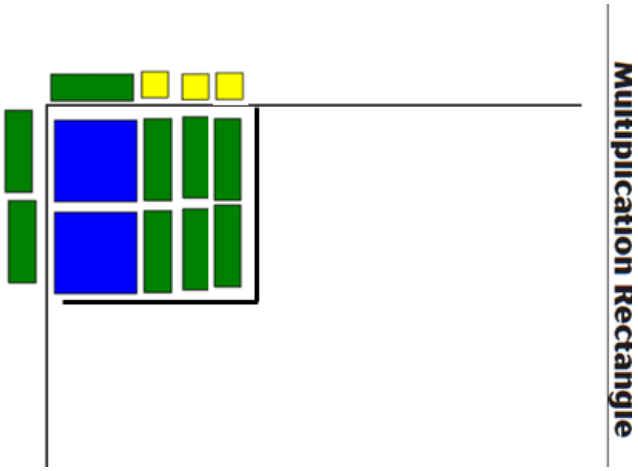
Using Algebra Tiles to Provide a Concrete Model for Factoring



Neptune Public Schools
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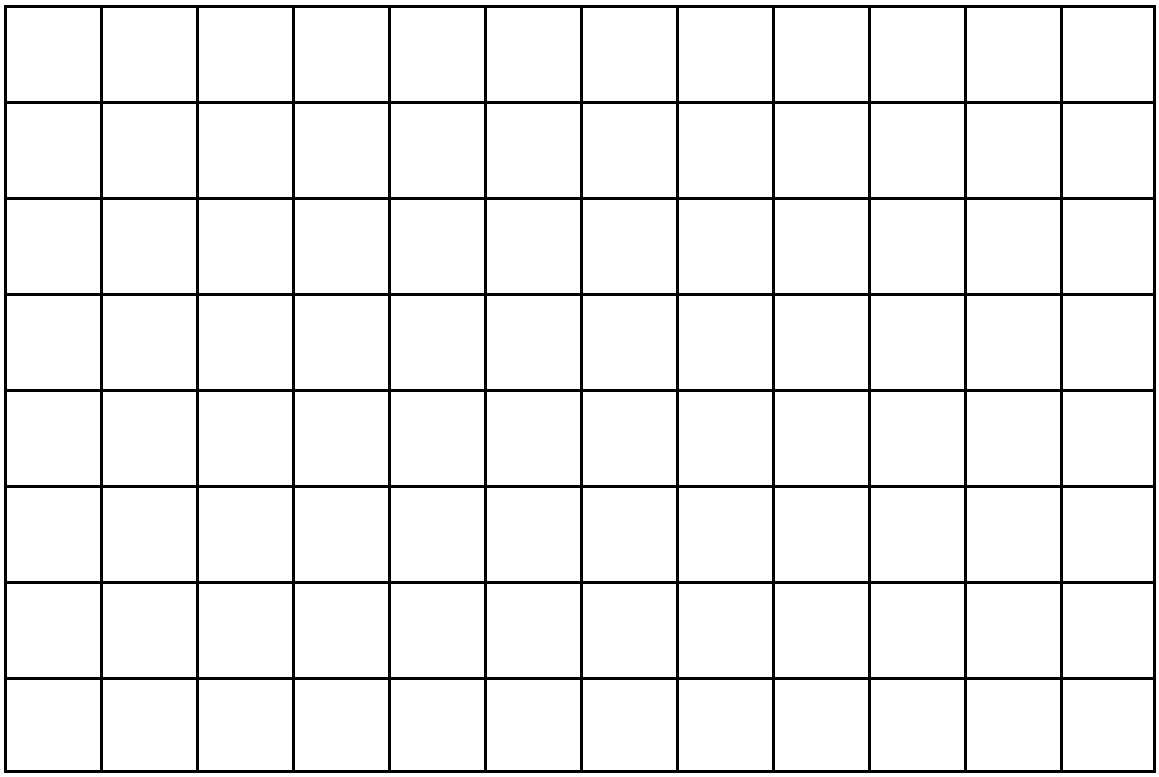
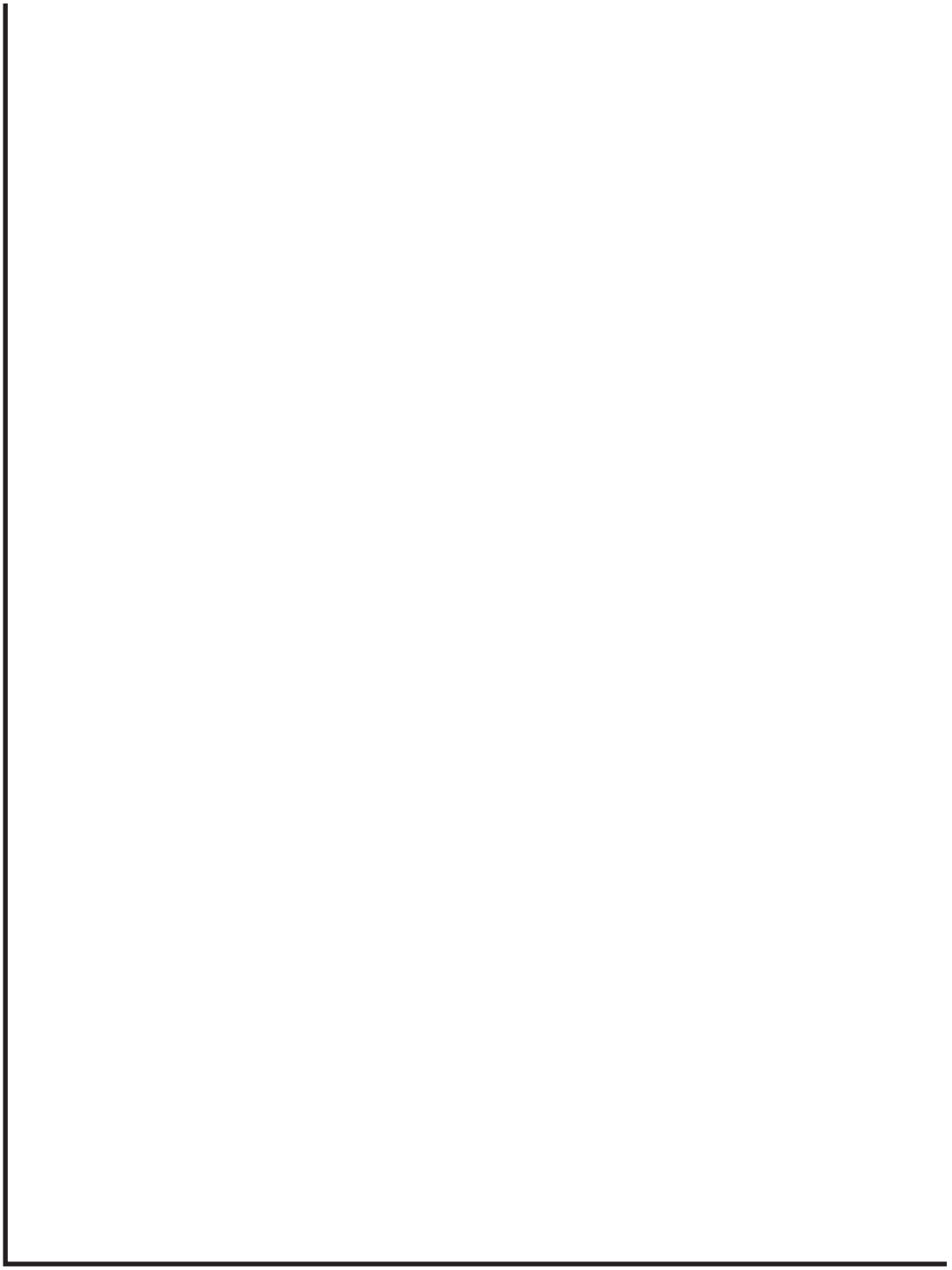
Jim Rahn
LL Teach, Inc.
www.llteach.com
www.jamesrahn.com

Distributive Property Template

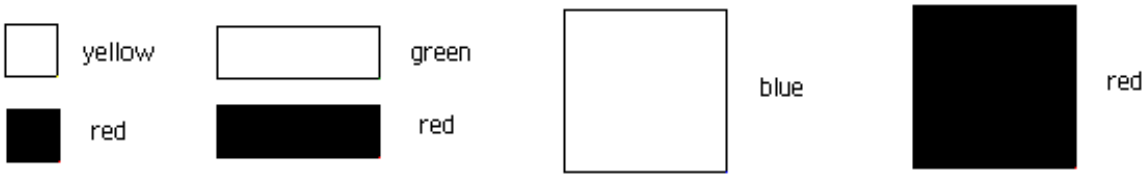


Algebra Table

Multiplication Rectangle



Multiplying with Variables
Part I



Each of the pieces of the algebra models represent an algebraic expression:

small yellow square - 1 unit tile
green rectangle - x tile
aqua square - x^2 tile

small red square - negative 1 unit tile
red rectangle - negative x tile
red square - negative x^2 tile

1. To complete 2(x+1) show two groups of (x + 1). Form a rectangle with the pieces on the table. How long is your rectangle? How wide is your rectangle? Notice how the length and width of the rectangle are part of 2(x+1). What is the algebraic name for the inside of your rectangle? Draw a picture of your rectangle:

2. To complete $3(x+{}^{-}2)$ show three groups of $(x+{}^{-}2)$. Form a rectangle with the pieces on the table. How long is your rectangle? How wide is your rectangle? What is the algebraic name for the inside of your rectangle? Draw a picture of your rectangle:

3. To complete $2(x^2+x)$, what should you show on the table?

4. To complete $3(x^2+x+1)$, what should you show on the table?

5. To complete $3(x^2 + {}^-2x + 2)$, what should you show?

6. To complete $4(2x^2 + x + {}^-1)$, what should you show?

7. Without using algebra tiles, what is the algebraic name for the following products?

$4(2x + {}^-5) =$

$2(3x + 8) =$

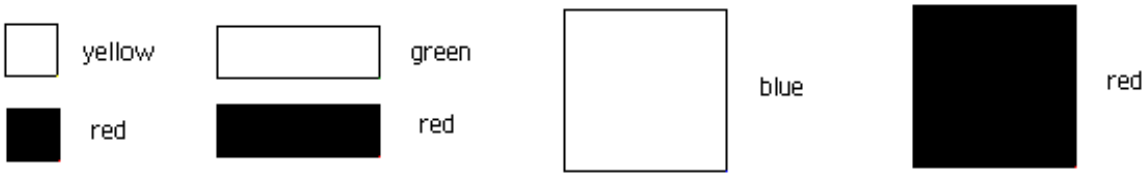
$5(3x + {}^-2) =$

$4(x^2 + {}^-3x + 4) =$

$6(2x^2 + {}^-5x + {}^-3) =$

$7({}^-4 + 8x + {}^-2x^2) =$

Multiplying with Variables
Part II



Each of the pieces of the algebra models represent an algebraic expression:

small yellow square - 1 unit tile
green rectangle - x tile
aqua square - x^2 tile

small red square - negative 1 unit tile
red rectangle - negative x tile
red square - negative x^2 tile

1. Use the multiplication rectangle to make a rectangle whose dimensions are x by 2x+ 1. Place an x tile on the left side and tiles that represent 2x + 1 across the top as illustrated at the right. Fill in the rectangle to show its area. Draw a picture of your rectangle at the right. What is the algebraic name for the inside of your rectangle?

2. Use the multiplication rectangle to multiply $x(x+2)$. Fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

3. Use the multiplication rectangle to multiply $2x(2x+3)$. Fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

4. Use the multiplication rectangle to multiply $(x+1)(2x+1)$. Fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

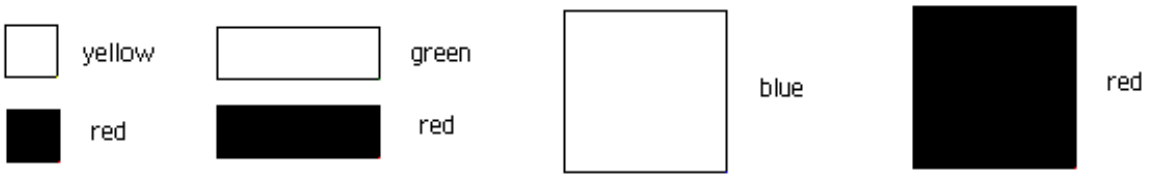
5. Use the multiplication rectangle to multiply $(2x+1)(x+3)$. Once you have set up the dimensions fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

6. Use the multiplication rectangle to multiply $(x+2)(2x+3)$. Once you have set up the dimensions fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

7. Study the picture from problems 4-6. Is there a way you can predict how many x rectangles will be in your final rectangle? Can you predict their location? Can you predict the number of blue unit squares that will be in your final rectangle? Can you predict their location? Try to predict what the rectangle will look like for $(2x+3)(x+4)$. Draw the picture without using the tiles. Write the algebraic expression for the rectangle.

8. Draw the picture for the multiplication of $(3x+1)(x+2)$. Write the algebraic expression for the rectangle.

Multiplying with Variables using Algebra Tiles
Part III



Each of the pieces of the algebra models represent an algebraic expression:

small yellow square - 1 unit tile
green rectangle - x tile
aqua square - x^2 tile

small red square - negative 1 unit tile
red rectangle - negative x tile
red square - negative x^2 tile

1. Use the multiplication rectangle to multiply $(x+1)(x+2)$. Once you have set up the dimensions fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

2. Use the multiplication rectangle to multiply $(x+{}^{-}1)(x+2)$. Once you have set up the dimensions fill in the rectangle. Watch the colors of the tiles. This problem involves a negative sign. Draw a picture of your rectangle. Can you simplify the rectangle by using zero pairs? What is the algebraic name for the inside of your rectangle?

3. Use the multiplication rectangle to multiply $(x+1)(2x+{}^{-}3)$. Once you have set up the dimensions fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

4. Use the multiplication rectangle to multiply $(x+{}^{-}1)\left(2x+{}^{-}1\right)$. Once you have set up the dimensions fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

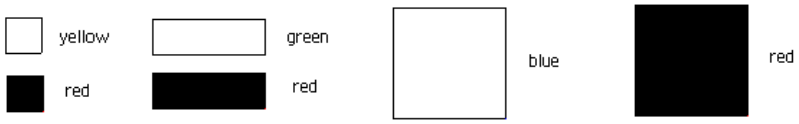
5. Use the multiplication rectangle to multiply $(2x+{}^{-}1)(x+3)$. Once you have set up the dimensions fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

6. Use the multiplication rectangle to multiply $(x+{}^{-}2)(2x+3)$. Once you have set up the dimensions fill in the rectangle. Draw a picture of your rectangle. What is the algebraic name for the inside of your rectangle?

7. Study the picture from problems 4-6. Is there a way you can predict how many x rectangles will be in your final rectangle? Can you predict their location? Can you predict the number of blue unit squares that will be in your final rectangle? Can you predict their location? Try to predict what the rectangle will look like for $\left(2x+{}^{-}3\right)\left(x+4\right)$. Draw the picture without using the tiles. Write the algebraic expression for the rectangle.

8. Draw the picture for the multiplication of $\left(3x+{}^{-}1\right)\left(x+2\right)$. Write the algebraic expression for the rectangle.

Factoring Polynomials - First Lesson



Color Key for Algebra Tiles

First let's review how we multiply.

1. Multiply (x+2)(x+3) using the multiplication rectangle. Represent x+2 along the top and x+3 along the left. Make a rectangle whose dimensions are equal to these two binomials. Draw a picture of your rectangle at the right.

Notice where the x square piece is located. Notice where the x pieces are located. Notice where the unit pieces are located.

2. Try to draw the picture of the multiplication rectangle for (x+4)(x+1) without using the tiles.

3. Draw the picture of the multiplication rectangle for (2x+1)(x+3) without using the tiles.

4. Describe the pieces you would use to build the multiplication rectangle for (x+2)(2x+3) and where they would be located in the multiplication rectangle. Try to do this without building the rectangle. (Use the rectangle only if you have having difficulty picturing the rectangle.

Now let's try to reverse the process.

5. Use the following pieces: one x^2 piece, four x pieces, and three unit pieces. Form a rectangle from these eight pieces. Draw a picture of your multiplication rectangle at the right.

What polynomial is represented by the rectangle?

Describe the polynomial represented by these eight pieces.

Describe the dimensions of your rectangle.

6. Use the following pieces: one x^2 piece, four x pieces, and four unit pieces. Form a rectangle from these nine pieces. Draw a picture of your multiplication rectangle at the right.

What polynomial is represented by the rectangle?

Describe the polynomial represented by these nine pieces.

Describe the dimensions of your rectangle.

7. Use the following pieces: one x^2 piece, five x pieces, and six unit pieces. Form a rectangle from these twelve pieces. Draw a picture of your multiplication rectangle at the right.

What polynomial is represented by the rectangle?

Describe the polynomial represented by these twelve pieces.

Describe the dimensions of your rectangle.

8. Use the following pieces: one x^2 piece, seven x pieces, and six unit pieces. Form a rectangle from these fourteen pieces. Draw a picture of your multiplication rectangle at the right.

What polynomial is represented by the rectangle?

Describe the polynomial represented by these fourteen pieces.

Describe the dimensions of your rectangle.

9. Use the following pieces: one x^2 piece, nine x pieces, and eight unit pieces. Form a rectangle from these eighteen pieces. Draw a picture of your multiplication rectangle at the right.

What polynomial is represented by the rectangle?

Describe the polynomial represented by these eighteen pieces.

Describe the dimensions of your rectangle.

10. Use the following pieces: one x^2 piece, six x pieces, and eight unit pieces. Form a rectangle from these fifteen pieces. Draw a picture of your multiplication rectangle at the right.

What polynomial is represented by the rectangle?

Describe the polynomial represented by these fifteen pieces.

Describe the dimensions of your rectangle.

Which piece helped you most to create the rectangle?

Describe how this number of pieces helped you know how to form your rectangle?

Now try something a little harder:

11. Use the following pieces: two x^2 piece, seven x pieces, and three unit pieces. Form a rectangle from these twelve pieces. Draw a picture of your multiplication rectangle at the right.

What polynomial is represented by the rectangle?

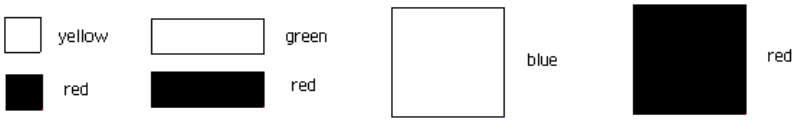
Describe the polynomial represented by these twelve pieces.

Describe the dimensions of your rectangle.

Summarizing Your Ideas

Suppose you want to determine the two factors whose product is $2x^2+11x+9$. Describe how you think about the arrangement of the twenty-two tiles so it will make a rectangle.

Factoring Polynomials - Second Lesson



Color Key for Algebra Tiles

In the first lesson you learned to use the algebra tiles to factor a general polynomial of the form ax^2+bx+c .

1. Find the factors for $1x^2+5x+4$. Draw the rectangle in the space at the right.

What do you notice about the 8 unit tiles?

What do you notice about the 6x tiles?

What are the factors of $1x^2+5x+4$?

2. Find the factors for $1x^2+4x+4$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

What are the factors of $1x^2+4x+4$?

3. Find the factors for $1x^2+5x+6$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

What are the factors of $1x^2+5x+6$?

4. Find the factors for $1x^2 + 7x + 6$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

What are the factors of $1x^2 + 7x + 6$?

5. Find the factors for $1x^2 + 7x + 12$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

What are the factors of $1x^2 + 7x + 12$?

6. Find the factors for $1x^2 + 8x + 12$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

What are the factors of $1x^2 + 8x + 12$?

7. Find the factors for $1x^2 + 13x + 12$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

What are the factors of $1x^2 + 13x + 12$?

8. Find the factors for $2x^2+3x+1$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

What are the factors of $2x^2+3x+1$?

9. Find the factors for $3x^2+4x+1$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

What are the factors of $3x^2+4x+1$?

10. Find the factors for $4x^2+4x+1$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

What are the factors of $4x^2+4x+1$?

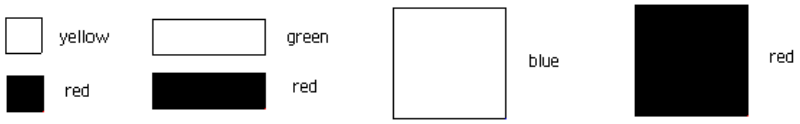
Which piece helped you most to create the rectangle?

Describe how this number of pieces helped you know how to form your rectangle?

Summarizing Your Ideas

Suppose you want to determine the two factors whose product is $5x^2+6x+1$. Describe how you think about the arrangement of the twelve tiles so it will make a rectangle.

Factoring Polynomials - Third Lesson



Color Key for Algebra Tiles

In the first two lessons you learned to use the algebra tiles to factor a general polynomial of the form $ax^2 + bx + c$, but now let's use negative tiles

1. Find the factors for $1x^2 + 4x + 3$. Draw the rectangle in the space at the right.

What do you notice about the 8 unit tiles?

What do you notice about the 6x tiles?

2. Find the factors for $1x^2 - 4x + 3$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

3. Find the factors for $1x^2 - 5x + 6$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

4. Find the factors for $1x^2 - 7x + 6$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

5. Find the factors for $1x^2 - 7x + 12$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

6. Find the factors for $1x^2 - 6x + 12$. Draw the rectangle in the space at the right.

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

Summarize what you have learned so far

Describe what you have noticed from the last 6 exercises.

7. Find the factors for $1x^2 - 2x - 3$. Draw the rectangle in the space at the right.
Check the signs for all tiles. Could you make a rectangle with the six pieces? Remember you can add more tiles to the rectangle by adding zero pairs. How many zero pairs do you want to add?

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

8. Find the factors for $1x^2 + 2x - 3$. Draw the rectangle in the space at the right. Check the signs for all tiles. Could you make a rectangle with the six pieces? Remember you can add more tiles to the rectangle by adding zero pairs. How many zero pairs do you want to add?

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

10. Find the factors for $1x^2 + 4x - 5$. Draw the rectangle in the space at the right. Check the signs for all tiles. Could you make a rectangle with the six pieces? Remember you can add more tiles to the rectangle by adding zero pairs. How many zero pairs do you want to add?

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

11. Find the factors for $1x^2 - 4x - 5$. Draw the rectangle in the space at the right. Check the signs for all tiles. Could you make a rectangle with the six pieces? Remember you can add more tiles to the rectangle by adding zero pairs. How many zero pairs do you want to add?

How is this rectangle the same as the previous rectangle?

How is this rectangle different from the previous rectangle?

Summarizing Your Ideas

Suppose you want to determine the two factors whose product is $x^2 + 8x - 9$. Describe how you think about the arrangement of the eighteen tiles so it will make a rectangle.

james.rahn@verizon.net