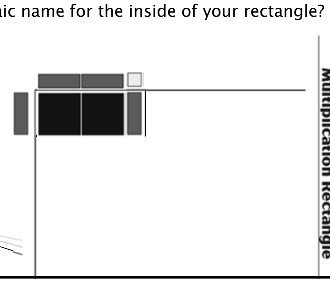
To complete $3(x^2 + x + 1)$ what should you show on the table?

What is this equal to?

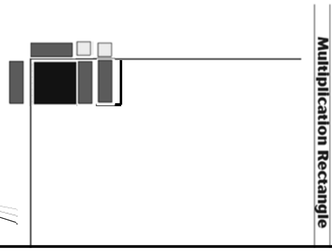


Multiplying Binomials

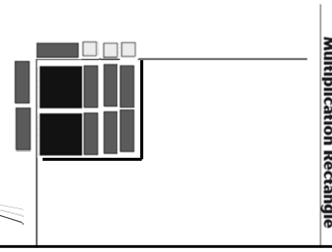
- Use the multiplication rectangle to make a rectangle whose dimensions are x by 2x+ 1.
- Place an x tile on the left side and tiles that represent 2x + 1 across the top as illustrated at the right.
- Fill in the rectangle to show it's area.
- Draw a picture of your rectangle at the right. What is the algebraic name for the inside of your rectangle?



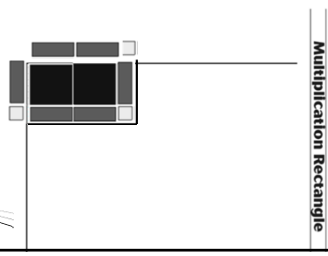
- Use the multiplication rectangle to multiply $x(x+2)$.
  - Fill in the area of the rectangle.
  - Draw a picture of your rectangle.
  - What is the algebraic name for the inside of your rectangle?



- Use the multiplication rectangle to multiply $2x(x+3)$
  - Fill in the rectangle.
  - Draw a picture of your rectangle.
  - What is the algebraic name for the inside of your rectangle?



- Use the multiplication rectangle to multiply $(x+1)(2x+1)$
  - Fill in the rectangle.
  - Draw a picture of your rectangle.
  - What is the algebraic name for the inside of your rectangle?



Multiply

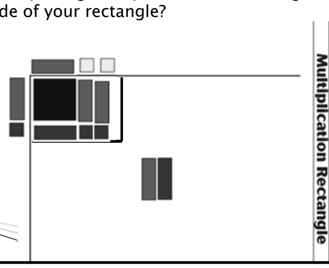
$(2x+1)(x+3)$

$(x+2)(2x+3)$

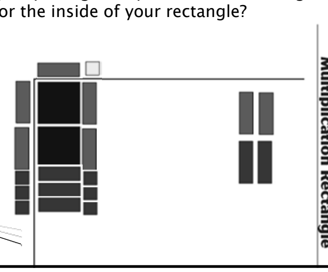
- Study the picture of the last few problems. Is there a way you can predict how many x rectangles will be in your final rectangle?
- Can you predict their location?
- Can you predict the number of blue unit squares that will be in your final rectangle?
- Can you predict their location?
- Try to predict what the rectangle will look like for $(2x+3)(x+4)$.
- Draw the picture without using the tiles.
- Write the algebraic expression for the rectangle.

- Draw the picture for the multiplication of $(3x+1)(x+2)$.
- Write the algebraic expression for the rectangle.

- Use the multiplication rectangle to multiply $(x + {}^{-}1)(x + 2)$.
- Once you have set up the dimensions fill in the rectangle. Watch the colors of the tiles. This problem involves a negative sign.
- Draw a picture of your rectangle. Can you simplify the rectangle by using zero pairs? What is the algebraic name for the inside of your rectangle?



- Use the multiplication rectangle to multiply $(x + 1)(2x + {}^{-}3)$.
- Once you have set up the dimensions fill in the rectangle. Watch the colors of the tiles. This problem involves a negative sign.
- Draw a picture of your rectangle. Can you simplify the rectangle by using zero pairs? What is the algebraic name for the inside of your rectangle?



- Use the multiplication rectangle to multiply $(x + -1)(2x + -1)$.
- Use the multiplication rectangle to multiply $(2x + -1)(x + 3)$.
- Use the multiplication rectangle to multiply $(x + -2)(2x + 3)$.

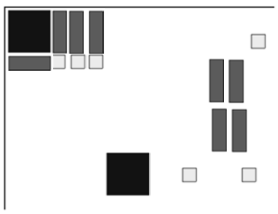
- Study the picture from the last few problems.
- Is there a way you can predict how many x rectangles will be in your final rectangle?
- Can you predict their location?
- Can you predict the number of blue unit squares that will be in your final rectangle?
- Can you predict their location?
- Try to predict what the rectangle will look like for $(2x + {}^{-}3)(x + 4)$.
- Draw the picture without using the tiles. Write the algebraic expression for the rectangle.

- Draw the picture for the multiplication of $(3x + -1)(x + 2)$.
- Write the algebraic expression for the rectangle.

Factoring Polynomials

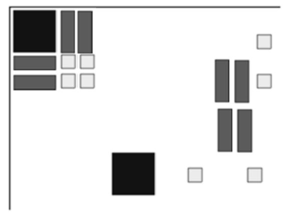
Multiplication Rectangle

- Use the following pieces: one x^2 piece, four x pieces, and three unit pieces.
- Form a rectangle from these eight pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these eight pieces.
- Describe the dimensions of your rectangle.



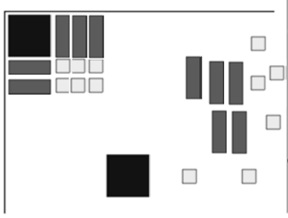
Multiplication Rectangle

- Use the following pieces: one x^2 piece, four x pieces, and four unit pieces.
- Form a rectangle from these nine pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these nine pieces.
- Describe the dimensions of your rectangle.



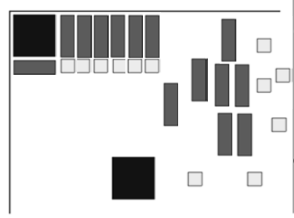
Multiplication Rectangle

- Use the following pieces: one x^2 piece, five x pieces, and six unit pieces.
- Form a rectangle from these twelve pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these twelve pieces.
- Describe the dimensions of your rectangle.



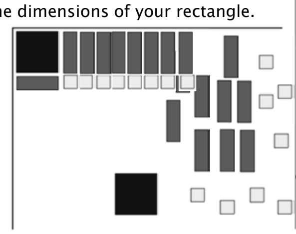
Multiplication Rectangle

- Use the following pieces: one x^2 piece, seven x pieces, and six unit pieces.
- Form a rectangle from these fourteen pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these fourteen pieces.
- Describe the dimensions of your rectangle.



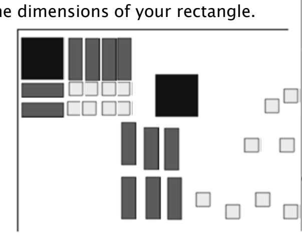
Multiplication Rectangle

- Use the following pieces: one x^2 piece, nine x pieces, and eight unit pieces.
- Form a rectangle from these eighteen pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these eighteen pieces.
- Describe the dimensions of your rectangle.

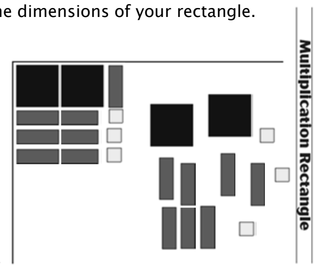


Multiplication Rectangle

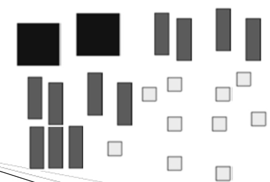
- Use the following pieces: one x^2 piece, eight unit pieces, a different number of x pieces.
- How many x pieces will you need to be able to form a rectangle from these pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these more than nine pieces.
- Describe the dimensions of your rectangle.



- Use the following pieces: two x^2 piece, seven x pieces pieces, and three unit pieces.
- Form a rectangle from these pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these pieces.
- Describe the dimensions of your rectangle.



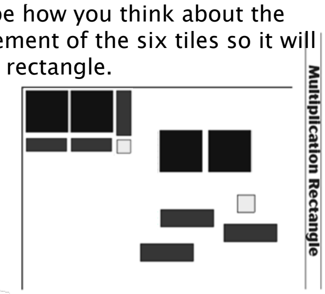
- Suppose you want to determine the two factors whose product is $2x^2 + 11x + 9$.
- Describe how you think about the arrangement of the twenty-two tiles so it will make a rectangle.



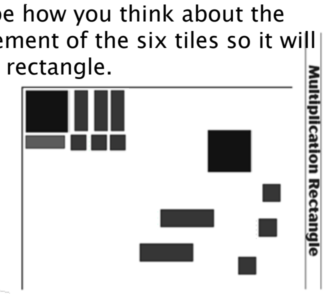
- Find the factors for $1x^2 + 5x + 4$
- Find the factors for $1x^2 + 4x + 4$
- Find the factors for $1x^2 + 5x + 6$
- Find the factors for $1x^2 + 7x + 6$

- Find the factors for $1x^2 - 5x + 4$
- Find the factors for $1x^2 - 4x + 4$
- Find the factors for $1x^2 - 5x + 6$
- Find the factors for $1x^2 - 7x + 6$

- Suppose you want to determine the two factors whose product is $2x^2 - 3x + 1$.
- Describe how you think about the arrangement of the six tiles so it will make a rectangle.



- Suppose you want to determine the two factors whose product is $1x^2 - 2x - 3$.
- Describe how you think about the arrangement of the six tiles so it will make a rectangle.



Do you think activities like we looked at today will help students

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

And build the understanding described in the

COMMON CORE STATE STANDARDS INITIATIVE

PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Key Takeaways from the High School Common Core State Standards Initiative in Mathematics

- The high school standards call on students to practice applying mathematical ways of thinking to real world issues and challenges; they prepare students to think and reason mathematically.
- The high school standards set a rigorous definition of college and career readiness, by helping students develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees regularly do.
- The high school standards emphasize mathematical modeling, the use of mathematics and statistics to analyze empirical situations, understand them better, and improve decisions. For example, the draft standards state: