

Algebra Workshop

Algebra Tiles Part II

Developing an Understanding for Recursive Sequences and the Underpinnings of Slope

Learning to Write Mathematical Phrases and Equations

Palmyra High School

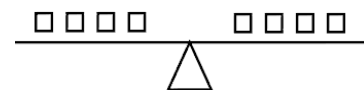
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Thinking about Keeping a Scale Balanced

- For each question start with the balance scale set up as illustrated at the right.
- Then perform the action described.
- Decide if the action creates a balanced scale or unbalanced scale. If the scale is unbalanced show which way the balance will tilt.



Action	Balanced or Unbalanced	Picture of Balance
1. Three red squares are added to the right side.	1.	1.
2. One yellow and one red square are added to the right side.	2.	2.
3. One yellow squares is removed from the left side and one yellow square is removed from the right side.	3.	3.
4. Two red squares are added to the right side of the scale and two yellow squares are added to the left side.	4.	4.
5. Multiply the number of items on each side by two.	5.	5.
6. Two red squares and two yellow squares are added to the left side of the scale.	6.	6.
7. A red square is added to the left and a yellow square is removed from the right.	7.	7.
8. The number of items on each side is cut in half.	8.	8.
9. Two yellow squares are removed from the left and two yellow squares are added to the right side of the scale.	9.	9.
10. Two red squares are removed from the left and two red squares are removed from the right side of the scale.	10.	10.
11. One zero pair is added to the left side and one zero pair is added to the right side of the scale.	11.	11.
12. Two yellow squares are added to the right side and two red squares are added to the left side of the scale.	12.	12.
13. One red square is added to each side of the scale.	13.	13.
14. Double the number of squares on the left and divide the number of squares on the right by two.	14.	14.

Summarize the steps or actions that lead to a balanced scale.

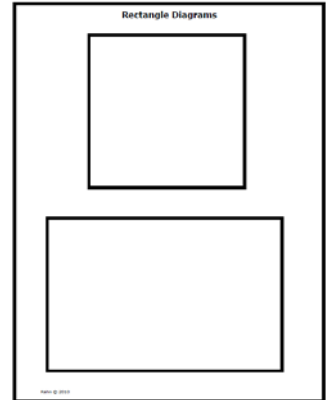
Completing the Square with The Area Model

Numbers like 64 are called perfect squares because they are the squares of integers, in this case 8 or -8. The trinomial $x^2 + 6x + 9$ is $(x+3)^2$. So it is also called a perfect square.

Identify which of these trinomials are perfect squares?

- A. $x^2 + 16x + 64$ B. $x^2 - 18x + 81$
 C. $x^2 + 12x + 16$ D. $x^2 - 12x + 36$

Use the Rectangle Diagrams Template to show the area model for each of the trinomials that are perfect squares.



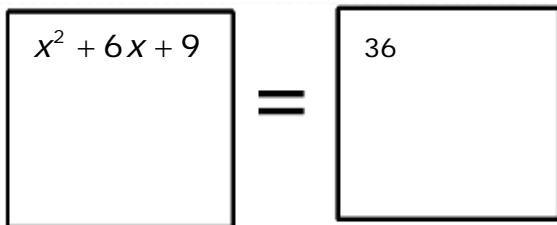
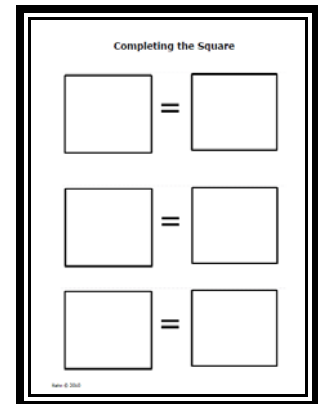
Study each of the following trinomials. Draw the model for each trinomial. If a number is missing, use your model to figure out the missing number.

- A. $x^2 + 1x + 1$ B. $x^2 - 6x + 9$ C. $x^2 + \underline{\hspace{1cm}} x + 36$
 D. $x^2 - \underline{\hspace{1cm}} x + 121$ E. $x^2 + 14x + \underline{\hspace{1cm}}$ F. $x^2 - 10x + \underline{\hspace{1cm}}$

What is the connection between the middle term and the last term that makes it possible to form a square?

Trinomials can also be placed in an equation such as

$x^2 + 6x + 9 = 36$. Use the Completing the Square Template to show this equivalence. Represent the trinomial in the square on the left and the constant in the square on the right. If the two squares have the same area then their dimensions must be equal also. Find the dimensions of each square. Don't forget both the negative and positive values.



$$\begin{aligned} (x + 3)^2 &= \pm 6 \\ x + 3 &= \pm 6 \\ x &= \pm 6 - 3 \\ x &= +3 \text{ or } -9 \end{aligned}$$

Solve each of the following equations using the Completing the Square Template.

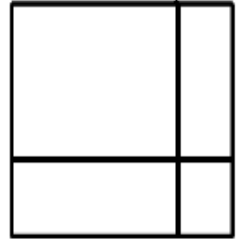
A. $x^2 + 12x + 16 = 25$

B. $x^2 - 12x + 36 = 25$

C. $x^2 + 18x + 81 = 100$

D. $x^2 - 14x + 49 = 64$

Study the square on the left in each of the problems. Describe any patterns you notice in the area of each section and the coefficients from the trinomial.

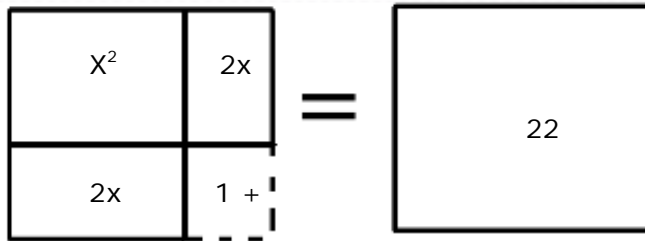


Suppose the trinomial was $x^2 + 16x + 64$. Label this square with the appropriate area and dimensions.

What if the trinomial is not a perfect square? Can a problem like this still be solved?

Suppose $x^2 + 4x + 1 = 22$.

- Begin to set up the two squares, but notice on the left that the area of the small square is too small.
- How much should it be?
- How much can we add to both sides of this equation?
- Since this is an equation, add the amount of area you need to both sides. Now complete the problem.



- After adding 3 square units to both sides, the shape on the left will be a square and you will have:

$$x^2 + 4x + 1 + 3 = 22 + 3$$

$$x^2 + 4x + 4 = 25$$

$$(x + 2)^2 = 25$$

$$x + 2 = \pm 5$$

$$x = -2 \pm 5$$

Solve each of these problems using the Completing the Square Template:

A. $x^2 + 6x + 1 = 28$

B. $x^2 - 16x + 30 = 15$

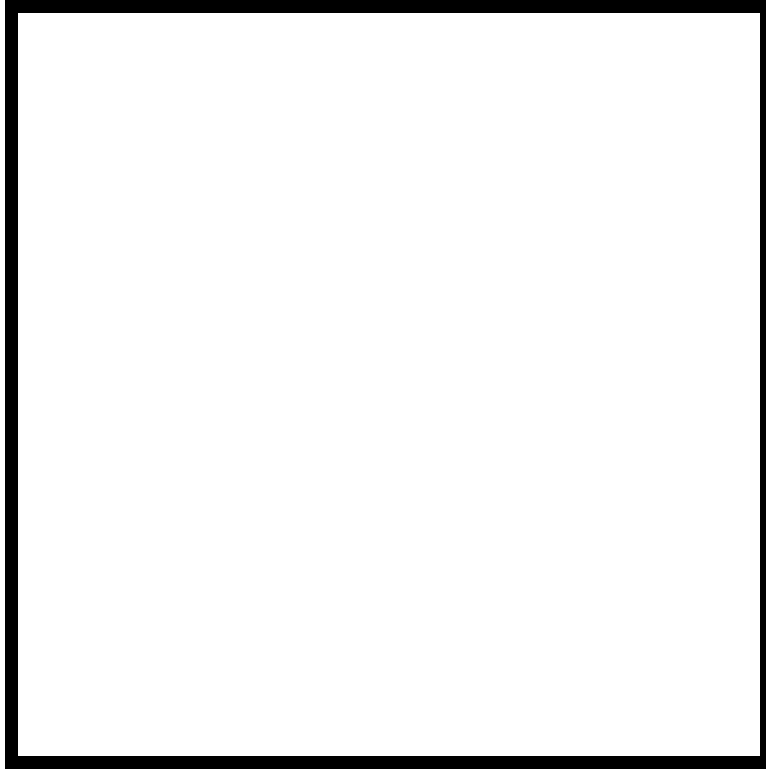
C. $x^2 + 12x + 6 = 70$

D. $x^2 - 18x + 60 = 43$

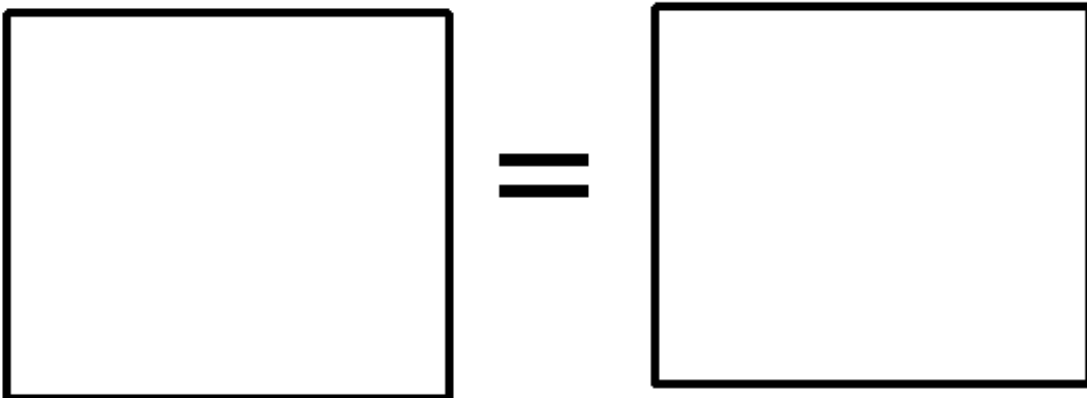
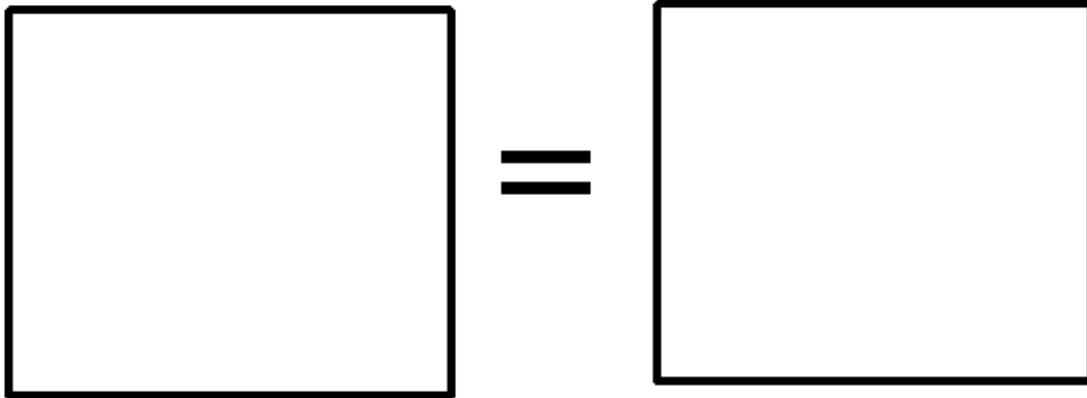
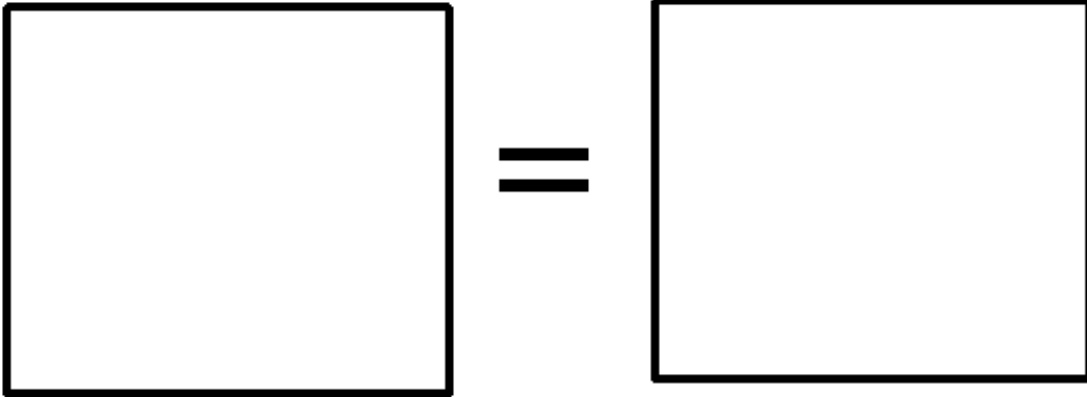
D. $x^2 + 4x + 1 = 0$

E. $x^2 + 3x + 2 = 8\frac{3}{4}$

Rectangle Diagrams



Completing the Square



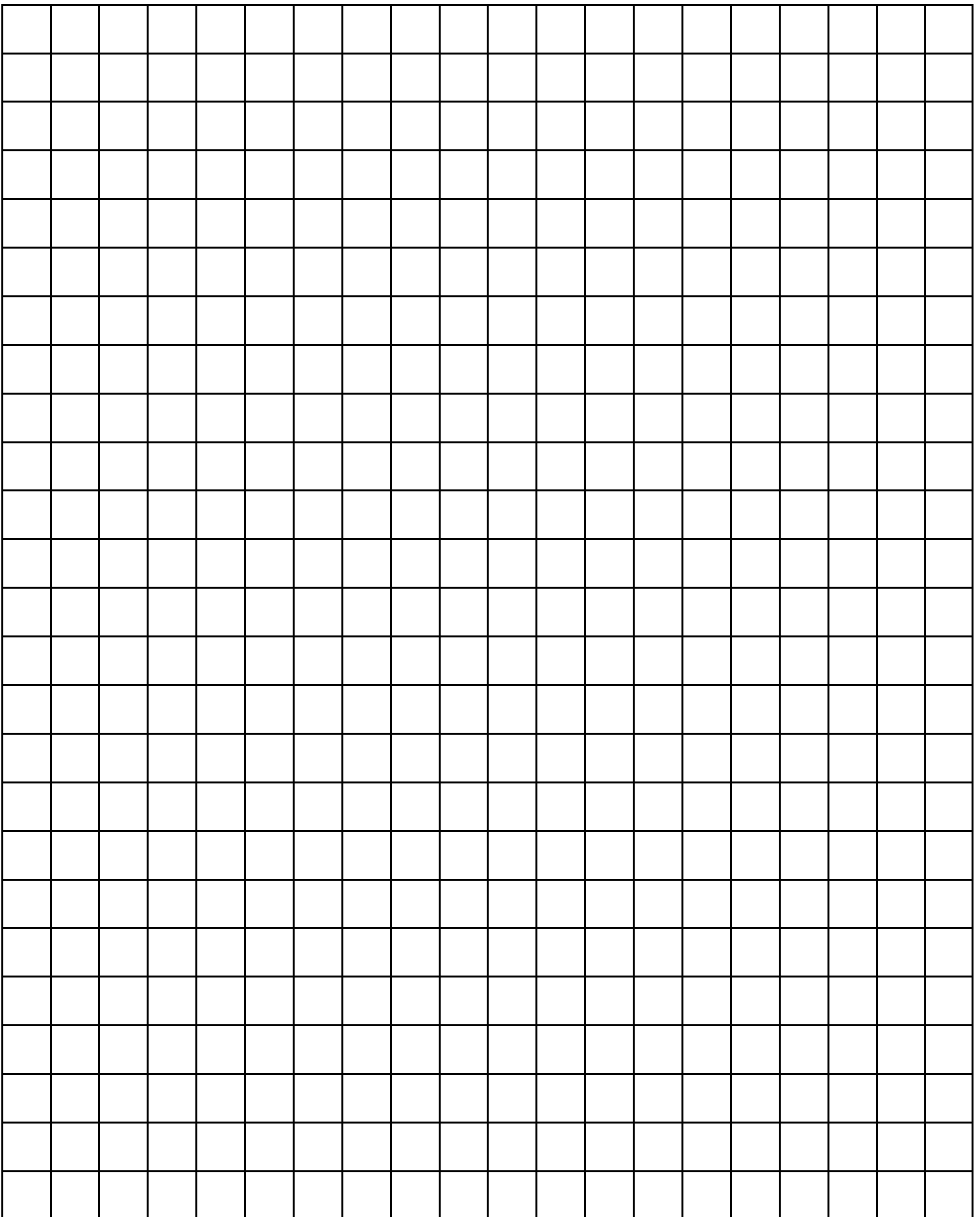
Building a Connection between Rate of Change and Straight Lines

- A sports car leaves High Point and heads south for Cape May.
- At the same time an overloaded van leaves Atlantic City and a pickup truck leaves Cape May and head north toward High Point.
- The sports car is traveling at 72 mph, the pickup truck is traveling 66 mph, and the overloaded van is traveling at 48 mph.
- When and where will they pass each other?
- Change the rates of change to miles per minute so we can study the problem in smaller increments.



- Make a table to record the distance from Cape May for each vehicle every minute.
- After completing the first couple of rows, change the intervals to 10 minute intervals until you have covered 4 hours.
- Write a recursive sequence would model each car's distance from Cape May?
- Define what x and y will represent.
- Make a paper graph for this traveling situation.
- What do you notice about the points that represent each vehicle?

- What is the starting position of each vehicle? Where is this on the table? Where is it on the graph?
- How does the vehicle's speed effect the graph?
- How can you tell which line represents the van?
- Where are the vehicles when the van meets the first vehicle heading north?
- How can you tell if the pickup truck or sports car is traveling faster from the graph?
- Which vehicle arrives at its destination first? How much later do the other vehicles reach their destination?
- Are you making any assumptions about each of the vehicles as you answer the questions?
- Write an equation that represents each vehicles distance from Cape May by referring to your recursive sequence.
- Enter these equations in your graphing calculator. How do these graphs compare to your paper graph?



Description	Expression

Just Undo It

Description	Sequence	Expression	Undo	Result
Pick a Number			