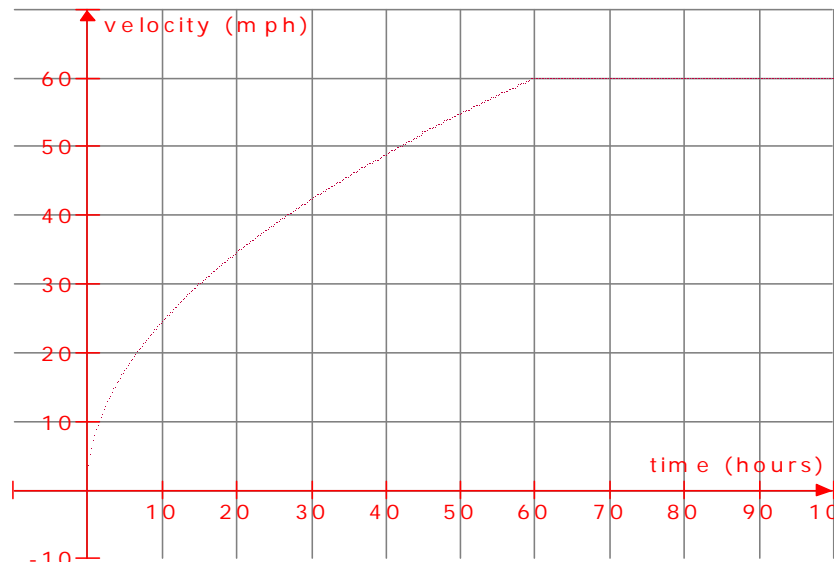


Supporting Our Calculus Program through Reasoning and Making Sense



Red Bank Regional High School

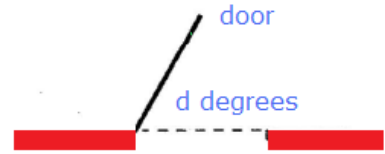
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Introducing Instantaneous Rate of Change

The diagram shows a door with an automatic closer.

At time $t = 0$ seconds someone pushes the door. It swings open, slows down, stops, starts closing, then slams shut at time $t = 7$ seconds. As the door is in motion the number of degrees, d , it is from its closed position depends on t .



Sketch a reasonable graph of d versus t .

Compare your graph with a graph drawn by a neighbor.

What characteristics are possessed by both graphs.

How do your graphs differ?

Suppose that d is given by the equation $d = 200t(2-t)$. Use your calculator to graph this function for $0 \leq t \leq 7$.

Does your range make sense?

What common characteristics does this graph have to your graph?

Make a sketch of your graph.

Make a table of values of d for each second from $t = 0$ through $t = 10$. Round to the nearest 0.10 degree.

Looking at only the chart, does the door appear to be opening, closing, or standing still at $t = 1$ second? Explain your reasoning.

Using the graph, does the door appear to be opening, closing, or standing still at $t = 1$ second? Explain your reasoning.

During the time interval $[0, 1]$ seconds, what is the average rate of change of the door's angle?

Time	Degrees
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Create a table of values for the angle of the door's opening over the interval $[0, 1]$ seconds with steps of 0.1, what is the average rate of change of the angle of door.

Time	Degrees
0	
0.1	
0.2	
0.3	
0.4	
0.5	
0.6	
0.7	
0.8	
0.9	
1.0	

During the time interval $[0.9, 1]$ seconds, what is the average rate of change of the door's angle?

How does this average rate of change compare with the average rate of change on the interval $[0,1]$? Does this make sense?

Are you surprised by the amount of difference? Explain your reasoning.

Create a table of values for the angle of the door's opening over the interval $[0.9, 1]$ seconds with steps of 0.01, what is the average rate of change of the angle of door.

Time	Degrees
0.9	
0.91	
0.92	
0.93	
0.94	
0.95	
0.96	
0.97	
0.98	
0.99	
1.0	

During the time interval $[0.99, 1]$ seconds, what is the average rate of change of the door's angle?

How does this average rate of change compare with the average rate of change on the interval $[0.9,1]$? Does this make sense?

Are you surprised by the amount of difference? Explain your reasoning.

Can you predict the rate of change of the opening of the door's angle at time $t = 1$?

How could you get a better approximation?

Continue until you can predict the door's rate of change of the angle to a hundredths of a second.

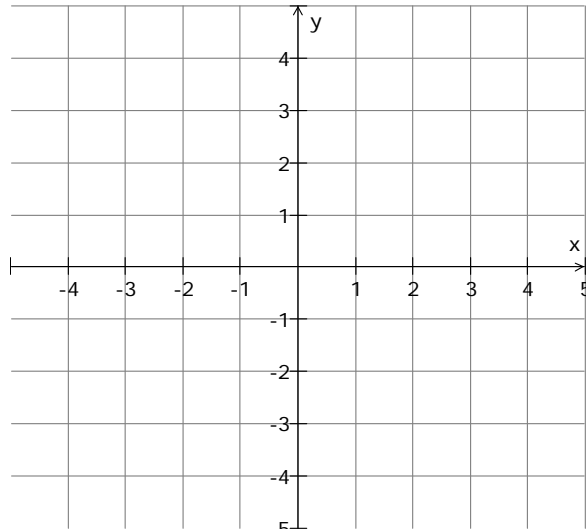
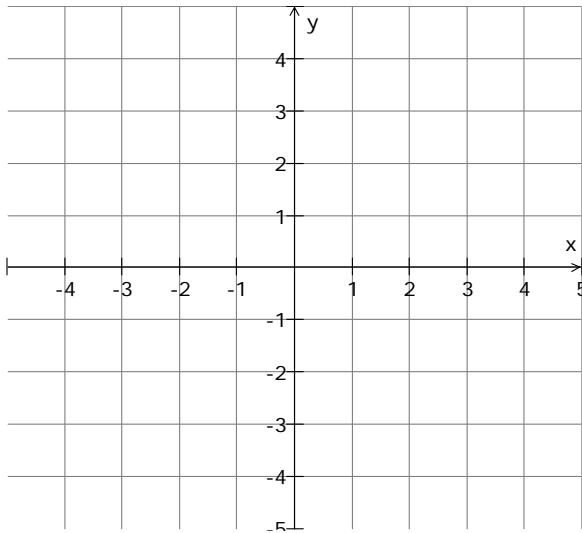
Describe in your own words how you can find the instantaneous rate of change of the door at exactly time 1 second.

Graphs of Functions

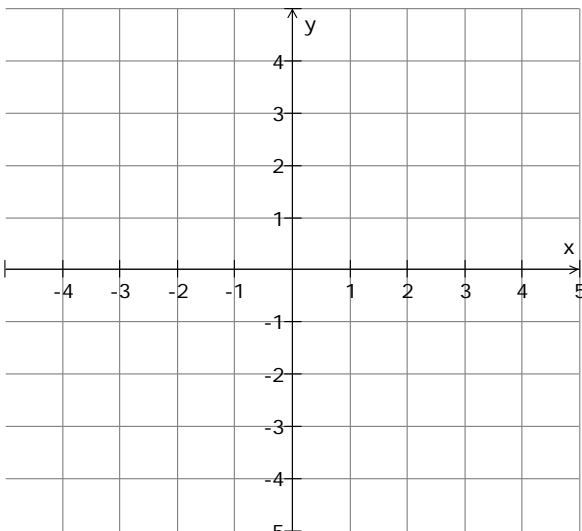
For each of the following functions:

- Draw a sketch of the each graph without using the graphing calculator. Check your answer by using the graphing calculator.
- Create a set of table values for each function in the neighborhood of 1: $0.5 \leq x \leq 1.5$
- Describe whether the function is increasing, decreasing, or not changing when $x = 1$.
- If the function is increasing or decreasing at $x = 1$, use the graph and/or table values to decide if the function is increasing or decreasing at a rate that is slow or fast.

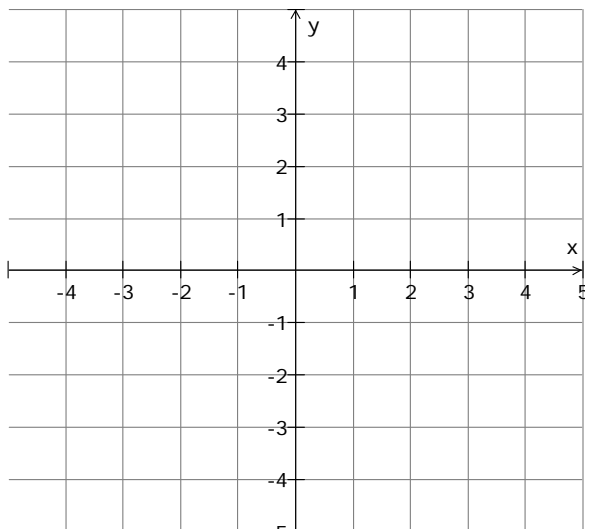
1. $y = 2^{-x}$



2. $y = 2 \sin\left(\frac{\pi}{2}x\right)$

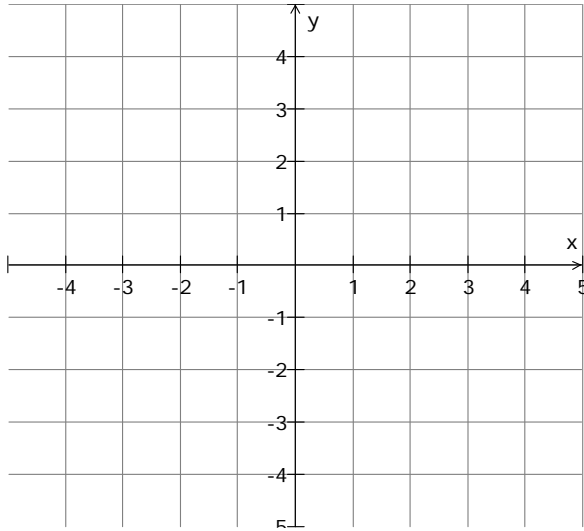


4. $y = \sin(2\pi x)$



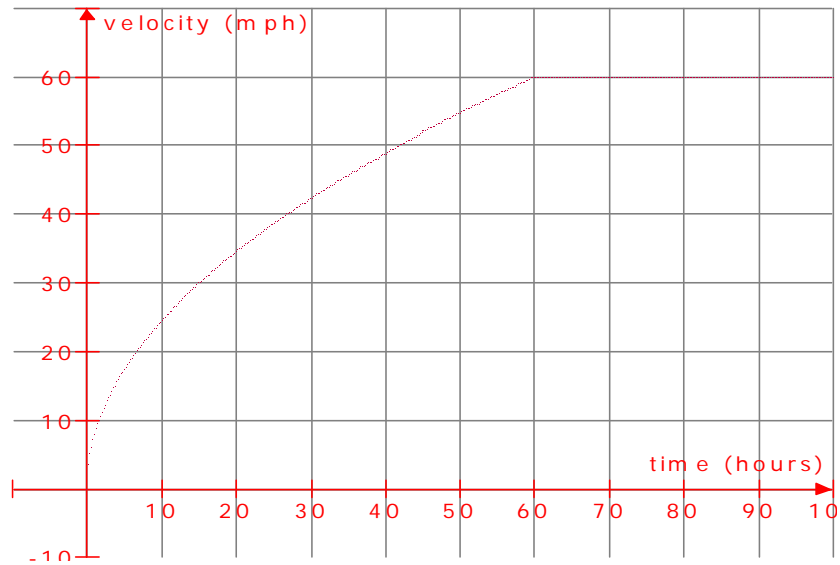
3. $y = \frac{1}{2}x^2 + 1\frac{1}{2}$

5. $y = \left| \frac{1}{x} \right|$



Learning to Work with Area

As you pull out on the highway you gradually increase your speed according the graph below. Then you notice your speedometer approaching 60 miles an hour so you tap the break and bring your car under control and travel at a constant rate of 60 miles an hour.



How far do you travel between time $t=60$ minutes and $t=100$ minutes?

Explain how you found this answer. How is this distance represented in the graph? Shade this region in on the graph.

How many feet are represented by one shaded in square. Explain your reasoning.

If a square was shaded in as illustrated at the right, to the nearest 0.1, what portion of the square is shaded?



How many feet would be represented by this shaded in portion?

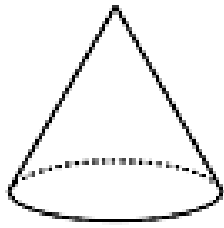
Find the distance traveled from $t=0$ to $t = 60$ minutes

Find out how far your car traveled in the first 100 minutes of the trip.

The distance traveled by your car is represented by the bounded area under the velocity graph and above the time axis. This area is called the **definite integral of the velocity** from time $t=0$ to $t= 100$ minutes.

You have just found a geometric method to calculate a definite integral of the velocity from

t=0 to t= 60 minutes.



A right circular cone is placed on its circular base. The cone is sliced parallel to the base, at points x distance from the base, to create cross sections that are circles. The area of each circular cross section is graphed at the right as a function of its distance from the

vertex. Find the volume of the right circular cone by determining the area bounded by the graph and the x axis.

What is the largest area of a cross sectional area?

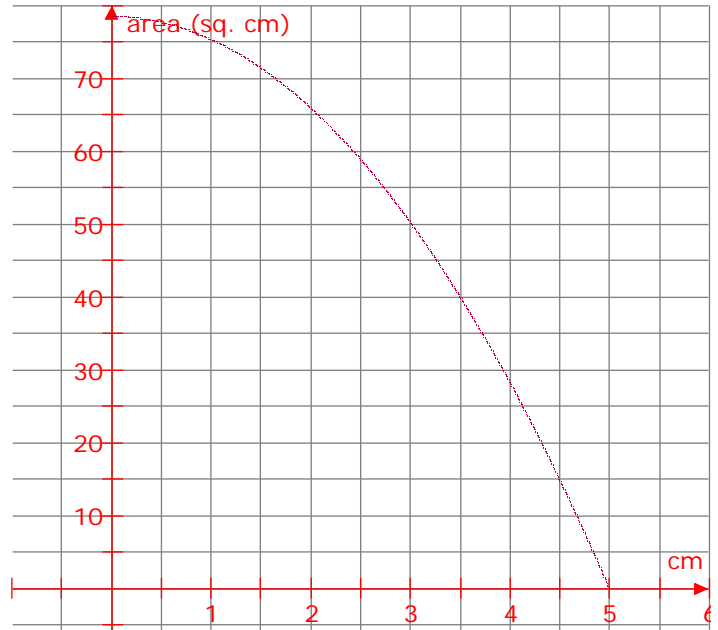
What is the area of the circle created 2 cm from the base?

At what distance from the base was a 40 square centimeter circle cut?

What are the units for the definite integral of the area with respect to x in this problem?

What does the definite integral represent in this problem?

Estimate the definite integral of y , the area, with respect to x for $0 \leq x \leq 5$. Show work that leads to your answer.



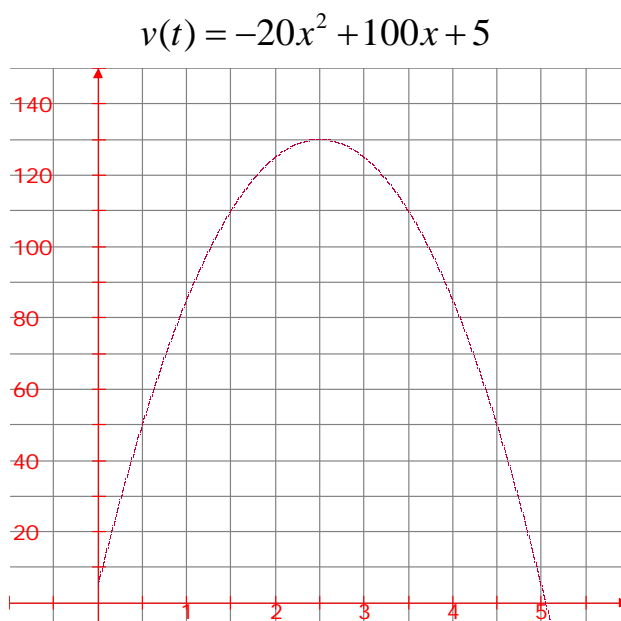
Finding Definite Integrals Another Way

The velocity of a particle is represented by the graph at the right. You are interested in determining how far the particle has traveled during the time interval $0 \leq t \leq 4$.

Subdivide the interval $0 \leq t \leq 4$ into 4 equal widths. Draw trapezoids in each interval by connecting two consecutive function values. Find the height of the velocity graph at the endpoints of each interval. Record them in the given table.

Find an approximation for the definite integral of $v(t)$ with respect to t by finding the sum of the area of the four trapezoids.

What does this definite integral represent?



x	0	1	2	3	4
$v(t)$					

Increase the number of intervals to 8. Find a second approximation for the definite integral of $v(t)$ with respect to t by finding the sum of the area of the eight trapezoids.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$v(t)$									

What do you notice about your two answers for the definite integral of this velocity from $0 \leq t \leq 4$.

How could you improve your answer for the definite integral of the velocity from $0 \leq t \leq 4$?

Can you approximate the particle's rate of change of velocity when $t=5$? Was the particle speeding up or slowing down at that time. Explain your reasoning.

At what time is the particle stopped. Explain your reasoning.

Introduction to Limits

Create a graph of $y = \frac{-3x^2 + 8x - 5}{x - 1}$.

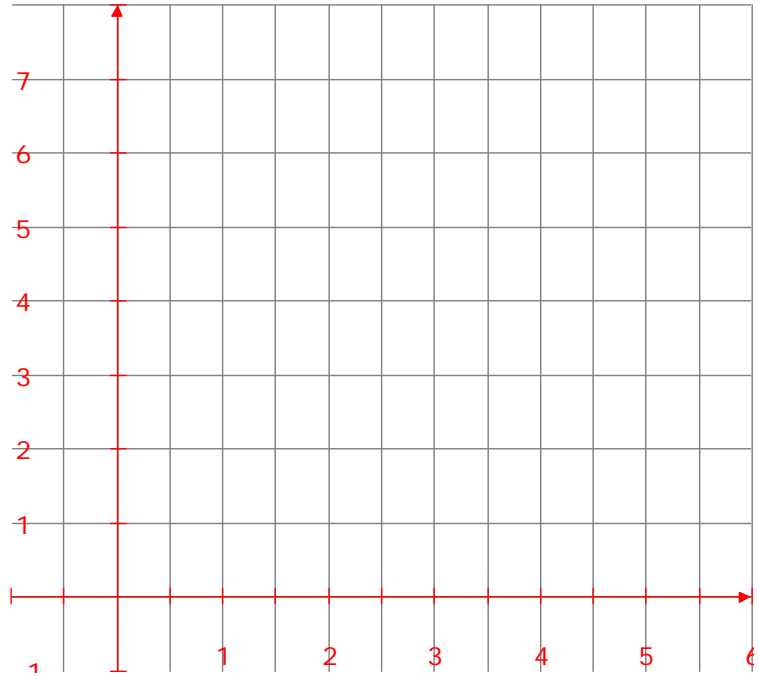
Did you expect this graph? Why or why not? Set your window so $0 \leq x \leq 4.7$. Did this change your graph?

Insert $x = 1$ into the equation. What is the value of y ? Explain the value you see.

Is this graph just jumping over one point or many points?

Use the trace key and explain what you are seeing.

Build a set of table values in the neighborhood around 1.



x	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
y											

Are the table values jumping over just one value or many values?

When x between 0.9 and 1.1, between what two values do the y values stay between?

Let's move in a little closer on 1. Build a set of table values between $0.96 \leq x \leq 1.04$.

x	0.96	0.97	0.98	0.99	1.0	1.01	1.02	1.03	1.04
y									

When $0.96 \leq x \leq 1.04$, between what two values do all the y values stay?

Let's move in a little closer on 1. Build a set of table values between $0.996 \leq x \leq 1.004$.

x	0.996	0.997	0.998	0.999	1.0	1.001	1.002	1.003	1.004
y									

When $0.996 \leq x \leq 1.004$, between what two values do all the y values stay?

What value do you believe the function is jumping over when you are close to $x = 1$?

We call this the limit of the function $y = \frac{-3x^2 + 8x - 5}{x - 1}$ when x is approaching 1.

Study your last set of table values. Try to complete this statement:

"If x is within _____ units of 1 (but not equal to 1), then $f(x)$ is within 0.001 unit of 2,"