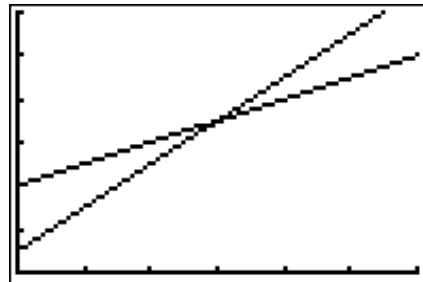


Building Understand for Solving a System of Equations through Applications

X	Y ₁	Y ₂
0	1.5	2.5
1	1.5	2.5
2	1.5	2.5
3	1.5	2.5
4	1.5	2.5
5	1.5	2.5
6	1.5	2.5
7	1.5	2.5
8	1.5	2.5
9	1.5	2.5
10	1.5	2.5

Y₁ = .5



Red Bank Regional High School

May 4, 2010

Jim Rahn
www.jamesrahn.com
james.rahn@verizon.net

- If y = distance the walker is from the starting line and x = time walked, write an equation that fits each walkers data.

$$y_{\text{walker1}} = \underline{\hspace{4cm}}$$

$$y_{\text{walker2}} = \underline{\hspace{4cm}}$$

We call these two equations a system of equations because they are describing two different situations that relate the same two variables.

- Enter the equations into the graphing calculator. Enter y_{walker1} in $y1$ and y_{walker2} in $y2$. Create the graph of these equations with the data. How does the data and the lines compare? How is the line different from the data?
- Turn off the scatter plots. Re-graph the lines.
- Trace along line $y1$. What do these points on the graph represent?
- Trace along line $y2$. What do these points on the graph represent?
- Find the approximate point where the lines intersect. Explain the real-world meaning of the intersection point.
- Check that the coordinates of the point of intersection satisfy both of your equations.
- When you create two graphs that represent the distance traveled by two walkers and the graphs intersect at one point, what does it mean?
- Create a set of table values for $y1$ and $y2$ using the table feature of the calculator. Set table start = 0 and $\Delta t_{bl}=1$. Does this table of value look familiar? Find the point of intersection using the table.
- How is the point of intersection found using a table of values?
- How is the point of intersection found using a scatter plot for the collected data?
- How is the point of intersection found using the graph of two equations?
- Explain in your own words what the meaning of the point of intersection is for set of table values, scatter plot, or graph of two equations.

Questions to think about:

- Suppose that walker 1 walks faster than 1 m/s. Select a new rate of change that is faster and change the equation for walker 1 to reflect this new rate of change. Describe how the graph is different. Locate the point of intersection for this new situation using the graph and table. What happens to the point of intersection?
- Suppose that two people walk at the same speed and direction from different starting marks. Change the equations to reflect the rate of change of both walkers so they are equal. Check both the graph and table for this new situation. What does the table tell you? What does the graph tell you? What happens to the point of intersection?
- Suppose that two people walk at the same speed in the same direction from the same starting mark. Change the equations to reflect that each walker is walking at the same rate of change and starting at the same location. Change the type of line for $y2$ to a bold line. What does this graph of this situation look like? How many points satisfy this system of equations?

Creating tables and graphs can often take a lot of time and produce solutions that are only approximate points of intersection. Let's explore one more way to solve a system of equations. Return to the original system of equations.

$$y_{\text{walker1}} = \underline{\hspace{2cm}}$$

$$y_{\text{walker2}} = \underline{\hspace{2cm}}$$

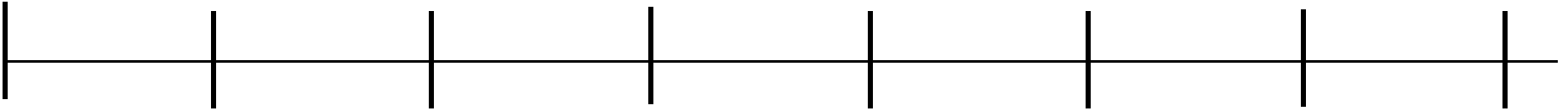
Notice that each equation describes the distance of each walker from the starting point.

- At the point of intersection, what will be true about the distance of each walker from the starting point?
- Since the two distances are equal then the expressions that represent those two distances must also be equal. Equate the two expressions for the distance of each walker from the starting point.

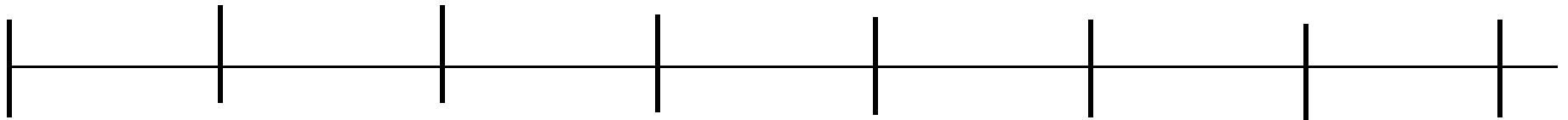
- By creating this new equation you have eliminated one of the variables in these equations. Complete the solving of this new equation for the value of x .

- What is the significance of this value of x ?
- How far has each walker walked for this value of x ?
- You have just found the solution to the system of equations by working with the equations analytically or algebraically rather than using tables and graphs.

First Walker



Second Walker



Path for Two Walkers

Application Problems

1. Gail leaves the trailhead at dawn to hike 12 mi toward the lake, where her friend Laura is camping. At the same time, Laura starts her hike toward the trailhead, Gail is walking uphill so she averages only 1.5 mi/h, while Laura averages 2.5 mi/hr walking downhill. When and where will they meet?



- Define variables for time and distance from the trailhead.
 - Write a system of two equations to model this situation.
 - Solve this system by creating a table and finding the values for the variables that make both equations true. Then locate this solution on a graph.
 - Check your solution and explain its real-world meaning.
2. Use the calculator table feature to find the solution to these systems of equations. Confirm the solution by using a graph.

A.
$$\begin{cases} y = -9 + 2x \\ y = 3 + 0.5x \end{cases}$$

B.
$$\begin{cases} y = 4x - 5.5 \\ y = -3x + 5 \end{cases}$$

C.
$$\begin{cases} y = 100 + 10x \\ y = 50 + 20x \end{cases}$$

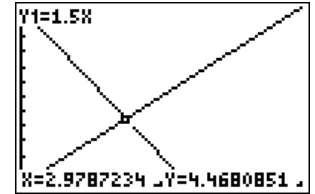
3. The total tuition for students at South College and North College consists of student fees plus costs per credit. Some classes have different credit values. The table shows the total tuition for programs with different numbers of credits at each college.

Total Tuition		
Credits	South College	North College
1	55	47
3	115	111
6	205	207
9	295	303
10	325	335
12	385	399

- A. Write a system of equations that represents the relationship between credit hours and total tuition for each college.
- B. Find the solution to this system of equations and check it.
- C. Which method did you use to solve this system? Why?
- D. What is the real-world meaning of the solution?
- E. When is it cheaper to attend South College? North College?

Building Understanding for Solving a System of Equations by Substitution

A solution to a system of equations can be found graphically and by using tables. Often these methods can be very tedious and may sometime lead to only an approximate solution.



To find the exact solutions, you'll need to work algebraically with the equations. Let's look at the substitution method.

On a rural highway a police officer sees a motorist run a red light at 50 mph and begins pursuit. At the instant the police officer passes through the intersection at 60 mph, the motorist is 0.2 mi down the road. When and where will the office catch up to the motorist?

X	Y1	Y2
1.5	2.25	0.25
2	3	0.5
2.5	3.75	0.75
3	4.5	1
3.5	5.25	1.25
4	6	1.5
4.5	6.75	1.75

X=3

- If d = the distance from the intersection and t = time traveled, write an equation in two variables to model this situation.
 - When the police car catches up to the motorist what will be true about the distance of the motorist from the intersection and the distance of the police car from the intersection?
 - Replace the d value in one equation with the d value from the other equation. Solve the equation for time.
-
- What is the meaning of this time?
 - How far is each vehicle from the intersection at this time?
 - Check your answer by looking at a graph and a set of table values for both equations.

Can we always use the Substitution Method?

So far you have seen equations written in intercept form.

These equations make it easy to use the substitution method since they are already both solved for y .

$$y = 900 - 6x$$

$$y = 1000 - 10.3x$$

Sometimes it is necessary to place equations in intercept form before using substitution.

A pharmacist has 5% saline (salt) solution and 20% saline solution. How much of each solution should be combined to create a bottle of 90 ml of 10% solution.

Let x represent the amount of 5% solution you have and y represent the amount of 20% solution you have. First write an equation for the total amount of liquid in each container. Then write a second equation that describes the amount of salt in each solution.

Solve the one equation for x or y and then substitute it into the other equation.

What is the meaning of this solution.

One More Problem:

This system of equations models the profits of two home-based Internet companies.

$$\begin{cases} P = -12000 + 2.5N \\ P = -5000 + 1.6N \end{cases}$$

The variable P represents profit in dollars, and N represents hits to the company's website.

- Use the substitution method to find an exact solution.
- Is an approximate or exact solution more meaningful in this model?

Developing an Understanding for the Elimination Method for Solving a System of Equations

We have solved a system of equations using graphs, tables, and substitution. The substitution method was introduced to provide you an analytical (or algebraic) way to work with a system of equations to find a solution to the system. In this activity you will explore using another analytic (or algebraic) way to solve a system of equations.

First let's think about some things we have done with equations.

When you add equal quantities to each side of an equation, the resulting equation is equivalent and has the same solution as the original equation.

$$\begin{array}{r} y - 4 = 12 \\ + \quad 4 = 4 \\ \hline y = 16 \end{array}$$

Recall that $y=16$ is a solution to the first equation and the last equation.

$$\begin{array}{r} 3y - 2x = 1 \\ + \quad 2x = 2x \\ \hline 3y = 1 + 2x \end{array}$$

Recall that $(1,1)$, $(4,3)$, and $(10,7)$ are solutions to first equation and the last equation.

so

$$\begin{array}{r} a = b \\ + \quad c = d \\ \hline a + c = b + d \end{array}$$

Frank is thinking of two numbers, but he won't say what they are. Instead he plays a game. He tells you that the sum of the two numbers is 163 and that their difference is 33. Find the two numbers.

If the two numbers are x and y , you can write two equations based on Frank's statements.

- Write an equation based on the first description:
- Write an equation based on the second description:

From your equations, which is the larger number? How can you tell?

Use the property described above to add the equations together. You should see that this results in an equation with just one variable. Solve for the value of that variable.

Use this value to find the value of the other number. What is the solution to Frank's problem?

How long is a paper clip? How wide is a penny?

- To write one equation, lay one paper clip along the long side of the paper and then add enough pennies to complete the 11 inch length. Write a statement that describe the 11" length of the paper using paper clips and pennies. Replace the word paper clips with C and pennies with P. You now have your first equation.
- To write the other equation for the system, lay two paper clips along the shorter side of the paper and add pennies to complete the 8.5 inch length. Write a statement that describe the 8.5" length of the paper using paper clips and pennies. Replace the word paper clips with C and pennies with P. You now have your second equation.
- Study your two equations. Notice that each variable has different coefficients on each equation.
- What could you do to both sides of one of the equations so that the variable C is eliminated when the two equations are added?
- Perform this operation to both sides of the one equation and then add the equations together.
- Once you have eliminated the C, solve the resulting equation for P.
- Substitute your P value back into either equation to solve for C.
- Find another way to solve this system by elimination.
- Explain the real-world meaning of the solution.
- Describe other experiments in measuring that you can solve using a system of equations.



From Science:

A molecule of hexane, C_6H_{14} , has six carbon atoms and fourteen hydrogen atoms.

Its molecular weight in grams per mole, the sum of the atomic weights of carbon and hydrogen is 86.178.

The molecular weight of octane, C_8H_{18} , is 114.232 grams per mole.

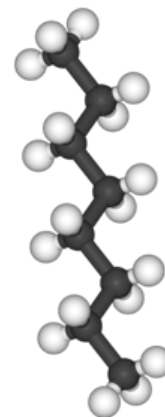
Octane has eight carbon atoms and eighteen hydrogen atoms per molecule.

Find the atomic weights of carbon and hydrogen.

hexane



octane



Try These:

1. Suppose you had the system of equations
$$\begin{cases} 3x - 2y = 11 \\ -x + 6y = 7 \end{cases}$$

Show how you could solve this system by eliminating the x variable.

Show how you could solve this system by eliminating the y variable.

Show how you could use substitution to solve this system of equations.