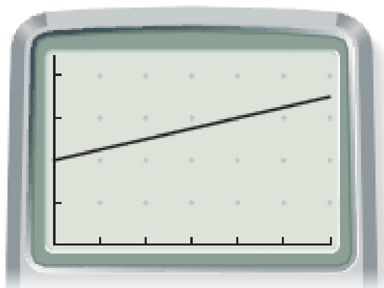
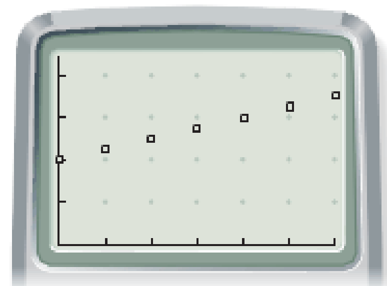
Solving Linear Equations in Algebra I

Inches	Centimeters
0	0
1	2.54
2	
	35.56
17	





Red Bank Regional High School

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Jim Rahn
www.jamesrahn.com
james.rahn@verizon.net

- What must all Algebra I students know about solving and working with Linear Equations?

  Students should

  - ✓ Understand the big idea of equivalence

    Numbers, expressions, functions, or equations have many different – but equivalent – forms. These forms differ in their efficacy and efficiency in interpreting or solving a problem, depending on the context.

    Algebra extends the properties of numbers to rules involving symbols to transform an expression, function, or equation into an equivalent form and substitute equivalent forms for each other.

    Solving problems algebraically typically involves transforming one equation to another equivalent equation until the solution becomes clear.

  - ✓ Understand the big idea of linearity

    The relationship between two quantities can often be represented graphically by a linear function.

    Linear functions can be used to show a relationship between two variables with a constant rate of change

    Linear functions can be used to show the relationship between two quantities that vary proportionately.

    Linear functions can also be used to model, describe, analyze, and compare sets of data.

    Understanding linear functions should be prominent in the Algebra I content.

  - ✓ Modeling real situations with variables
  - ✓ Use appropriate tools such as algebra tiles and graphing calculators, and spreadsheets regularly
  - ✓ Understand that geometric objects can be represented algebraically (lines can be described using coordinates)
  - ✓ Understand that algebraic expressions can be interpreted geometrically (systems of equations and inequalities can be solved graphically)

Building Understanding for Writing and Evaluating Expressions

Algebra should become a language through which we can describe various situations.

What is three plus five times two?

How would you have to write
4 + 3•2
so the answer is 14?

Let's try performing a string of operations to see what we get. On paper:

- Start with 6.
  Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.
- Start with 20.
  Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.
- Start with -4
  Multiply 2 times a starting number, then add 6, divide this result by 2, and then subtract your answer from 10.

Description	Expression

You Create a Trick

- Create your own trick that has at least 5 stages.
- Test it on your calculator with at least four different numbers to make sure all the answers are the same.
- When you think your trick works, test it on your other group members.

Analyzing a Number Trick

$$2\left(\frac{x+3}{2}+5\right)-x+8$$

- Write in words the number trick that is described above.
- Test the number trick to be sure you get the same result no matter what number you choose.
- Can you explain why this number trick work?

Daxun, Lacy, Claudia, and Al are working on a number trick. Here are the number sequences their number trick generates:

Description	Daxun's sequence	Lacy's sequence	Claudia's sequence	Al's sequence
Pick the starting number.	14	−5	−8.6	x
	19	0		
	76	0		
	64	−12		
	16	−3		
	2	2		

a.

Describe the stages of this number trick in the first column.
b. Complete Claudia's sequence.
c. Write a sequence of expressions for Al in the last column.

Solving an Equation

- Choose a secret number.
- Now choose four more non-zero numbers and in any random order
- add one of them,
- multiply by another,
- subtract another, and
- divide by the final number
- Record in words what you did and your final result on the communicator with a blank Building and Evaluating an Expression or Equation template. (Do not record your secret number.)
- Switch communicators and have another students find your secret number.

Building and Undoing an Expression or Equation

Description / Sequence	Expression	Undo	Result

Start with an Equation and Solve it

$$\frac{3+2(x-4)}{5}+6=11$$

Here is an equation. What is it saying? Use the Building and Undoing an Expression or Equation Template to first build the equation, then we'll solve I it.

Simplifying the Technique of Solving an Equation

Use the Undoing an Equation Template to simplify the steps in solving this equation:

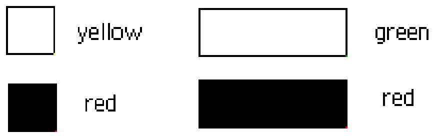
$$7+\frac{(x-3)}{4}=42$$

Undoing an Equation Template

Equation:		
Description	**Undo**	**Result**
Pick x		

Understanding Solving Equations by Balancing a Scale Part I

Each of the pieces of the algebra models represent an algebraic expression:



small yellow square - 1 unit tile
green rectangle - x tile
blue square - x^2 tile

small red square - negative 1 unit tile
red rectangle - negative x tile
red square - negative x^2 tile

Algebra tiles can be used to model solving equations.

Figure 1 illustrates a ***balanced*** scale. This is because 4 yellow square tiles balances with 4 square yellow square tiles. Build this scale in front of you.

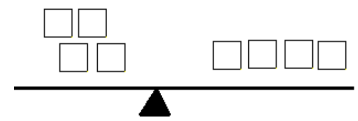


Figure 1

1. Let's discover some things we can do to balanced scale that will keep it balanced. Check off all changes that keep the scale in figure 1 balanced.

___What would happen if you added 2 yellow squares tiles to both sides of figure 1?

___What would happen if you added 1 red square tile to both sides of figure 1?

___What would happen if you added 1 red square to the left side and one yellow square tile to the right side of figure 1?

___What would happen if you added double the number of tiles on both sides of figure 1?

___What would happen if you removed one yellow square from the left side and added one red square to the right side of figure 1?

___What would happen if you cut the number of tiles in half on each side of figure 1?

___What would happen if you doubled the left side and divided the right side by 2 in figure 1?

___What would happen if you added one red square to the left side only in figure 1?

___What would happen if you added one yellow square to the right side only in figure 1?

___What would happen if you added red square to the left and removed one yellow square from the right in figure 1?

Look over your results to the 10 things you did to the balanced scale. In a few sentences or phrases describe what you can do to a balanced scale that keep the scale balanced.

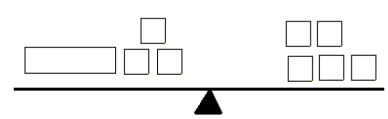


Figure 2

2. Figure 2 illustrates a **balanced** scale. Build this on your scale. How many red or yellow squares would the green rectangle be equal to?

Using one of the ideas from above, we can show that the green rectangle is equal to 2 yellow squares. Show at least two ways this can be accomplished.

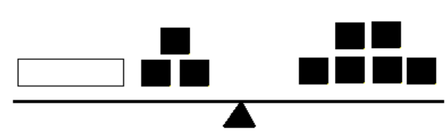


Figure 3

3. Set up figure 3 on your balance scale. Again use one of the ideas from question 1 to find at least two ways you can determine the value of the one green rectangle.

4. Set up figure 4 on your balance scale. Again use one of the ideas from question 1 to find at least two ways you can determine the value of the one green rectangle. Explain why your answer makes sense.

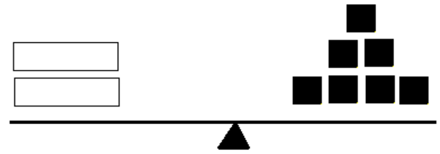


Figure 4

5. Set up figure 5 on your balance scale. Again use one of the ideas from question 1 to find at least two ways you can determine the value of the one green rectangle. Explain why your answer makes sense.

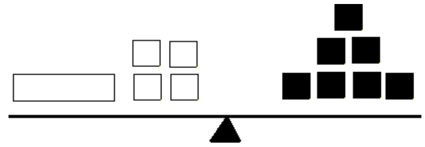


Figure 5

6.   Set up figure 6 on your balance scale. Again use one of the ideas from question 1 to find at least two ways you can determine the value of the one green rectangle. Explain why your answer makes sense.

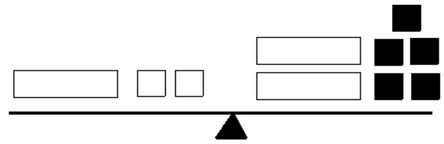


Figure 6

7. Set up figure 7 on your balance scale. Again use one or more of the ideas from question 1 to find at least two ways you can determine the value of one green rectangle. (Notice there are not green rectangles in this figure.) Explain why your answer makes sense.

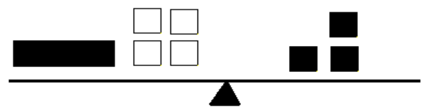


Figure 7

8. Set up figure 8 on your balance scale. Again use one or more of the ideas from question 1 to find at least two ways you can determine the value of one green rectangle. (Notice there are not green rectangles in this figure.) Explain why your answer makes sense.

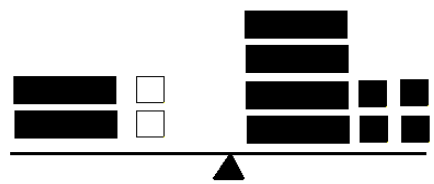


Figure 8

9   Set up figure 8 on your balance scale. Again use one or more of the ideas from question 1 to find at least two ways you can determine the value of one green rectangle. Explain why your answer makes sense.

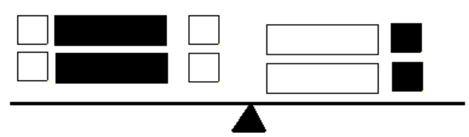


Figure 9

10. Set up figure 8 on your balance scale. Again use one or more of the ideas from question 1 to find at least two ways you can determine the value of one green rectangle. Explain why your answer makes sense.

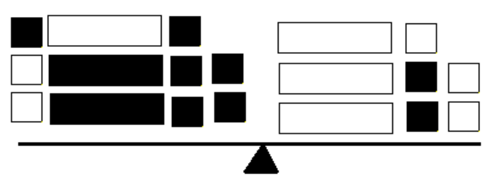
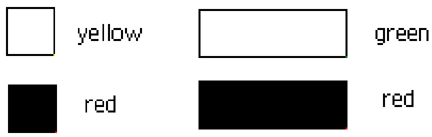


Figure 10

Understanding Solving Equations by Balancing a Scale Part II

Each of the pieces of the algebra models represent an algebraic expression:



small yellow square - 1 unit tile
green rectangle - x tile
blue square - x^2 tile

small red square - negative 1 unit tile
red rectangle - negative x tile
red square - negative x^2 tile

Algebra tiles can be used to model solving equations.

1. Recall that you can keep the ***balanced*** scale in Figure 1 balanced by performing certain steps. Check off those steps that produced a balanced scale.

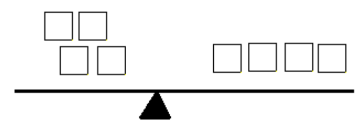


Figure 1

___Removing one yellow square from the left side and adding one red square to the right side of figure 1?

___Adding red square to the left side and removing one yellow square from the right side in figure 1?

___Adding 2 yellow squares tiles to both sides of figure 1?

___Adding 1 red square tile to both sides of figure 1?

___Adding 1 red square to the left side and one yellow square tile to the right side of figure 1?

___Adding double the number of tiles on both sides of figure 1?

___Adding one yellow square to the right side only in figure 1?

___Adding one red square to the left side only in figure 1?

___Cutting the number of tiles in half on each side of figure 1?

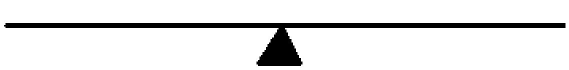
___Doubling the left side and dividing the right side by 2 in figure 1?

Set up each equation on the balance scale. Draw a sketch of your balance scale. Then use one of the techniques you have learned that keeps the scale balanced to find the value of x or the green rectangle. Check your answer to make sure it makes sense.

2. x + -3 = -4

X = _____

3. 2x + -3 = 5



X = _____

4. $^{-}2x + {}^{-}3 = {}^{-}4 + -1x$

X = _____

5. $^{-}3 + x = 2x + 1$

X = _____

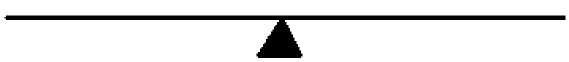
6. $^{-}4 = 2(x + 2)$

X = _____

7. $x + 4 = {}^{-}2x + {}^{-}2$

X = _____

8. $3x + {}^{-}3 = 2(x + 1)$



X = _____

Understanding Solving Equations by Balancing a Scale Part III

Each of the pieces of the algebra models represent an algebraic expression:

small yellow square - 1 unit tile
green rectangle - x tile
blue square - x^2 tile

small red square - negative 1 unit tile
red rectangle - negative x tile
red square - negative x^2 tile

Algebra tiles can be used to model solving equations.

1. Recall that you can keep the ***balanced*** scale in Figure 1 balanced by performing certain steps. Describe all the ways a balanced scale can be kept balanced.

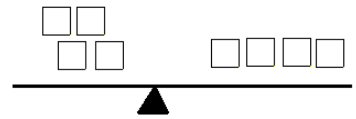


Figure 1

Instead of using the scale this time we will perform all the steps algebraically by working with the equation.

For example, if the equation was 1 + 2x + 3 = 7 you would have built the balance scale in figure 2.
One step you might do first is combine the like terms

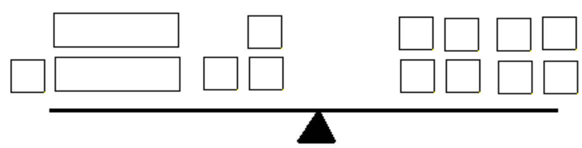


Figure 2

This would result in figure 3. Figure 3 says that 2x+4 = 8.
Now you might think about remove 4 yellow squares from both sides.

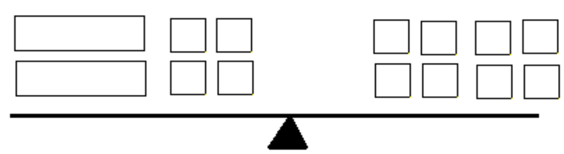


Figure 3

This would leave you with figure 4. Figure 4 says that 2x = 4.
Then you would have divided both sides into two equal groups so the green rectangle equals 2 yellow squares or x = 2.

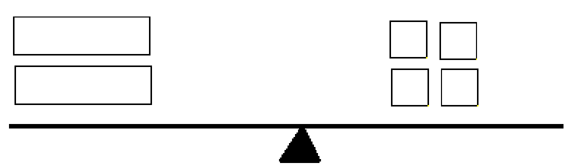


Figure 4

This time our steps will be more algebraic, but based upon what we did with the balance scale.

$1 + 2x + 3 = 8 \Leftrightarrow$ combine like terms
$2x + 4 = 8 \Leftrightarrow$ remove 4 from both sides
$2x = 4 \Leftrightarrow$ divide both sides into 2 equal groups
x = 2

Think about what each of these equations looks like on a balance scale. Solve each equation by using one of the steps that produces a balanced scale.

2. $2x + 3 + {}^{-}1x = 5$ X = _____

3. $3x + 4 + 1x = 8$ X = _____

4. $9 = 1x + {}^{-}3 + 3x$ X = _____

5. ${}^{-}3 = 3x + 2 + 2x$ X = _____

6. $2x + {}^{-}4 + 3x = {}^{-}2 + 3x + 3$ X = _____

7. $x + 4 = {}^{-}2x + {}^{-}2$ X = _____

8. $6 + {}^{-}5x + {}^{-}4 = 6 + {}^{-}2x + 2$ X = _____