# Developing a Concrete Model for Algebraic Concepts with Algebra Tiles

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## Addition of Integers

## The Zero Property

## Addition of Integers

- We will define **addition as adding to the table.**
- The first number will tells me what I start with on the table and the second number tells me what I add to the table.
  - 3 yellow + 2 yellow means I start with 3 yellow and I add 2 more yellow to the table.
- Complete Experiments 1—5 using the tiles.
- Remember that every pair of a yellow unit square and a red unit square is equal to zero and can be removed from the table without changing the sum on the table.
  - For each problem, record both the number and the color of the tile left.
  - You may use a R for red and a Y for yellow.

## Understanding the concept of zero

- Let each red unit square represent the opposite of each yellow unit square. Therefore when one red unit square and one yellow unit square are placed on a table together they cancel each other out and represent zero.
- Demonstrate two other representations for zero.
- Represent zero using a total of 6 tiles.

## Analyzing the Data

1. What do you notice about the colors used in the problems in Experiment 1? What do you notice about the colors found in the answers?
2. What do you notice about the colors used in the problems in Experiment 2? What do you notice about the colors found in the answers?
3. What do you notice about the colors used in the problems in Experiment 3? What do you notice about the colors found in the answers?
4. What do you notice about the colors used in the problems in Experiment 4? What do you notice about the colors found in the answers?
5. What do you notice about the colors used in the problems in Experiment 5? What do you notice about the colors found in the answers?

6. How do the problems in Experiments 1 and 2 differ from those in Experiments 3, 4, and 5?
7. Describe a pattern that exists between the problems and the answers in Experiments 1 and 2.
8. Describe a pattern that exists between the problems and the answers in Experiments 3 and 4.
9. Why do you think there are no tiles left in any of the answers for the problems in Experiment 5?
10. Create a set of rules that would help someone find the total number of tiles for each problem in Experiments 1-6.

## Applying What You Know

DIRECTIONS: Think about the tiles to find answers to each of the following:

1. (-6) + (2) = _____
2. (-3) + (-2) = _____
3. (-12) + (8) = _____
4. 4 + (7) =_____

## Using Symbols to Replace Tiles

- Because writing the words "yellow" or "un-shaded" and "red" or "shaded" is time consuming, symbols for yellow (un-shaded) and red (shaded) can be used. So that all students will use the same symbols, a yellow square tile will be represented by placing a positive (+) sign in front of a number or no sign at all. The symbol for red square tile will be a negative (-) sign.
  - Example 1: Three red unit squares will be recorded as (-3).
  - Example 2: Four yellow unit squares will be recorded as (+4) or (4).

Based on your knowledge of yellow and red tiles, create a set of rules that might help you to find sums of integers that would be too large to complete easily with actual tiles. Write the rules you create in your journal.

- To show that we are beginning with a certain number of tiles and then placing additional tiles on the table, we will use the plus (+) sign between the two different sets of tiles. Use an equals (=) sign to separate a problem from its answer.
- In the space to the right of each problem in Experiments 1—5, use symbols (+, —, =) to represent each problem and its answer.
  - Example 3: Nine yellow unit squares and two yellow unit squares would be recorded as (+9) + (+2) = +11.

Using only what you know about collecting tiles, determine ONLY THE SIGN of the answer for each of the following. Be able to connect the concept of the tile to how you determined the sign of the answer.

a. 4234 + 987=_____
b. -981 + (599) =_____
c. -1562 + (-222) = _____
d. 96 + (-873) = _____

- Based on your knowledge of yellow and red tiles, determine ONLY THE SIGN for each of the following. Then use a fraction capable calculator to compute each of the problems below. Verify the sign you predicted for the answer.

## Subtraction of Integers

| Problem | Sign You Predict | Sign Shown on Calculator |
|---|---|---|
| a. (-0.4) + (-0.05) | | |
| b. 3.032 + (-8.123) | | |
| c. -12.030 + 16.003 | | |
| d. 0.121 + 2.1021 | | |
| e. $\left(\frac{1}{4}\right)+\frac{5}{6}$ | | |
| f. $\left(-\frac{3}{4}\right)+\left(\frac{1}{8}\right)$ | | |
| g. $\left(-1\frac{3}{5}\right)+\left(-\frac{3}{4}\right)$ | | |
| h. $\left(-\frac{5}{8}\right)+\frac{13}{16}$ | | |

## Representing a Number in More than One Way

Represent the number 4 on the table.

- Then show each of the following:
  - 4 + 2 + (-2)
  - 5 + (-1)
  - 4 + (-1) + 1
  - -5+9
- Which expressions incorporates the use of the zero rule of addition?

Based on your knowledge of yellow (un-shaded) and red (shaded) tiles, predict ONLY THE SIGN of the answers for each of the problems below.

| Problem | Sign You Predict |
|---|---|
| a. -9 + 17 + (-18) | |
| b.-10 + (-11) + (-3) | |
| c. 14 + 18 + 25 | |
| d. -10 +15 +(10) +(-17) | |
| e. 12 + (-16) + (-32) + 4 | |
| f. 12 + (-4) + 18 + (-8) | |

## Subtraction of Integers

- We will define **subtraction as removing from the table.**
- You will notice that the questions ask you to remove certain tiles from the table. This is subtraction.
- Remove 3 yellow unit squares from 4 yellow unit squares means I start with 4 yellow unit squares and I remove 3 yellow unit squares from the table.
- For each problem in Experiments 1-6, record the results of the problem in the space provided. Your answer should include the number of tiles remaining after the operation is performed, along with the color of the tiles. You may use a R for red unit squares and Y for yellow unit squares.

## Analyzing the Data

1. Describe general strategies that were employed to solve the problems in Experiments 3-6.
2. How did the solutions in Experiments 3-6 differ from those in Experiments 1 and 2?
3. How are the problems in Experiments 1-6 similar to the addition problems you solved in Addition of Integer?
4. What rule could you create that would help you subtract signed numbers easily?

**Applying What You Know**

Use the rule you created for subtraction to complete these problems

1. (-7) - (3) =_____
2. (3) - (-7) =_____
3. (4) - (5) =_____
4. 5 - 4 =_____
5. -9 - (-3) = _____
6. -10 - (-11)=_____
7. 4 - (-2) =_____
8. (-3) - (5) _____
9. (-13)- (10) = __
10. 9 - (-6) = _____

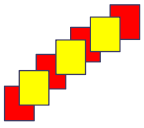
## Using Symbols to Replace the Tiles

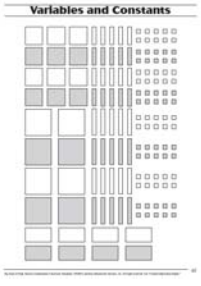
- Because writing the words "yellow" and "red" is time consuming, symbols for the colors can be used. So that all students in your class will use the same symbols, a red unit squares will be represented by placing a negative (-) sign in front of a number. The symbol for yellow unit squares will be a positive (+) sign or no sign at all.
  - Example 1: Three red unit squares will be recorded as (-3).
  - Example 2: Four yellow unit squares will be recorded as (+4) or (4).

- Use a calculator to check your answers to problems 1-10. Discuss errors with other members of your group to discover strategies that will yield correct answers. Record your answers to the following questions in your journal: If you made any errors, what kind did you make? What strategies can you use to avoid making the same kind of mistake in the future?

- To show different sets of tiles being subtracted, a minus (-) sign is placed between the two numbers representing the tiles. Use an equal (=) sign to separate a problem from its answer.
- In the space to the right of each problem in Experiments 1-6, use symbols (+, —, =) to represent each problem and its answer.
- Example 3: Remove 2 red unit squares from 5 red unit squares. (-5) - (-2) = - 3

## Using the Variables and Constanct Template





## Variables and Constants

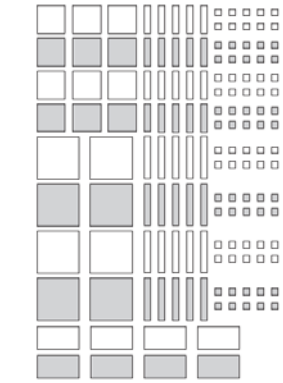
- Picture:

$^{+}3 + {}^{+}2$

$^{-}3 + {}^{-}4$

$^{+}3 + {}^{-}5$

$^{+}7 + {}^{-}2$



## Variables and Constants

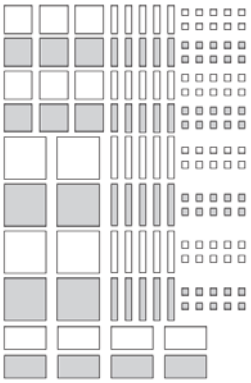
- Picture:

$^{+}3x - {}^{+}2x$

$^{-}3x - {}^{-}4x$

$^{+}3x - {}^{-}5x$

$^{+}7x - {}^{-}2x$



## Variables and Constants

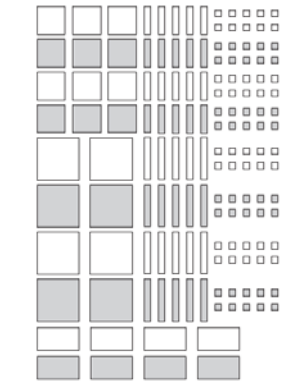
- Picture:

$^{+}3 - {}^{+}2$

$^{-}3 - {}^{-}4$

$^{+}3 - {}^{-}5$

$^{+}7 - {}^{-}2$



## Variables and Constants

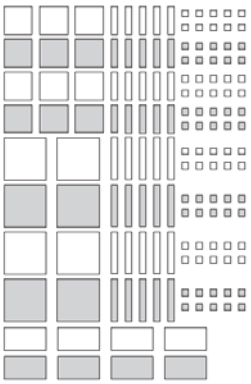
- Picture:

$^{+}3x^2 + {}^{+}2x^2$

$^{-}3x^2 + {}^{-}4x^2$

$^{+}3x^2 + {}^{-}5x^2$

$^{+}3x^2 + {}^{-}2x^2$



## Variables and Constants

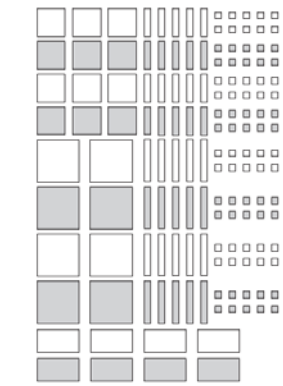
- Picture:

$^{+}3x + {}^{+}2x$

$^{-}3x + {}^{-}4x$

$^{+}3x + {}^{-}5x$

$^{+}7x + {}^{-}2x$



## Variables and Constants

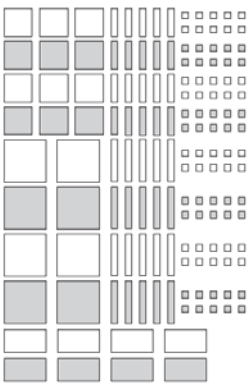
- Picture:

$^{+}3x^2 - {}^{+}2x^2$

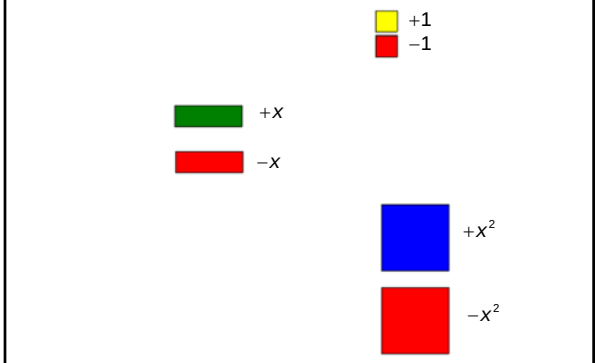
$^{-}3x^2 - {}^{-}4x^2$

$^{+}3x^2 - {}^{-}5x^2$

$^{+}3x^2 - {}^{-}2x^2$



# Writing Expressions with Algebra Tiles




Below are several symbolic representations for algebraic expressions. Show their concrete representation using algebra tiles.

$3x - 2$

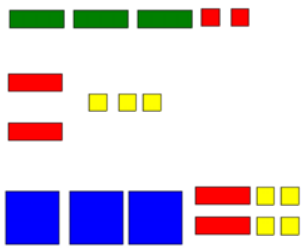
$x^2 - x$

$2x^2 + x + 1$

$3 - x^2$

$x^2 + 2x - 2$

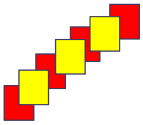
Pictured below are the concrete representation of several algebraic expressions. Write their symbolic representation.



Use Algebra Tiles as needed to complete the following problems. Simplify:

1. $2x + 3 + 5x - 4$
2. $2x^2 + 3x - 5 + 4x^2 + x$
3. $3x^2 + 2x - 4x^2 + 2 + 5x + 1$
4. $x^2 + 2x + x^2 + 3x^2 - 4x - x^2$
5. $2x^2 + 3 - 4x - 4x^2$
6. $2x^2 + 3x^2 + 5x - 2x$
7. $2(x^2 + 3)$
8. $3(x - 2)$
9. $4(x^2 + 3xy - 2)$
10. $3(x^2 - 5)$
11. $2(3x^2 + 4) - 2x^2$
12. $2(x - 1) + 4x + 3$

## Multiplication and Division of Integers



2. Study the problems and answers in Experiments 2 and 3.
   a. What color discs appear in every answer?
   b. What do you notice about each of the problems in Experiment 2?
   c. What do you notice about each of the problems in Experiment 3?

## Multiplication of Signed Integers

- Multiplication is often thought of as a shortcut for addition, or as thinking of groups of things.
  - We will define multiplication as either adding or removing groups of tiles from the table.
  - Begin each problem with an empty table. Then either remove or add tiles to that empty table.
- For each problem in Experiments 1-4, record the results in the space provided. Your answer should include the number of tiles in the result, along with the color of the tiles. You may use a Y yellow unit squares and a R for red unit squares.

3. Based on your observations, what rule could you create to help determine the sign of the product of TWO factors?

## Analyzing the Data

1. Study the problems and the answers in Experiments 1 and 4.
   - What color discs appear in every answer?
   - What do you notice about each of the problems in Experiment 1?
   - What do you notice about each of the problems in Experiment 4?

## Using Symbols to Replace the Tiles

- **DIRECTIONS:** Because writing the words “yellow” and “red” is time consuming, symbols for the colors can be used. So that all students in your class will use the same symbols, a red unit square tile will be represented by placing a negative (—) sign in front of a number. The symbol for yellow unit square tile will be a positive (+) sign or no sign at all.
- Example 1: Three red unit squares will be recorded as (-3).
- Example 2: Four yellow unit squares will be recorded as (+4) or (4).

- When using symbols to indicate multiplication, place the number of groups to be displayed first, then the number of tiles that are to be in each group second. In these multiplication problems it is customary to place each of the numbers in parentheses or separate them by a "•". Use a positive (+) sign to indicate that the groups are to be added and a minus sign (-) to indicate that groups of numbers are to be removed.

6. Based on the conclusions you reached in answering questions 2-5 predict ONLY THE SIGN of the answer to each of the following problems. Use your calculator to test your predictions.
   a. (-3)(-2)(-1) =_____
   b. (-2)(3)(4) =_____
   c. (5)(-2)(-5) =_____
   d. (4)(3)(5) =_____
   e. (-2)3 =_____
   f. (4)3 =_____

## Applying What You Know

1. Based on the rule you developed in Part 2, predict ONLY THE SIGN of the answer for each of the following problems. Use your calculator to verify the results.

a. $\left(-\frac{1}{4}\right)\left(\frac{3}{4}\right)=$ _____
b. (-0.01)(-0.2) =_____
c. (3.14)(2.02)= _____
d. $\left(5\frac{1}{2}\right)\left(-\frac{3}{16}\right)=$ _____
e. (−3)2 =_____
f. (5)2 =_____

Since $(-3)\bullet(+2) = -6$ we can write

$\frac{-6}{+2} = -3$ or $\frac{-6}{-3} = +2$ .

2. What will be sign of the product when three positive factors are multiplied together? Why?

3. What will be sign of the product when three negative factors are multiplied together? Why?

4. What will be sign of the product when two positive and one negative factor are multiplied together? Why?

5. What will be sign of the product when two negative and one positive factor are multiplied together? Why?

- Return to the Operations with Integers – Multiplication and write one division problem for each multiplication problem.
- Study the division problems you have written. What can you write about the sign of the quotient of...
  a. a positive divisor and a positive dividend?
  b. a negative divisor and a negative dividend?
  c. a positive divisor and a negative dividend?
  d. a negative divisor and a positive dividend?

❑ Write a rule that will help you determine the sign when two integers are divided.

The rules for signs in division problems work the same as those in multiplication. Therefore what can you conclude about the sign of the quotient of…

a. a positive divisor and a positive dividend?

b. a negative divisor and a negative dividend?

c. a positive divisor and a negative dividend?

d. a negative divisor and a positive dividend?

Test your theories by creating sample problems and entering them into your calculator.

- What number did you begin with?
- Did everyone use the same number?
- To represent or show the number you chose, I am going to place a green rectangle on the screen.
- The area of this rectangle will stand for the number you chose.

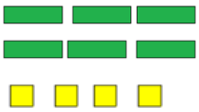
## Understand and apply the concept of a variable

- Step 1 was to double the number.
- To show this, one more rectangle will be placed on the screen.
- Now there are twice as many as when we started.

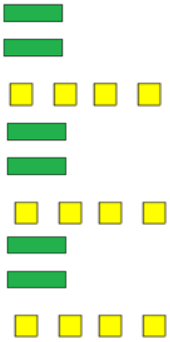
## Choose any whole number from 1 to 10

- Double the number you chose.
- Add 4 to this product.
- Multiply the resulting sum by 3.
- Divide this product by 6
- Subtract the original number.
- Multiply by 5.
- Turn to your partner and find out what they got for an answer.

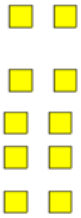
- Step 2 was to add 4.
- Adding 4 rectangles would not represent adding 4, it would represent adding 4 more of the value of the number that the student chose.
- So, we will add 4 unit squares to show 4.



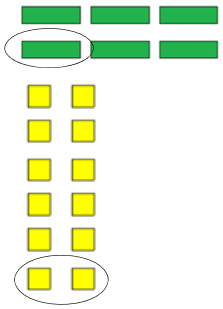
- Step 3 was to triple the amount from step 2.
- To do this, we need to triple the number of rectangles as well as the number of unit squares.
- This would leave us with 6 rectangles and 12 unit squares.



- Step 6 was to multiply by 5.
- Since there are 2 unit squares, 5 times as many would be a 10.
- Therefore, the answer is 10.



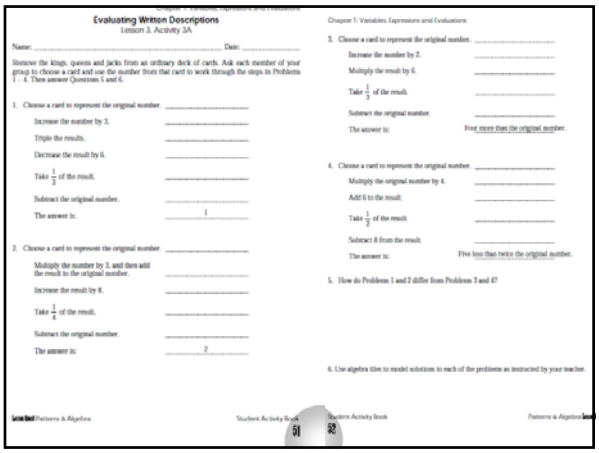
- Step 4 was to take 1/6 of the quantity we now had.
- This means we need to take 1/6 of 6 rectangles AND of 12 unit squares.
- This would leave 1 rectangle and 2 unit squares.



- The green rectangle represented, or stood for, each of the values that you chose.
- The value of the rectangle changed, or varied, depending on what number you chose.
- The green rectangle was not always the same value, this green rectangle can be referred to as a *variable*.
- The yellow squares, however, did not change value – it always represented the unit 1; it remained *constant*.

- Step 5 was to subtract the original number.
- Since each of the original numbers is represented by a rectangle, we can remove the rectangle.
- This leaves 2 unit squares on the overhead.

- Turn to the Evaluating Written Descriptions Activity in your handout.
- You are to work in a group of 4.
- Each person is to select a card from a deck of cards (remove the jacks, queens and kings)
- Red cards represent negative numbers and black cards represent positive numbers.
- Work each of the four exercises with your selected number.



**Evaluating Written Descriptions**
Lesson 3, Activity 3A

Name: _____ Date: _____

Remove the kings, queens and jacks from an ordinary deck of cards. Ask each member of your group to choose a card and use the number from that card to work through the steps in Problems 1 – 4. Then answer Questions 5 and 6.

1. Choose a card to represent the original number. _____
   Increase the number by 3. _____
   Triple the results. _____
   Decrease the result by 6. _____
   Take $\frac{1}{3}$ of the result. _____
   Subtract the original number. _____
   The answer is: 1

2. Choose a card to represent the original number. _____
   Multiply the number by 3, and then add the result to the original number. _____
   Increase the result by 8. _____
   Take $\frac{1}{4}$ of the result. _____
   Subtract the original number. _____
   The answer is: 2

3. Choose a card to represent the original number. _____
   Increase the number by 2. _____
   Multiply the result by 6. _____
   Take $\frac{1}{3}$ of the result. _____
   Subtract the original number. _____
   The answer is: Four more than the original number.

4. Choose a card to represent the original number. _____
   Multiply the original number by 4. _____
   Add 6 to the result. _____
   Take $\frac{1}{2}$ of the result. _____
   Subtract 8 from the result. _____
   The answer is: Five less than twice the original number.

5. How do Problems 1 and 2 differ from Problems 3 and 4?

6. Use algebra tiles to model solutions to each of the problems as instructed by your teacher.

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- Choose a card to represent the original number.
- Multiply the number by 3, and then add the result to the original number. Increase the result by 8.
- Take ¼ of the result.
- Subtract the original number.
- The answer is:1

Let's now model each question with your algebra tiles.

- Choose a card to represent the original number.
- Increase the number by 2.
- Multiply the result by 6.
- Take 1/3 of the result.
- Subtract the original number.
- The answer is: Four more than the original number.

- Choose a card to represent the original number.
- Increase the number by 3.
- Triple the results.
- Decrease the result by 6.
- Take 1/3 of the result.
- Subtract the original number.
- The answer is: 2

- Choose a card to represent the original number.
- Multiply the original number by 4.
- Add 6 to the result.
- Take ½ of the result.
- Subtract 8 from the result.
- The answer is: Five less than twice the original number.

- How do Problems 1 and 2 differ from Problems 3 and 4?

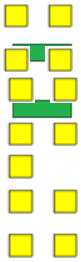
**Problem 1**

| Instruction | Result of Action Taken |
|---|---|
| Choose a card to represent the original number. | |
| Increase the number by 3. | |
| Triple the results. | |
| Decrease the result by 6. | |
| Take $\frac{1}{3}$ of the result. | |
| Subtract the original number. | |

**Problem 2**

| Instruction | Result of Action Taken |
|---|---|
| Choose a card to represent the original number. | |
| Multiply the number by 3, then add the results to the original number. | |
| Increase the result by 8. | |
| Take $\frac{1}{4}$ of the result. | |
| Subtract the original number. | |

- What is the value of this expression if $x$ (the rectangle) stood for 4.
- Each variable is replaced by 4 units.
- The value would be 13 because 4 + 4 + 5 = 13, or
- 2(4) + 5 = 13.



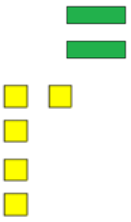
**Problem 3**

| Instruction | Result of Action Taken |
|---|---|
| Choose a card to represent the original number. | |
| Increase the number by 2. | |
| Multiply the result by 6. | |
| Take $\frac{1}{3}$ of the result. | |
| Subtract the original number. | |

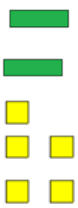
**Problem 4**

| Instruction | Result of Action Taken |
|---|---|
| Choose a card to represent the original number. | |
| Multiply that number by 4. | |
| Add 6 to that result. | |
| Take $\frac{1}{2}$ of the result. | |
| Subtract 8 from the result. | |

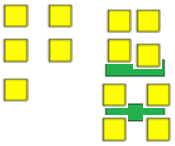
- Place 2 green rectangles and 5 yellow squares on your table.
- What is this a representation of?
- 2x + 5 or 5 + 2x



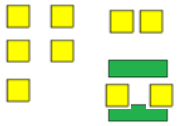
- Place the tiles shown at the right on your table.
- This represents the expression $2x + 5$ because there are two variables and 5 constants or 5 units.



- If the green rectangle was replaced with 4 what would be the value of the expression?



- If the green rectangle was replaced with 2 what would be the value of the expression?



## Now try to add these expressions by just describing the tiles you would use:

| Expression 1 | Expression 2 |
|---|---|
| 2x+3 | -x +-1 |
| -3x +5 | 2x+-2 |
| x+-3 | -4x+-3 |
| -2x +-2 | -3+4x |

## The value of the expression depends upon the value of the variable.

- Place 3 green rectangles and 2 yellow tiles on the table.
- What variable expression do these tiles represent?
- Evaluate this expression for x = -2
- Evaluate this expression for x = 3

## Now try to subtract

- First place expression one on the board. Then try to remove or subtract expression two from the board. (Hint: You might need some zero pairs to do this.)

| Expression 1 | Expression 2 |
|---|---|
| x+3 | -2x +-1 |
| -2x +5 | -3x+-2 |
| -x+3 | 4x+-3 |
| 3x +-2 | 3+-2x |

## Adding expressions together

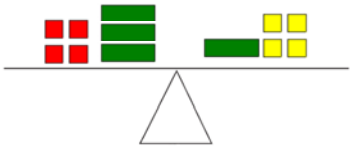
- First place expression one on the board. Then add to this board the expression two. What is the results?

| Expression 1 | Expression 2 |
|---|---|
| x+3 | -2x +-1 |
| -2x +5 | -3x+-2 |
| -x+3 | 4x+-3 |
| 3x +-2 | 3+-2x |

- Now try to subtract these expressions without using the tiles. Just describe what you would do:

| Expression 1 | Expression 2 |
|---|---|
| 2x+3 | -x +-1 |
| -3x +5 | 2x+-2 |
| x+-3 | -4x+-3 |
| -2x +-2 | -3+4x |

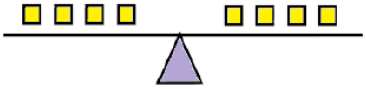
# Balancing Equations



8. The number of items on each side is cut in half. — balanced
9. Two yellow squares are removed from the left and two yellow squares are added to the right side of the scale. — **unbalanced**
10. Two red squares are removed from the left and two red squares are removed from the right side of the scale. — balanced
11. One zero pair is added to the left side and one zero pair is added to the right side of the scale. — balanced
12. Two yellow squares are added to the right side and two red squares are added to the left side of the scale. — **unbalanced**
13. One red square is added to each side of the scale.
14. Double the number of squares on the left and divide the number of squares on the right by two.

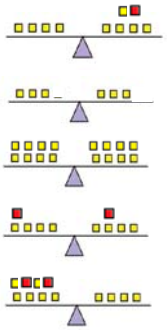
## Actions with Balanced Scales

- Tell what would happen to the balanced scale below if each of the actions listed are taken.
- Remember, the scale is reset after each action.



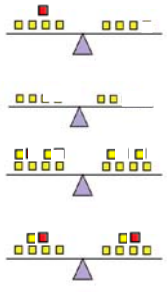
## Balanced Scale

1. One yellow and one red square are added to the right side.
2. One yellow squares is removed from the left side and one yellow square is removed from the right side.
3. Multiply the number of items on each side by two.
4. One red square is added to each side of the scale.
5. Two red squares and two yellow squares are added to the left side of the scale

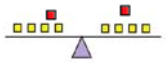


1. Three red squares are added to the right side. — **unbalanced**
2. One yellow and one red square are added to the right side. — balanced
3. One yellow squares is removed from the left side and one yellow square is removed from the right side. — balanced
4. Two red squares are added to the right side of the scale and two yellow squares are added to the left side. — **unbalanced**
5. Multiply the number of items on each side by two. — balanced
6. Two red squares and two yellow squares are added to the left side of the scale. — balanced
7. A red square is added to the left and a yellow square is removed from the right. — balanced

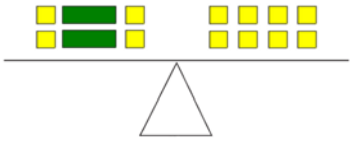
6. A red square is added to the left and a yellow square is removed from the right.
7. The number of items on each side is cut in half.
8. Two red squares are removed from the left and two red squares are removed from the right side of the scale.
9. One zero pair is added to the left side and one zero pair is added to the right side of the scale.



10. One red square is added to each side of the scale.



Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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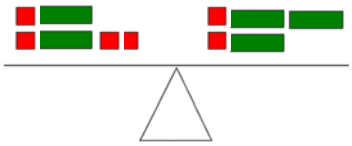
## Solving Equations

Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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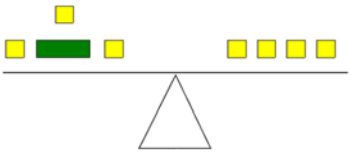
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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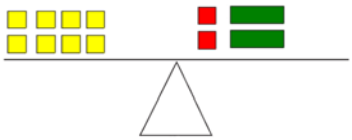
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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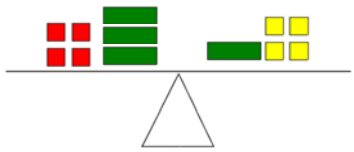
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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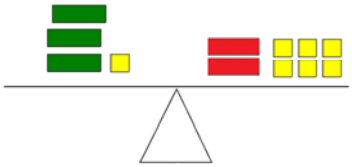
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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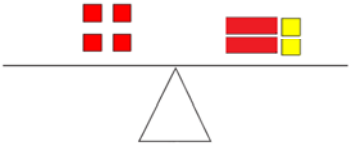
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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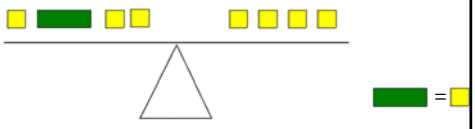
- Use the algebra models to represent
  $x + 3 = 4$
  on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



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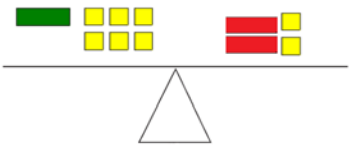
Use Algebra Tiles and the Balance Scale template to determine the answer to each of the problems below.
Give the value for *x and explain how you determined the answer.*



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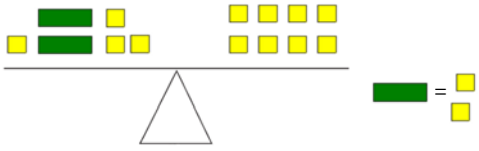
- Use the algebra models to represent
  $2x+4 = 8$
  on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



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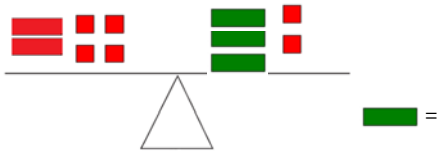
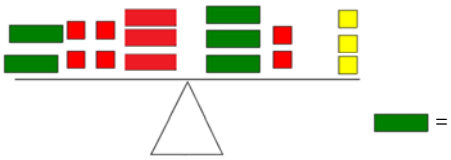
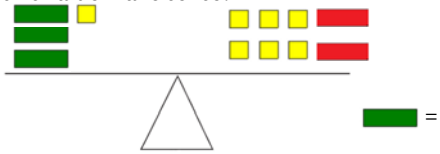
- Use the algebra models to represent
$$^{-}2x + {}^{-}4 = {}^{-}2 + 3x$$
on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



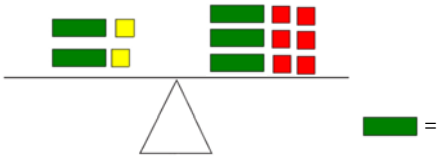

- Use the algebra models to represent
$$2x + {}^{-}4 + 3x = {}^{-}2 + 3x + 3$$
on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?



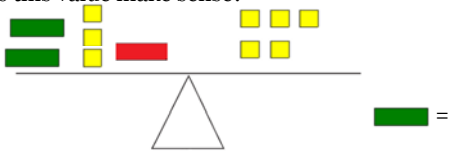

- Use the algebra models to represent
$$3x + 1 = {}^{-}2x + 6$$
on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?




- Use the algebra models to represent
$$2(x+1) = 3(x + {}^{-}2)$$
on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?




- Use the algebra models to represent
$$2x + 3 + {}^{-}1x = 5$$
on the equation balance. Find the value of x by doing the same thing to both sides of the balance until you have the x (green rectangle) by itself. Does this value make sense?




## Visualizing Multiplication of Binomials and Factoring with Algebra Tiles

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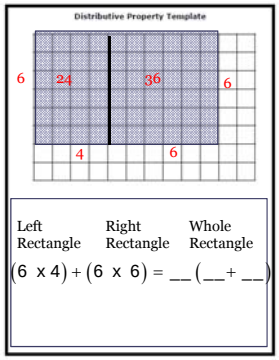
# Multiplication Using the Area Model

Draw a rectangle that measures 6 by 10 using the grid at the top of the page.

What is the area of the rectangle?

Separate the rectangle into two parts to illustrate 6 x 4 and 6 x 6.

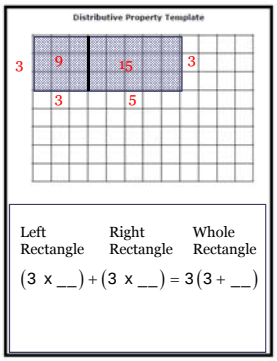



Complete the following statement:

Draw a rectangle that measures 3 by 8 using the grid at the top of the page.

What is the area of the rectangle?

Separate the rectangle into two parts by drawing a vertical line so that one rectangle has an area of 9.



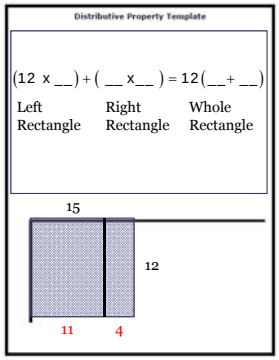

Write the dimensions of both sections and find the area of each.

Complete the following statement:

Use the partial rectangle at the bottom of the page to draw a rectangle that is 12 x 15.

What is the area of the rectangle?

Separate the rectangle into two parts to illustrate 12 x 4 and 12 x 11

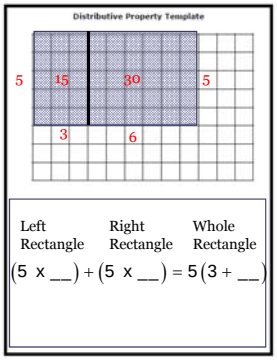



Complete the following statement:

Draw a rectangle that measures 5 by 9 using the grid at the top of the page.

What is the area of the rectangle?

Separate the rectangle into two parts by drawing a vertical line so that one part is 5 by 3



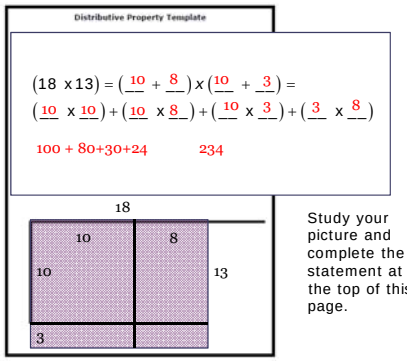

Write the dimensions of both sections and find the area of each.

Complete the following statement:

Use the partial rectangle at the bottom of the page to draw a rectangle that is 13 x 18.

Separate the 18 side into two parts 10 and 8 using a vertical line.

Separate the 13 side into two parts 10 and 3 by using a horizontal line.

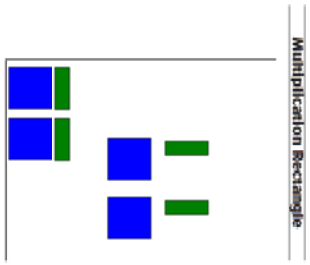



Study your picture and complete the statement at the top of this page.

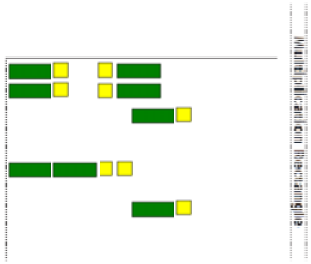
# Multiplying with Variables

To complete $2(x^2 + x)$ what should you show on the table?

What is this equal to?



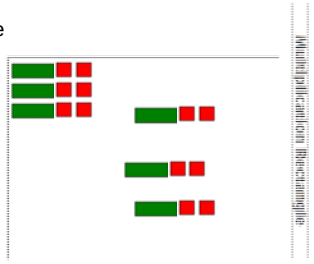

- To complete 2(x+1) show two groups of (x + 1). Form a rectangle with the pieces on the table.
- How long is your rectangle? How wide is your rectangle? Notice how the length and width of the rectangle are part of 2(x+1).
- What is the algebraic name for the inside of your rectangle?



To complete $3(x^2 + x + 1)$ what should you show on the table?

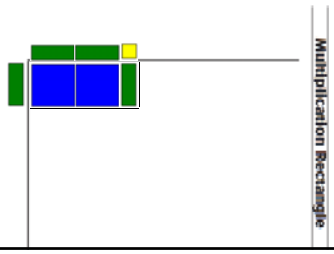
What is this equal to?

- To complete $3(x + {}^{-}2)$ show three groups of $(x + {}^{-}2)$.
- Form a rectangle with the pieces on the table.
- How long is your rectangle?
- How wide is your rectangle?
- What is the algebraic name for the inside of your rectangle?

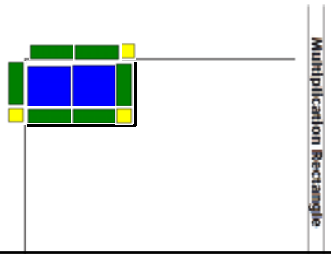


# Multiplying Binomials

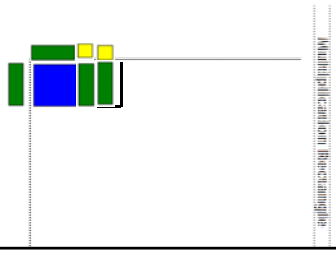
- Use the multiplication rectangle to make a rectangle whose dimensions are x by 2x+ 1.
- Place an x tile on the left side and tiles that represent 2x + 1 across the top as illustrated at the right.
- Fill in the rectangle to show it's area.
- Draw a picture of your rectangle at the right. What is the algebraic name for the inside of your rectangle?




- Use the multiplication rectangle to multiply $(x+1)(2x+1)$
- Fill in the rectangle.
- Draw a picture of your rectangle.
- What is the algebraic name for the inside of your rectangle?




- Use the multiplication rectangle to multiply $x(x+2)$ .
- Fill in the area of the rectangle.
- Draw a picture of your rectangle.
- What is the algebraic name for the inside of your rectangle?

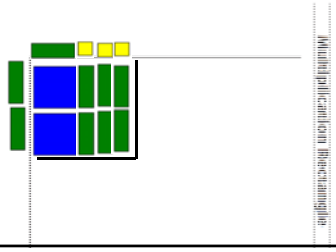


Multiply

$(2x+1)(x+3)$

$(x+2)(2x+3)$

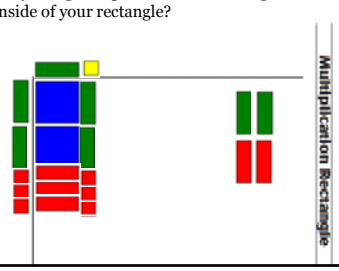
- Use the multiplication rectangle to multiply $2x(x+3)$
- Fill in the rectangle.
- Draw a picture of your rectangle.
- What is the algebraic name for the inside of your rectangle?



- Study the picture of the last few problems. Is there a way you can predict how many x rectangles will be in your final rectangle?
- Can you predict their location?
- Can you predict the number of blue unit squares that will be in your final rectangle?
- Can you predict their location?
- Try to predict what the rectangle will look like for $(2x+3)(x+4)$ .
- Draw the picture without using the tiles.
- Write the algebraic expression for the rectangle.

- Draw the picture for the multiplication of $(3x+1)(x+2)$
- Write the algebraic expression for the rectangle.

- Use the multiplication rectangle to multiply $(x+1)(2x+^{-}3)$.
- Once you have set up the dimensions fill in the rectangle. Watch the colors of the tiles. This problem involves a negative sign.
- Draw a picture of your rectangle. Can you simplify the rectangle by using zero pairs? What is the algebraic name for the inside of your rectangle?

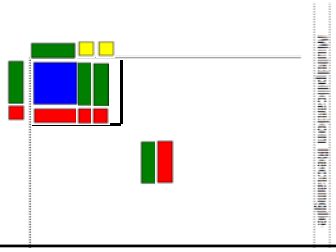



- Recall that each of the following is a representation for zero

- Show zero using 4 rectangles.
- Show zero using 6 small squares

- Use the multiplication rectangle to multiply $(x+^{-}1)(2x+^{-}1)$.
- Use the multiplication rectangle to multiply $(2x+^{-}1)(x+3)$.
- Use the multiplication rectangle to multiply $(x+^{-}2)(2x+3)$.

- Use the multiplication rectangle to multiply $(x+^{-}1)(x+2)$.
- Once you have set up the dimensions fill in the rectangle. Watch the colors of the tiles. This problem involves a negative sign.
- Draw a picture of your rectangle. Can you simplify the rectangle by using zero pairs? What is the algebraic name for the inside of your rectangle?



- Study the picture from the last few problems.
- Is there a way you can predict how many x rectangles will be in your final rectangle?
- Can you predict their location?
- Can you predict the number of blue unit squares that will be in your final rectangle?
- Can you predict their location?
- Try to predict what the rectangle will look like for $(2x+^{-}3)(x+4)$.
- Draw the picture without using the tiles. Write the algebraic expression for the rectangle.

- Draw the picture for the multiplication of $(3x + -1)(x + 2)$ .
- Write the algebraic expression for the rectangle.

## Using a Rectangle Diagram

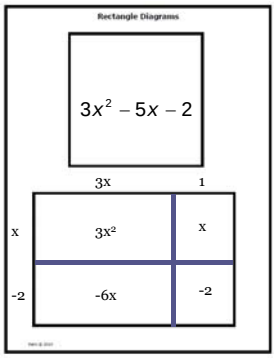
Multiply $(3x + 1)(x + -2)$

Subdivide the rectangle

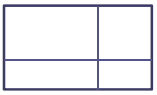
Label the dimensions

Calculate the areas

Write the expression




## Extending the Algebra Tile Model to an Rectangle Diagram



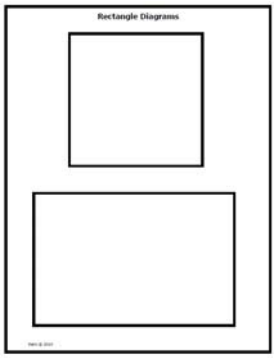
## Using a Rectangle Diagram

Multiply these:

$(4x + -1)(x + -3)$

$(1+-2x)(-1+2x)$

$(1+-x)(2+3x)$




## Using a Rectangle Diagram

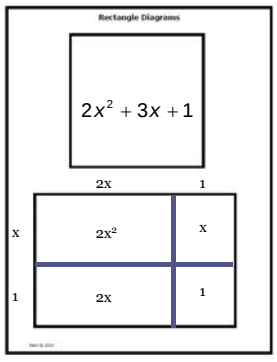
Multiply $(2x + 1)(x + 1)$

Subdivide the rectangle

Label the dimensions

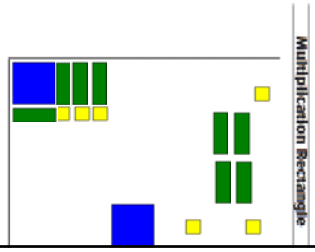
Calculate the areas

Write the expression

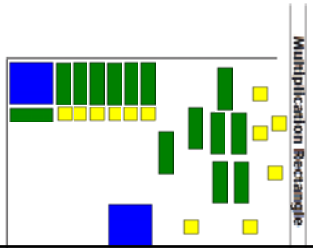



## Factoring Polynomials

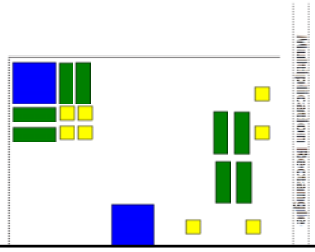
- Use the following pieces: one $x^2$ piece, four x pieces, and three unit pieces.
- Form a rectangle from these eight pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these eight pieces.
- Describe the dimensions of your rectangle.



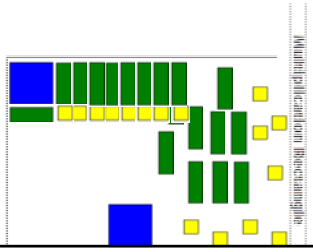

- Use the following pieces: one $x^2$ piece, seven x pieces, and six unit pieces.
- Form a rectangle from these fourteen pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these fourteen pieces.
- Describe the dimensions of your rectangle.



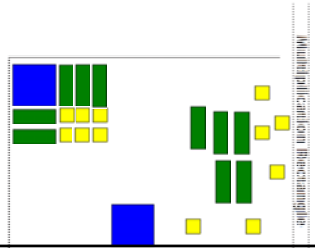

- Use the following pieces: one $x^2$ piece, four x pieces, and four unit pieces.
- Form a rectangle from these nine pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these nine pieces.
- Describe the dimensions of your rectangle.



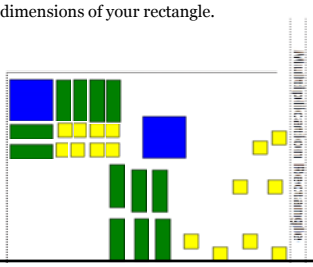
- Use the following pieces: one $x^2$ piece, nine x pieces, and eight unit pieces.
- Form a rectangle from these eighteen pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these eighteen pieces.
- Describe the dimensions of your rectangle.



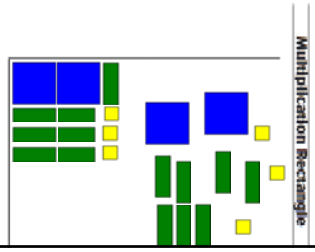
- Use the following pieces: one $x^2$ piece, five x pieces, and six unit pieces.
- Form a rectangle from these twelve pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these twelve pieces.
- Describe the dimensions of your rectangle.



- Use the following pieces: one $x^2$ piece, eight unit pieces, a different number of x pieces.
- How many x pieces will you need to be able to form a rectangle from these pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these more than nine pieces.
- Describe the dimensions of your rectangle.

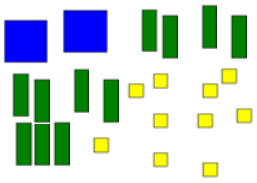


- Use the following pieces: two $x^2$ piece, seven x pieces pieces, and three unit pieces.
- Form a rectangle from these pieces.
- What polynomial is represented by the rectangle?
- Describe the polynomial represented by these pieces.
- Describe the dimensions of your rectangle.

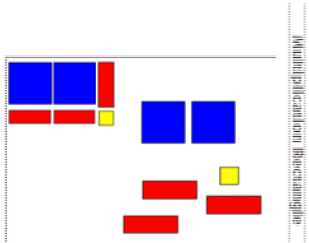



- Find the factors for $1x^2 - 5x + 4$
- Find the factors for $1x^2 - 4x + 4$
- Find the factors for $1x^2 - 5x + 6$
- Find the factors for $1x^2 - 7x + 6$

- Suppose you want to determine the two factors whose product is $2x^2 + 11x + 9$ .
- Describe how you think about the arrangement of the twenty-two tiles so it will make a rectangle.

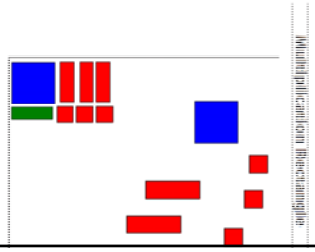


- Suppose you want to determine the two factors whose product is $2x^2 - 3x + 1$
- Describe how you think about the arrangement of the six tiles so it will make a rectangle.

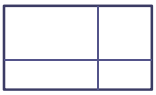


- Find the factors for $1x^2 + 5x + 4$
- Find the factors for $1x^2 + 4x + 4$
- Find the factors for $1x^2 + 5x + 6$
- Find the factors for $1x^2 + 7x + 6$

- Suppose you want to determine the two factors whose product is $1x^2 - 2x - 3$
- Describe how you think about the arrangement of the six tiles so it will make a rectangle.

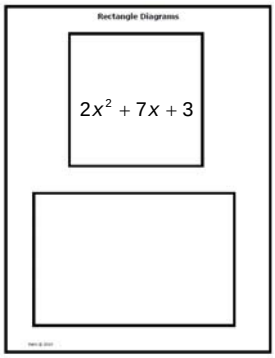


# Extending the Algebra Tile Model to an Rectangle Diagram



## Using a Rectangle Diagram

Factor $2x^2 + 7x + 3$




## Using a Rectangle Diagram

Factor $2x^2 + 3x + 1$

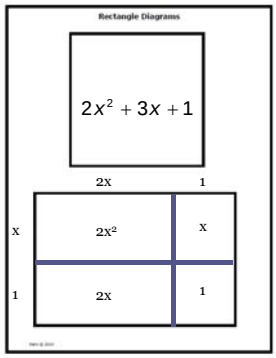
Subdivide the rectangle

Label the area at the bottom right

Label the area at the top left

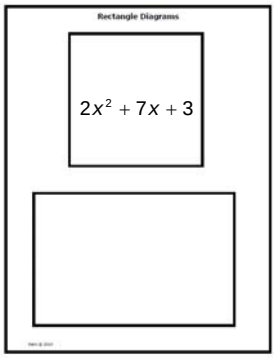
Think how you can separate the area of 3x

Label the dimensions



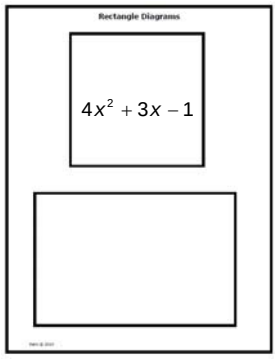

## Using a Rectangle Diagram

Factor $2x^2 + 7x + 3$



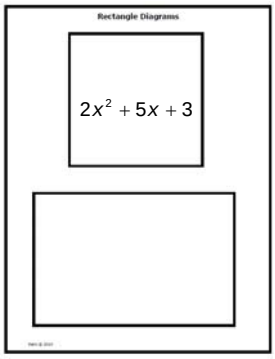

## Using a Rectangle Diagram

Factor $4x^2 + 3x + -1$




## Using a Rectangle Diagram

Factor $2x^2 + 5x + 3$



## Think about it

- Did the Distributive Property Template build understanding for the Distributive Property?
- Did the Algebra Tiles help you give meaning to 2(x+1)?
- Did the Algebra Tiles help you visualize multiplication of binomials?
- Did the Algebra Tiles help you visualize factoring trinomials?